VC 10/11 – T11
Optical Flow

Mestrado em Ciência de Computadores

Mestrado Integrado em Engenharia de Redes e Sistemas Informáticos

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Outline

- Optical Flow Constraint Equation
- Aperture problem.
- The Lucas & Kanade Algorithm

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Topic: Optical Flow Constraint Equation

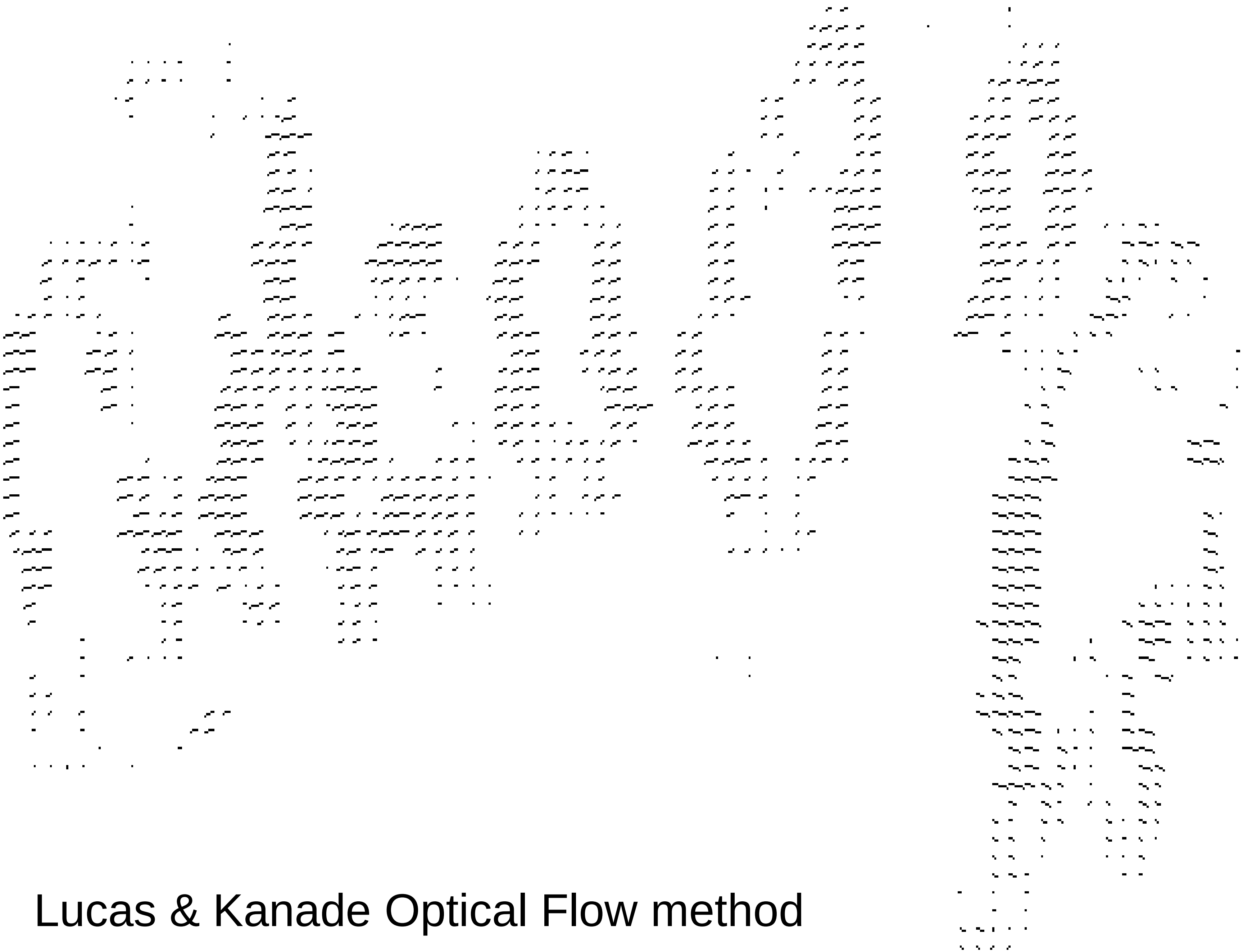
- Optical Flow Constraint Equation
- Aperture problem.
- The Lucas & Kanade Algorithm

Optical Flow and Motion

- We are interested in finding the movement of scene objects from time-varying images (videos).

- Lots of uses
  - Track object behavior
  - Correct for camera jitter (stabilization)
  - Align images (mosaics)
  - 3D shape reconstruction
  - Special effects

Where can i find motion?

