Constraint Logic Programming (CLP): a short tutorial

- What is CLP?
  - the use of a rich and powerful language to model optimization problems
  - modelling based on variables, domains and constraints

CLP

- Motivation:
  1. to offer a declarative way of modelling constraint satisfaction problems (CSP)
  2. to solve 2 limitations in Prolog:
     - Each term in Prolog needs to be explicitly evaluated and is not interpreted (evaluated):
       - `X + 1` is a term that is not evaluated in Prolog. It is only a syntactic representation.
       - a variable can assume one single value.
     - Uniform computation, but not that powerful: depth-first search, “generate-and-test”.
  3. to integrate Artificial Intelligence (AI) and Operations Research (OR)

CLP

- CLP can use Artificial Intelligence (AI) techniques to improve the search: propagation, data-driven computation, “forward checking” and “lookahead”.
- Applications: planning, scheduling, resource allocation, computer graphics, digital circuit design, fault diagnosis etc.
- Clients: Michelin and Dassault, French railway SNCF, Swissair, SAS and Cathay Pacific, HK International terminals, Eriksson, British Telecom etc.

CLP

- CLP joins 2 research areas:
  - Introduction of richer and powerful data structures to Logic Programming (e.g.: replace unification by efficient manipulation of constraints and domains).
  - Consistency techniques:
    “generate-and-test” x “constrain-and-generate”

CLP

- Systems:
  - CHIP, Dincbas and Van Hentenryck (ECRC)
  - OPL, Van Hentenryck
  - Xpress
  - CLP(R), Jaffar, Michaylov, Stuckey and Yap (Monash)
  - Prolog III, Colmerauer
  - ECLiPSe, Wallace (IC Parc)
  - Oz, Smolka (DFKI)
  - clp(FD), Diaz and Codognet (INRIA, France)
- The CLP(X) scheme:
  - “constraint solver”: replaces simple unification.
  - 2 popular domains: arithmetic and boolean.

Arithmetic domain: linear constraints

- Prolog can not solve `x-3 = y+5`.
- CLP(R): first language to introduce arithmetic constraints.
- Linear arithmetic expressions composed by: numbers, variables and operators (negation, addition, subtraction, multiplication and division).
- Example: `t1 R t2`, with R = { $>$, $\geq$, =, $\leq$, $<$, =}
- Popular decision procedures:
  - Gauss elimination.
  - Simplex (most popular):
    - average good behavior
    - popular
    - incremental

Arithmetic Domain: Linear Constraints

- Example:
- A meal consists of starter, main course and dessert
- database with various kinds of food and their caloric values
- Problem: produce a menu with light meals (caloric value < 10Kcal)

Arithmetic Domain: Linear Constraints

```
light_meal(A,M,D) :-
    I > 0, J > 0, K > 0,
    I + J + K =< 10,
    starter(A,I),
    main_course(M,J),
    dessert(D,K).

    meat(steak,5).
    meat(pork,7).
    fish(sole,2).
    fish(tuna,4).
    dessert(fruit,2).
    dessert(icecream,6).
```

```
main_course(M,I) :-
   meat(M,I).
main_course(M,I) :-
   fish(M,I).
starter(salad,1).
starter(soup,6).
```

Arithmetic Domain: Linear Constraints

- Intermediate results = compute states.
- 2 components: constraint store and continuation of objectives.
- Query: ◇ `light_meal(A,M,D).`
- `I + J + K =< 10, I > 0, J > 0, K > 0` ◇ `starter(A,I), main_course(M,J), dessert(D,K).`
- `A = salad, I = 1, 1 + J + K =< 10, 1>0, J>0, K>0` ◇ `main_course(M,J), dessert(D,K).`
- `A = salad, I = 1, M=M1, J=I1, 1 + J + K =< 10, 1>0, J>0, K>0` ◇ `meat(M1,I1), dessert(D,K).`
- `A = salad, I = 1, M=steak, J=5, M1=steak, I1 = 5, 1 + 5 + K =< 10, 1>0, 5>0, K>0` ◇ `dessert(D,K).`
- `A = salad, I = 1, M=steak, J=5, M1=steak, I1 = 5, D=fruit, K = 2, 1 + 5 + 2 =< 10, 1>0, 5>0, 2>0` ◇.

