Solvability of $N \times N - 1$ tile sliding problems

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NOTE: These conditions are valid for boards of width even!

- Let B be an initial board of dimension $N \times N$, with N even;
- Let the condition to reach from B to the final standard configuration be:

$$(Inv\%2 == 0) == (blankRow\%2 == 1)$$

where Inv is the number of inversions of B, and blankRow is the row of the blank space in B

Final standard configuration (STD):

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

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- Let Inv_i be the number of inversions of any initial configuration (C_i);
- Let Inv_f be the number of inversions of any final configuration (C_f);
- Let $blankRow_i$ be the row of the blank space in the initial configuration counting from bottom (first row starts from 1);
- Let $blankRow_f$ be the row of the blank space in the final configuration counting from bottom (first row starts from 1);

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- If there is a solution from C_i to the final standard config (STD) AND there is a solution from C_f to the final standard config (STD), then there is a solution from C_i to C_f and vice-versa. Conversely,
- If there is NO solution from C_i to the final standard config AND there is NO solution from C_f to the final standard config, then there is a solution from C_i to C_f and vice-versa.

Solvability of $N \times N - 1$ tile sliding problems: conditions

- Let $Cond_i$ be the result of condition:

$$(Inv_i\%2 == 0) == (blankRow_i\%2 == 1)$$

- Let $Cond_f$ be the result of condition:

$$(Inv_f\%2 == 0) == (blankRow_f\%2 == 1)$$

- There will be a solution from C_i to C_f and vice-versa, iff:

$$Cond_i == Cond_f$$