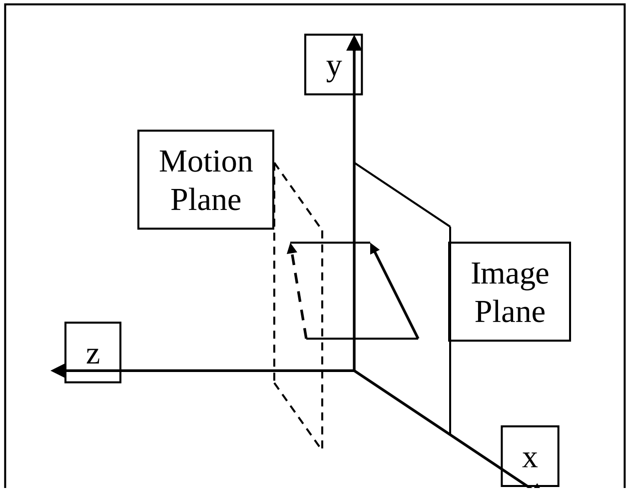


Lucas & Kanade Optical Flow method

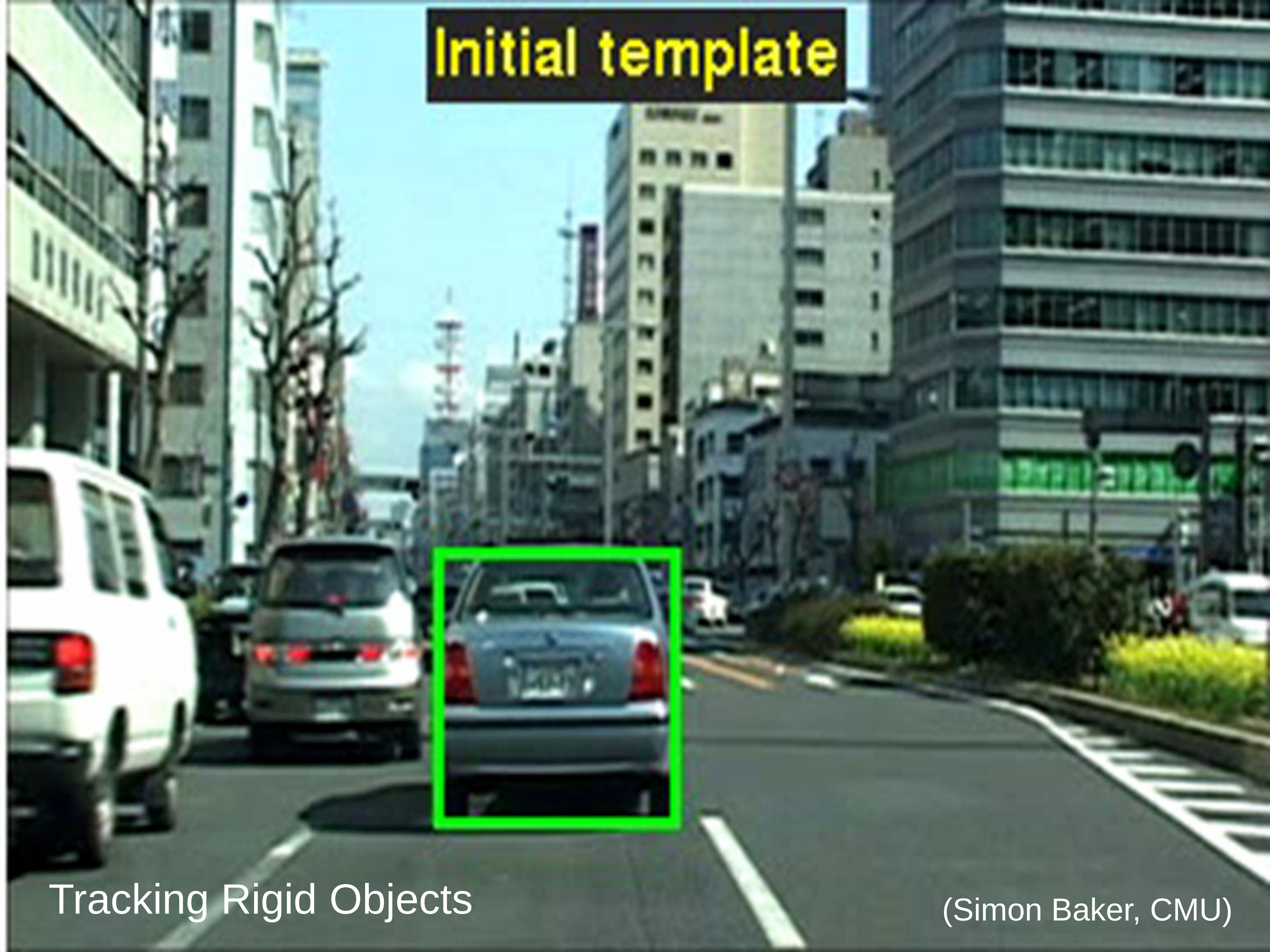
Optical Flow – What is that?

Optical flow is "the distribution of apparent velocities of movement of brightness patterns in an image" – Horn and Schunck 1980

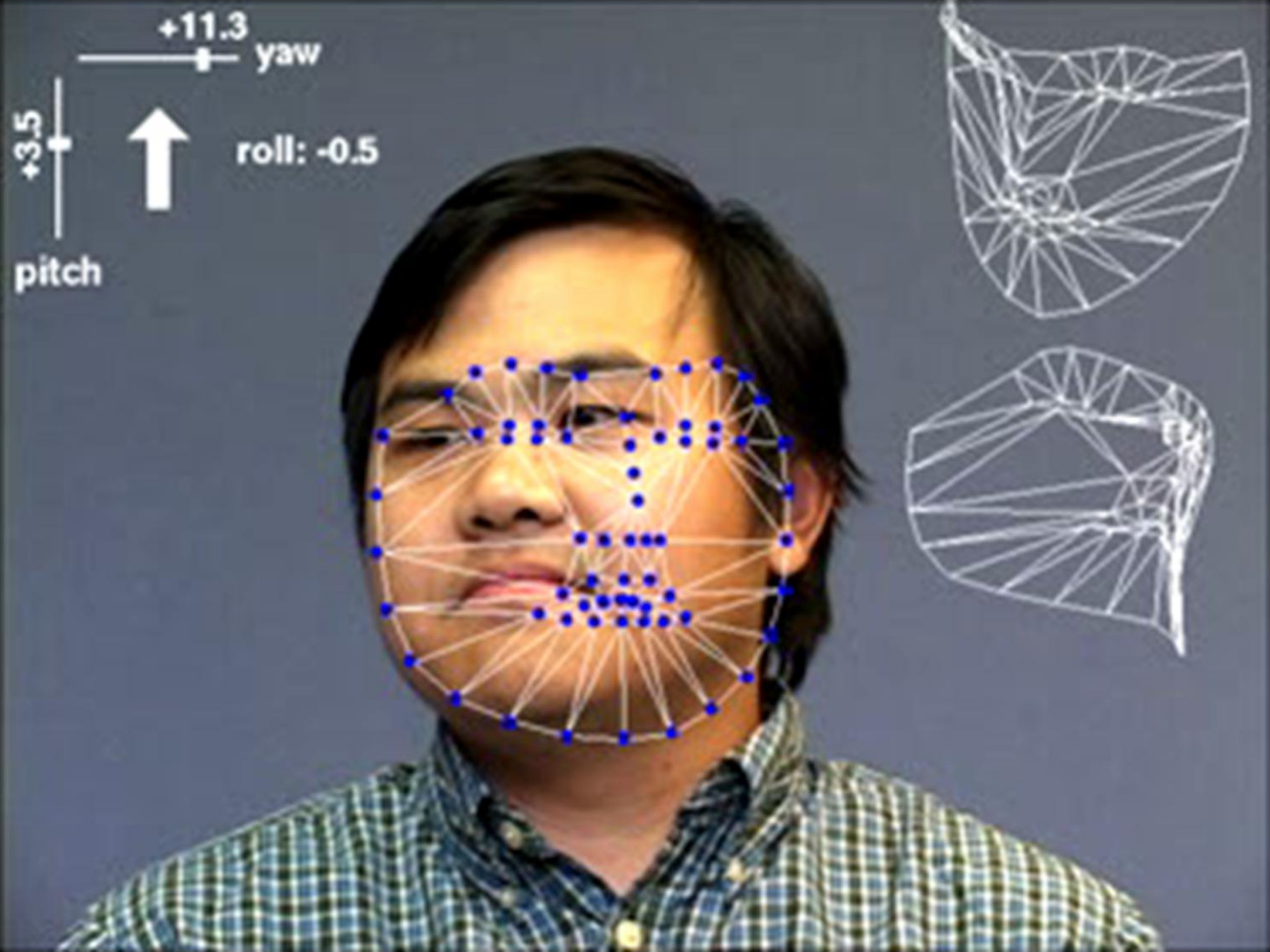
The optical flow field approximates the true motion field which is a "purely geometrical concept..., it is the [2D] projection into the image [plane] of [the sequence's] 3D motion vectors" – Horn and Schunk 1993