Arithmetic Domain: Linear Constraints

Inconsistent derivation :

- `A = pasta, I = 6, M=steak, J=5, M1=steak, I1 = 5, 6 + 5 + K =< 10, 5>0, 6>0, K > 0` $\diamond$ `dessert(D,K).`

Arithmetic Domain: non-linear Constraints

- Example: multiply 2 complex numbers: `(R1 + I*I1) * (R2 + I*I2).`

```
zmul(R1,I1,R2,I2,R3,I3) :-
    R3 = R1 * R2 + I1 * I2,
    I3 = R1 * I2 + R2 * I1.
```

- Query: ◇ `zmul(1,2,3,4,R3,I3)`
- Equations become linear.
- Solution: `R3 = -5`, `I3 = 10` (definite solution)
- Query: ◇ `zmul(1,2,R2,I2,R3,I3)`
- Solution:

```
    I2 = 0.2*I3 - 0.4*R3
    R2 = 0.4*I3 + 0.2*R3
    yes (undefined solution)
```

Arithmetic Domain: non-linear Constraints

- Same example: multiply 2 complex numbers:

  `(R1 + I*I1) * (R2 + I*I2)`

- Query: ◇ `zmul(R1,2,R2,4,-5,10), R2 < 3.`
- CLP(R): (do not solve non-linear equations)

```
R1 = -0.5*R2 + 2.5
3 = R1*R2
R2 < 3
Maybe
```

- applications of non-linear equations: computational geometry and financial applications (various algorithms used).

Boolean domain

- Main Application: digital circuit design (hardware verification) and theorem proof.
- Boolean terms: truth values (F or T), variables, logical operators, one single constraint: equality.
- various unification algorithms for boolean constraints.
- Solution: provides a decision procedure for propositional calculus (NP-complete).

Boolean domain

- Example: full adder (operators # (xor), * (and), + (or)

```
add(I1,I2,I3,O1,O2) :-
    X1 = I1 # I2,
    A1 = I1 * I2,
    O1 = X1 # I3,
    A2 = I3 * X1,
    O2 = A1 + A2.
```

- Query: $\diamond$ `add(a,b,c,O1,O2)`
- Solution: `O1 = a+b+c, O2 = (a * b) + (a * c) # (b * c)`

Consistency techniques

- Eliminate inconsistent ’labellings’ by constraint propagation information about the values of variables.
- examples: arc-consistency, forward checking, generalized propagation.
- Example: task scheduling.

```
    T2                    before(T1,T2).
   /                      before(T1,T3).
  /                       before(T2,T6).
T1                        before(T3,T5).
  \          T6           before(T4,T5).
   \        /             before(T5,T6).
    T3 --- T5             notequal(T2,T3).
          /
       T4
```

Consistency Techniques

- Example: $T1 \in \{1, 2, 3, 4, 5\}$, $T2 \in \{1, 2, 3, 4, 5\}$.
- before(T1,T2) –> apply consistency:
  - $T1 \in \{1, 2, 3, 4\}$
  - $T2 \in \{2, 3, 4, 5\}$
- Value 5 removed from T1, because there is no other value in T2 that can satisfy T1 $<$ T2.
- Value 1 removed from T2, for the same reason.