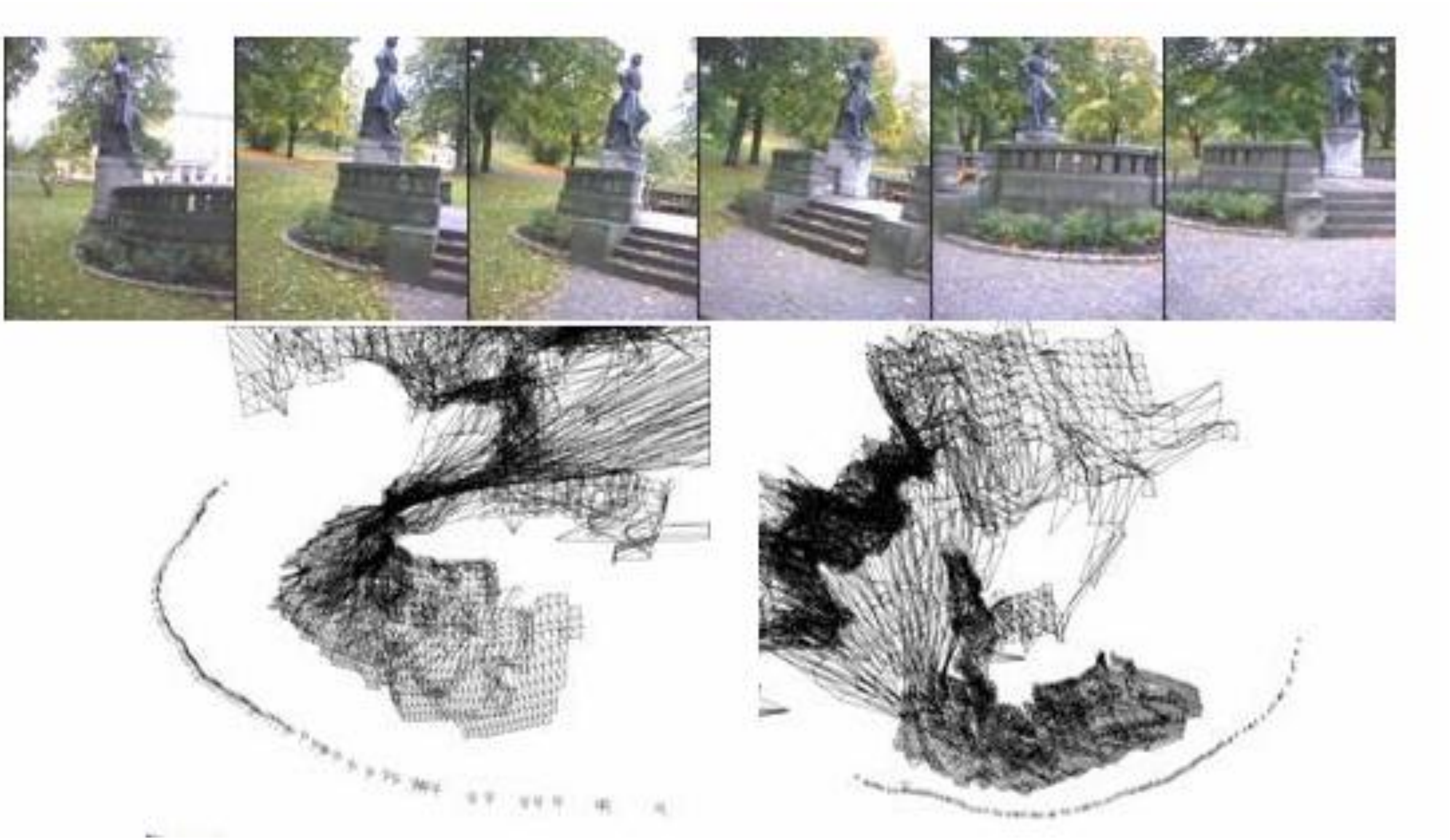
What can i use it for?


Initial template
Tracking Rigid Objects
(Simon Baker, CMU)

Tracking – Non-rigid Objects (Comaniciu et al, Siemens)


+11.3
yaw
+3.5
roll: -0.5
pitch

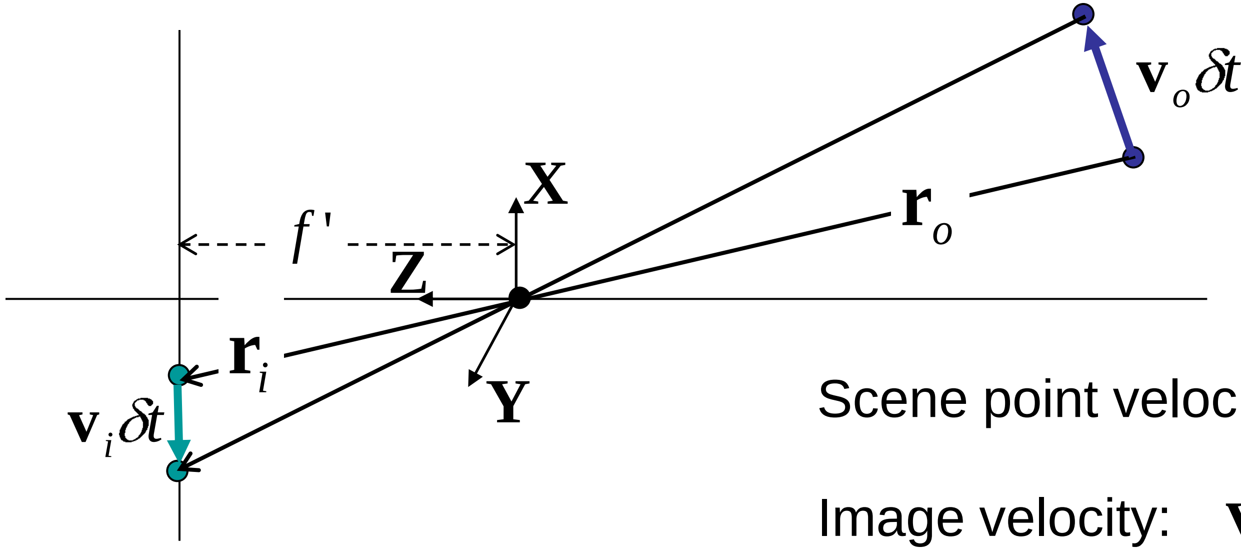
3D Structure from Motion



(David Nister, Kentucky)

Motion Field

- Image velocity of a point moving in the scene



Scene point velocity: $\mathbf{v}_o = \dfrac{d\mathbf{r}_o}{dt}$

Image velocity: $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt}$

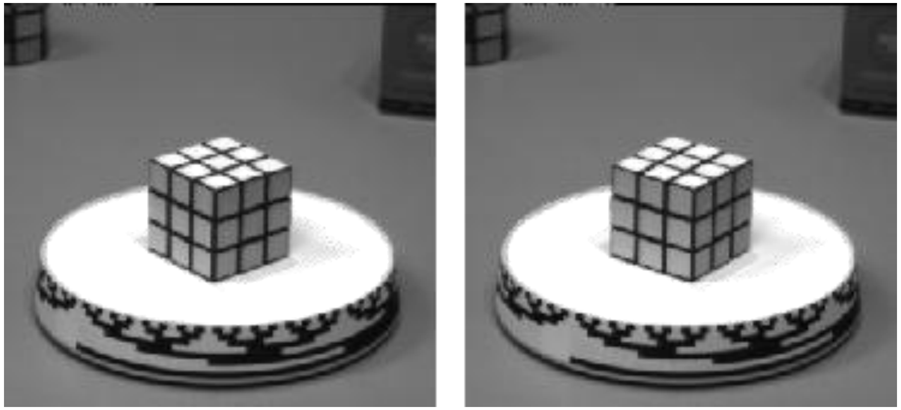
Perspective projection: $\dfrac{1}{f'}\mathbf{r}_i = \dfrac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{Z}}$

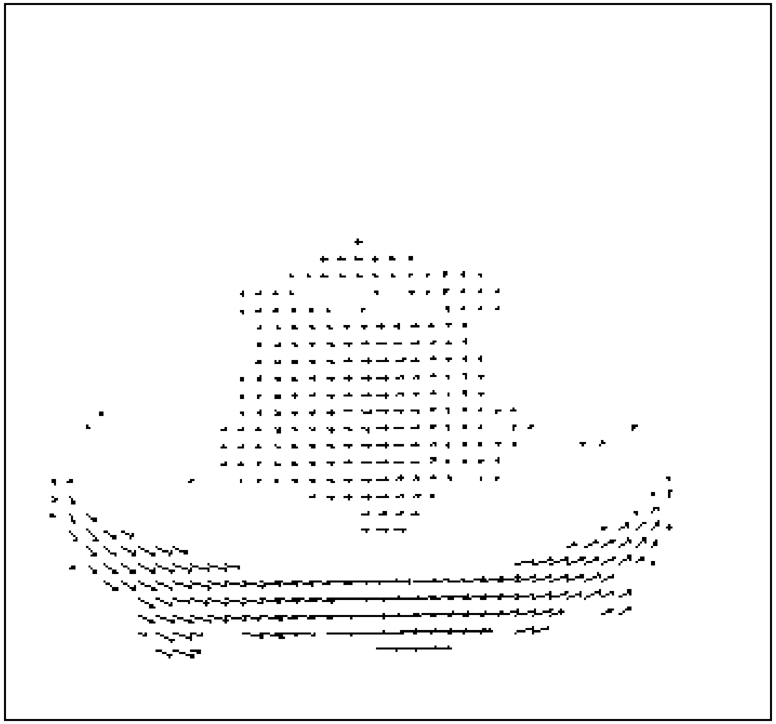
Motion field

$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f' \frac{(\mathbf{r}_o \cdot \mathbf{Z})\mathbf{v}_o - (\mathbf{v}_o \cdot \mathbf{Z})\mathbf{r}_o}{(\mathbf{r}_o \cdot \mathbf{Z})^2} = f' \frac{(\mathbf{r}_o \times \mathbf{v}_o) \times \mathbf{Z}}{(\mathbf{r}_o \cdot \mathbf{Z})^2}$$

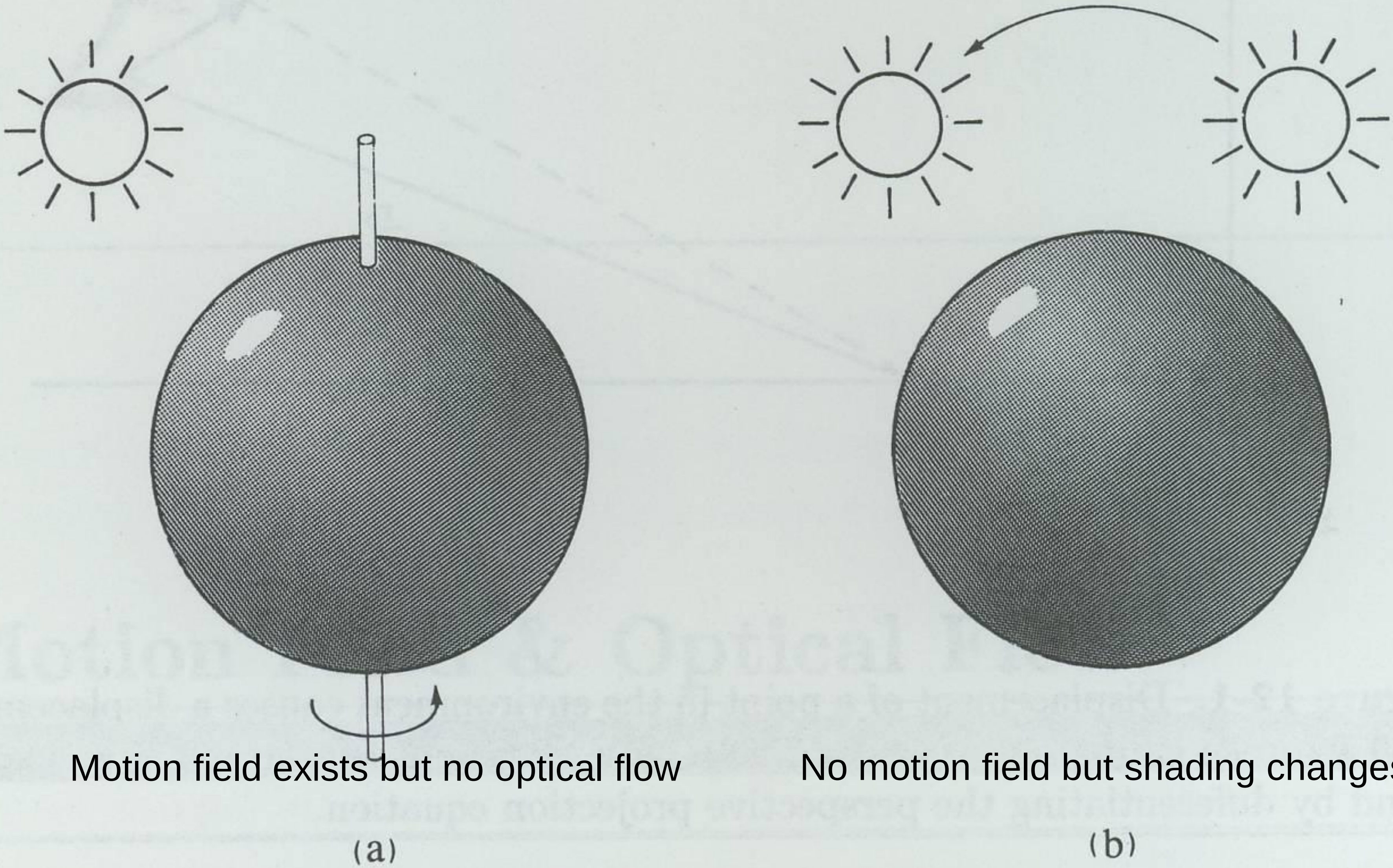
Optical Flow

- Motion of brightness pattern in the image
- **Ideally** Optical flow = Motion field