Arc Consistency

- $T1 \in \{1,2\}$, $T2 \in \{2,3,4\}$, $T3 \in \{2,3\}$, $T4 \in \{1,2,3\}$, $T5 \in \{3,4\}$, $T6 \in \{4,5\}$.
- Value 2 from T1 is chosen (T1 is “labelled” with value 2).
- Using propagation: $T2 \in \{3,4\}$ and $T3 \in \{3\}$
- $T2 \in \{4\}$ from notequal(T2,T3).
- Finally: $T1 \in \{2\}$,$T2 \in \{4\}$, $T3 \in \{3\}$, $T4 \in \{1,2,3\}$, $T5 \in \{4\}$, $T6 \in \{5\}$

Infinite Domains

- Utilization of maximum and minimum values to solve constraints in the linear domain.
- E.g.: x,y,z with domains [1..10] with constraint 2x + 3y + 2 $<$ z
- Removing inconsistent values:
  - 10 is the largest value for z, then: 2x + 3y $<$ 8
  - 1 is the smallest possible value for y, therefore: 2x $<$ 5
  - x can only assume values {1,2}
  - 3y $<$ 6, y $<$ 2, $y \in \{1\}$
  - z $>$ 7, $z \in \{8, 9, 10\}$

Basic programming techniques

- Define problem variables and their domains.
- Establish the constraints between variables.
- Search for solution.

```
?- [X,Y,Z]::1..10,
   2 * X + 3 * Y + 2 #< Z,
   indomain(X), indomain(Y), indomain(Z).
```

Algorithms and other examples

Forward Checking:

```
n = 8
(1) V1 = 1   ==> V2 = {3,4,5,6,7,8}
                 V3 = {2,4,5,6,7,8}
                 V4 = {2,3,5,6,7,8}
                 V5 = {2,3,4,6,7,8}
                 V6 = {2,3,4,5,7,8}
                 V7 = {2,3,4,5,6,8}
                 V8 = {2,3,4,5,6,7}

(2) V2 = 3   ==> V3 = {5,6,7,8}
                 V4 = {2,6,7,8}
                 V5 = {2,4,7,8}
                 V6 = {2,4,5,8}
                 V7 = {2,4,5,6}
                 V8 = {2,4,5,6,7}
```

Forward Checking: Example

```
                   +---+---+---+---+---+---+---+---+
                   | X |   |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
                   |   |   |   |   |   |   |   |   |
n-queens           +---+---+---+---+---+---+---+---+
n=8                |   | X |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
                   |   |   |   |   |   |   |   |   |
after V1=1         +---+---+---+---+---+---+---+---+
      V2=3         |   |   |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
                   |   |   |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
                   |   |   |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
                   |   |   |   |   |   |   |   |   |
                   +---+---+---+---+---+---+---+---+
```

Algorithms

- infinite domains: linear programming, simplex, revised simplex, convex hull, Gauss elimination.
- finite domains: forward checking, lookahead, arc-consistency.
- For finite domains, 2 problems:
  - choice of variable:
    - *most-constrained*: smallest domain
    - *most constraining*: mostly constrains domains of other variables
  - choice of a value for a variable:
    - *first-fail* principle.
    - *least constraining*: value that constrains less sets of values of other variables

Map Colouring

```
Map Coloring: 3 colours      +------------+
blue                         |B           |
green              +---------+            |
red                |A        |            |
                   |         |            |
          +--------+--+      |            |
          |C          +------+------+     |
          |           |             |     |
          +-+----+----+             |     |
            |    |                  +---+-+
            |    |E                 |   |
            |    +------------------+   +---------+
            |                           |         |
            |                           |         |
            |                           |         |
            |F                          |D        |
            +---------------------------+---------+
```

Heuristics

- most-constrained variable: allows to solve the n-queens problem with n equals 100
- pure forward checking: only solves for n = 30
- least-constraining: solves for n = 1000

References

- A comparative study of eight constraint programming languages, by Antonio Fernandez and Pat Hill, 2000
- Constraint Logic Programming - An Informal Introduction, 1993, by Thom Frühwirth, A. Herold, T. Le Provost, P. Lim, E. Monfroy, M. Wallace, V. Küchenhoff, Thierry Le Provost Pierre Lim
- Constraint logic programming: A survey J. Jaffar, M. J. Maher, 1994