Optical Flow ≠ Motion Field

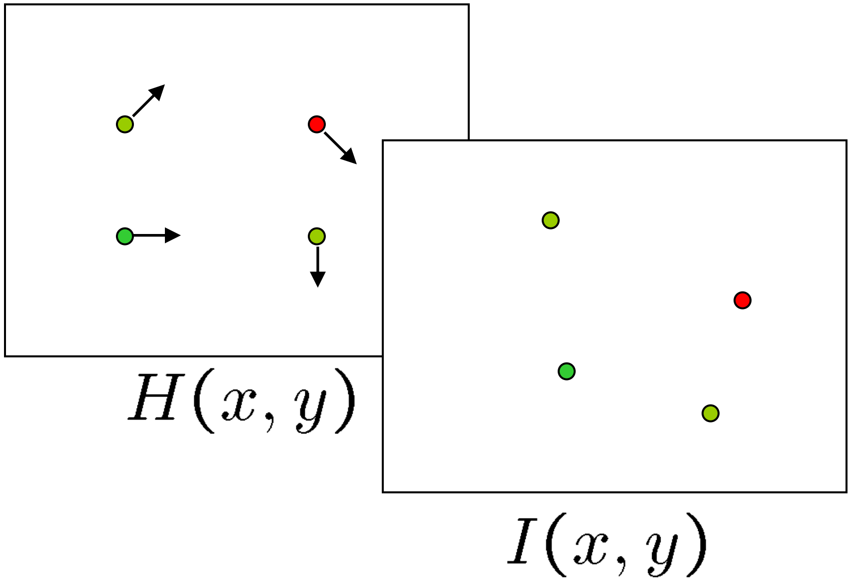


Motion field exists but no optical flow

No motion field but shading changes

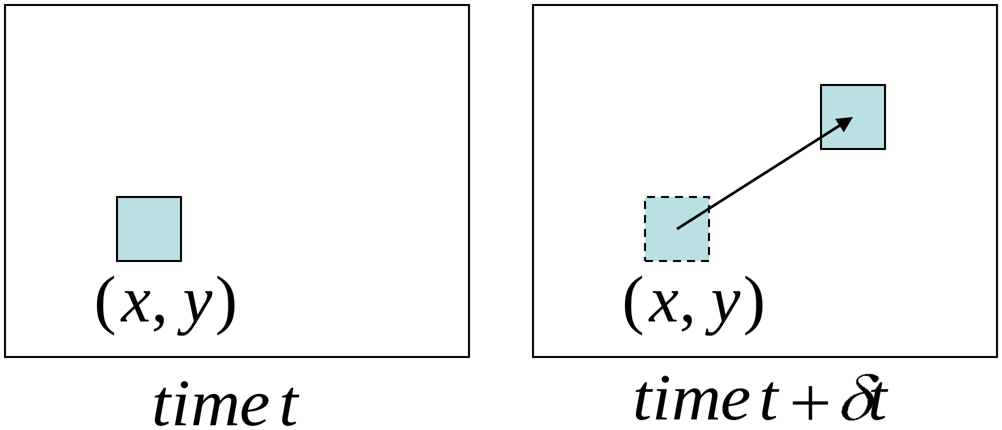
Problem Definition: Optical Flow

- How to estimate pixel motion from image H to image I?
  - Find pixel correspondences
    - Given a pixel in H, look for nearby pixels of the same color in I



- Key assumptions
  - **color constancy:** a point in H looks "the same" in image I
    - For grayscale images, this is **brightness constancy**
  - **small motion**: points do not move very far

Optical Flow Constraint Equation



$(x + u\,\delta t, y + v\,\delta t)$

Optical Flow: Velocities (u, v)

Displacement:

$$(\delta x, \delta y) = (u\,\delta t, v\,\delta t)$$

– Assume brightness of patch remains same in both images:

$$E(x + u\,\delta t, y + v\,\delta t, t + \delta t) = E(x, y, t)$$

– Assume small motion: (Taylor expansion of LHS up to first order)

$$E(x, y, t) + \delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} = E(x, y, t)$$

Optical Flow Constraint Equation

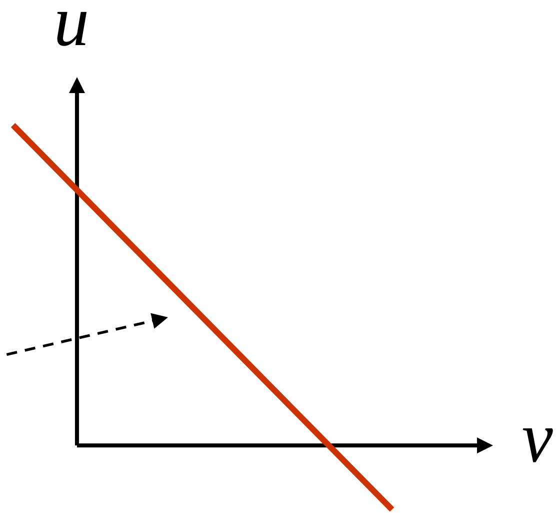
$$\delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} = 0$$

Divide by δt and take the limit $\delta t \rightarrow 0$

$$\frac{dx}{dt}\frac{\partial E}{\partial x} + \frac{dy}{dt}\frac{\partial E}{\partial y} + \frac{\partial E}{\partial t} = 0$$

Constraint Equation

$$E_x\, u + E_y\, v + E_t = 0$$



NOTE: (u,v) must lie on a straight line

We can compute E_x, E_y, E_t using gradient operators!

But, (u,v) cannot be found uniquely with this constraint!

Optical Flow Constraint

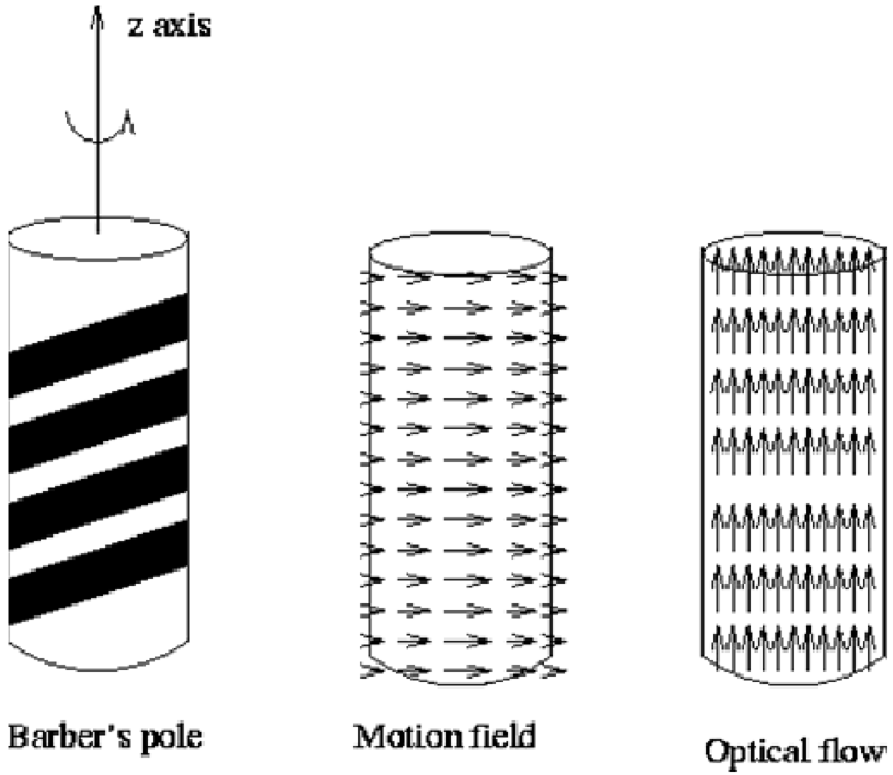
- Intuitively, what does this constraint mean?
  - The component of the flow in the gradient direction is determined.
  - The component of the flow parallel to an edge is unknown.

Topic: Aperture problem

- Optical Flow Constraint Equation
- Aperture problem.
- The Lucas & Kanade Algorithm

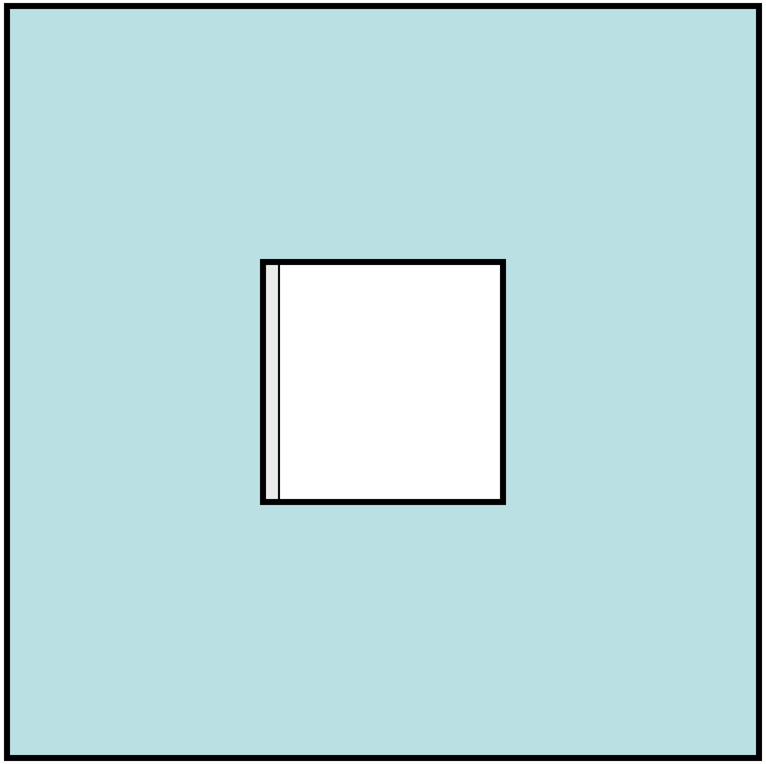
Optical Flow Constraint

Barber pole illusion



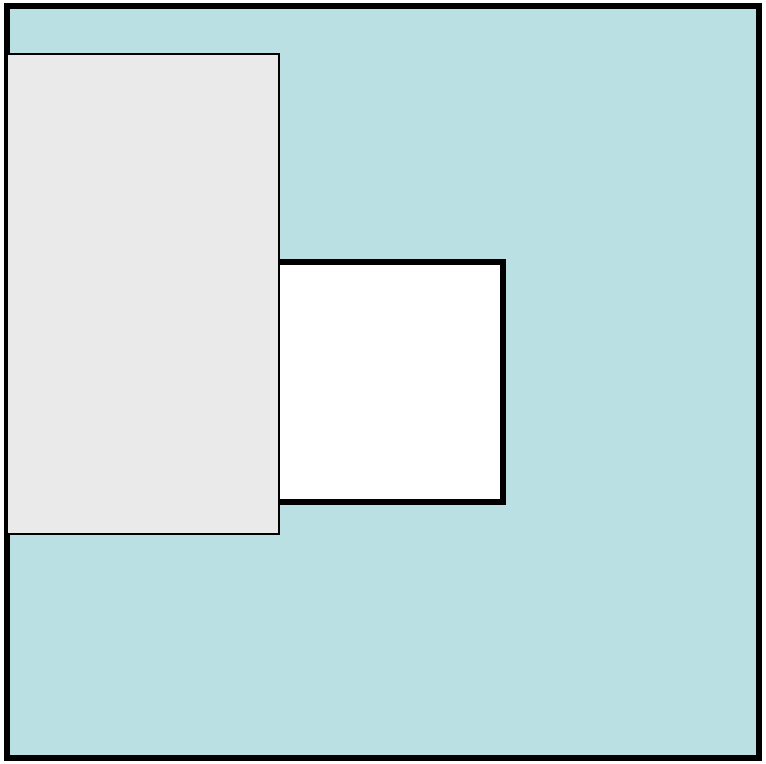
How does this show up visually? Known as the "Aperture Problem"

[Gary Bradski, Intel Research and Stanford SAIL]



Aperture Problem Exposed

[Gary Bradski, Intel Research and Stanford SAIL]



Motion along just
an edge is ambiguous

Computing Optical Flow

- Formulate Error in Optical Flow Constraint:

$$e_c = \iint_{image} (E_x u + E_y v + E_t)^2 \, dx \, dy$$

- We need additional constraints!

- Smoothness Constraint (as in shape from shading and stereo):

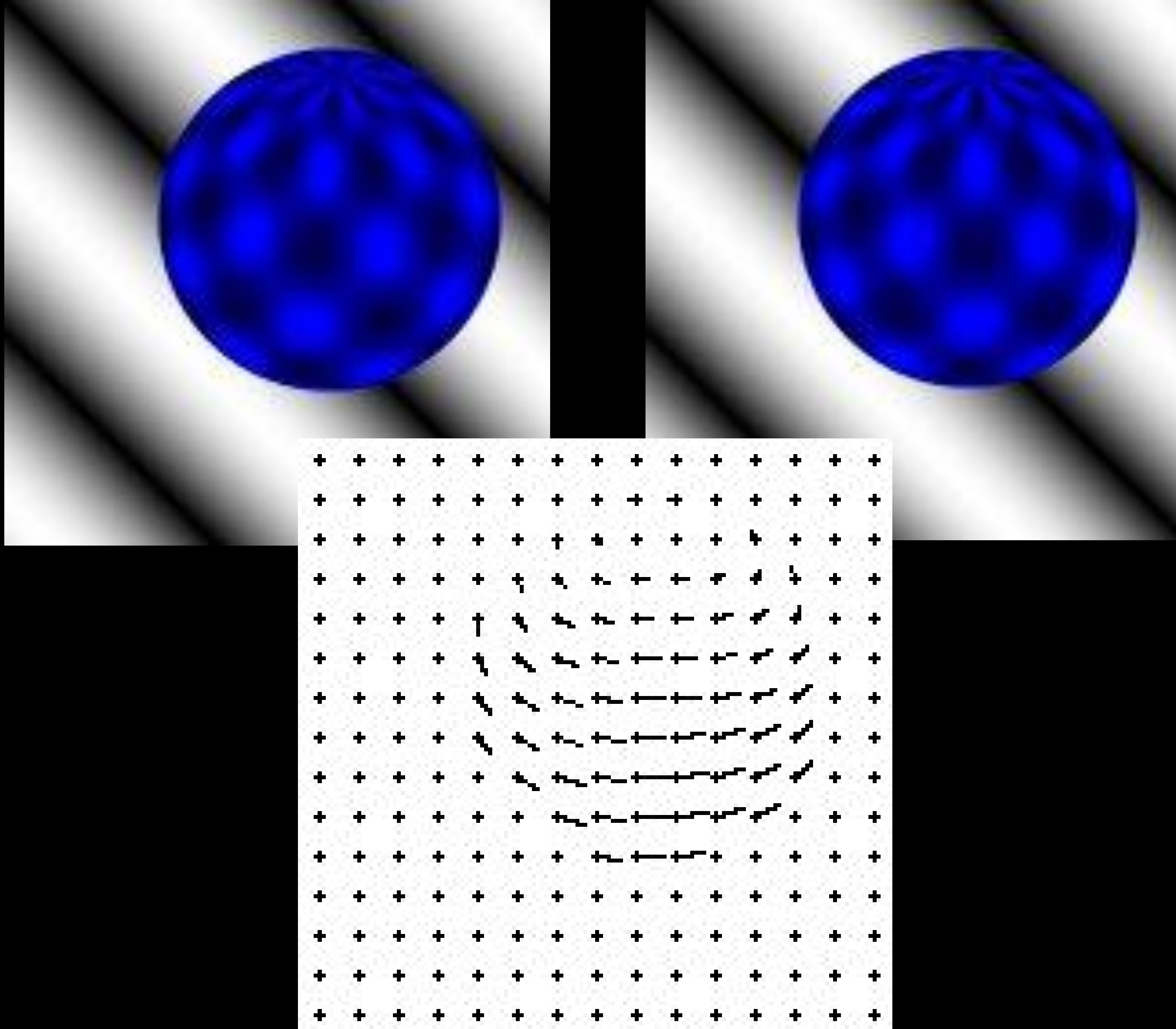
  Usually motion field varies smoothly in the image.
  So, penalize departure from smoothness:

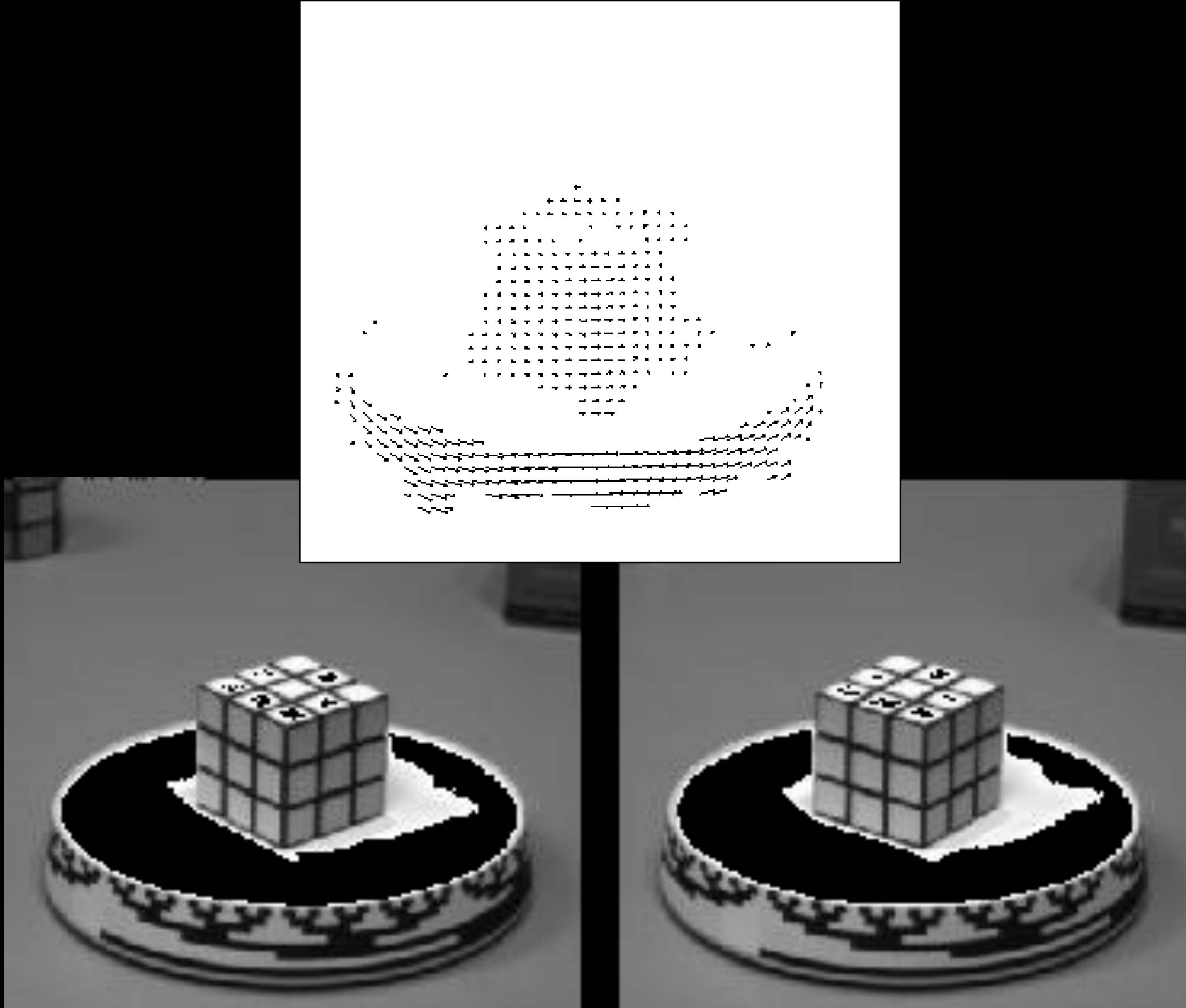
$$e_s = \iint_{image} (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \;\; dx \, dy$$

- Find (u,v) at each image point that MINIMIZES:

$$e = e_s + \lambda e_c$$

weighting factor

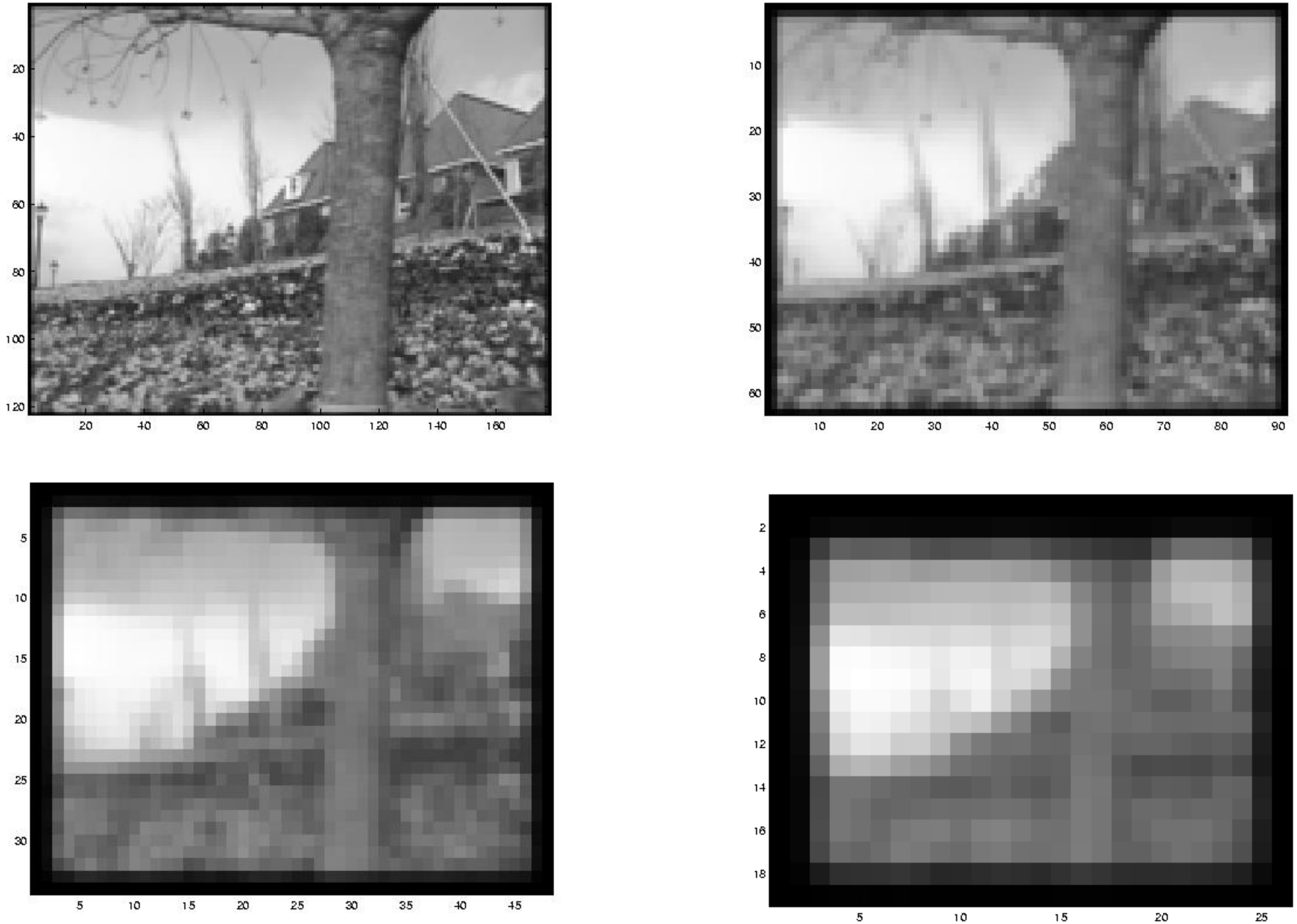




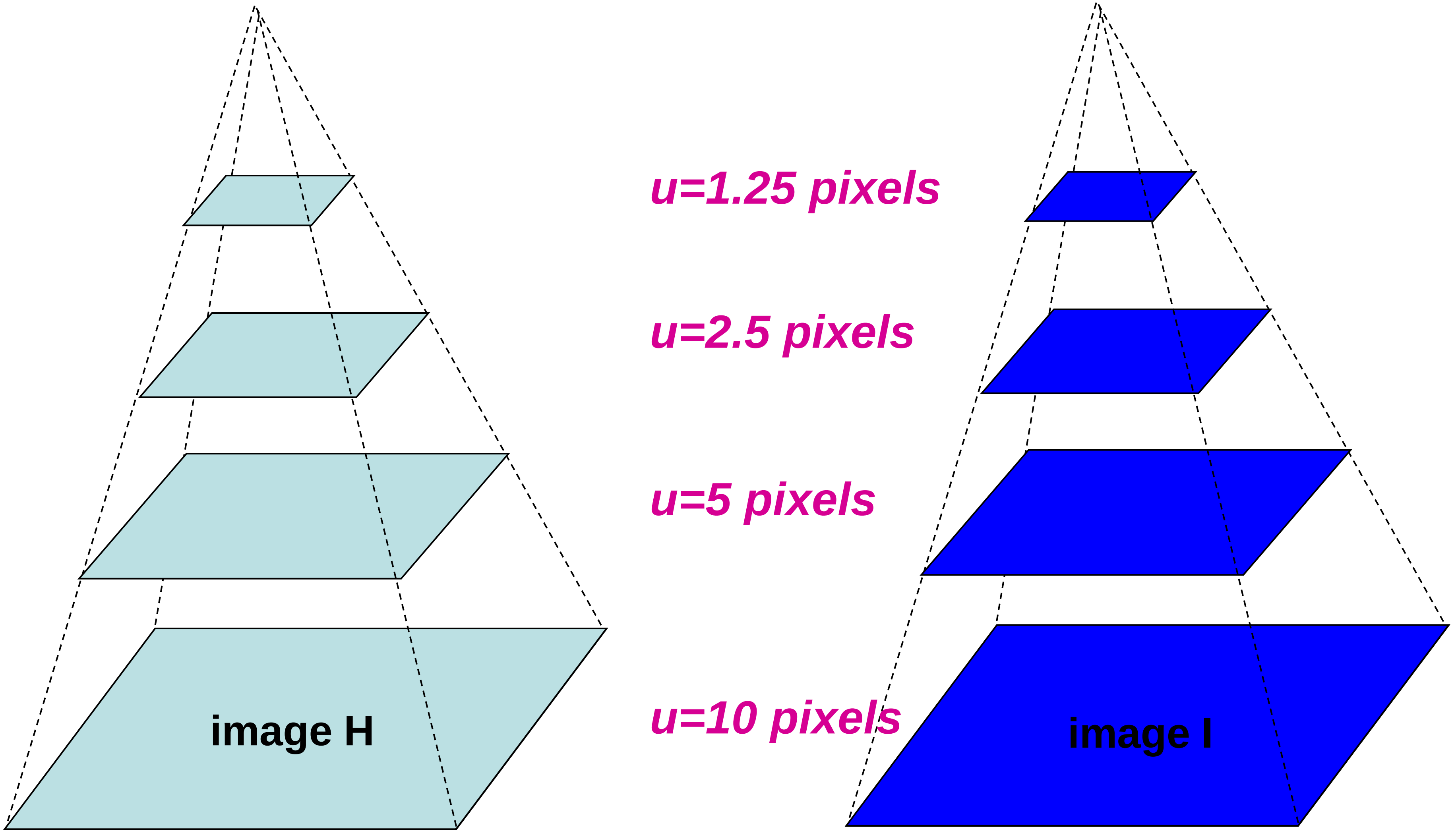
Revisiting the Small Motion Assumption

- Is this motion small enough?
  - Probably not—it's much larger than one pixel (2^{nd} order terms dominate)
  - How might we solve this problem?

Reduce the Resolution!



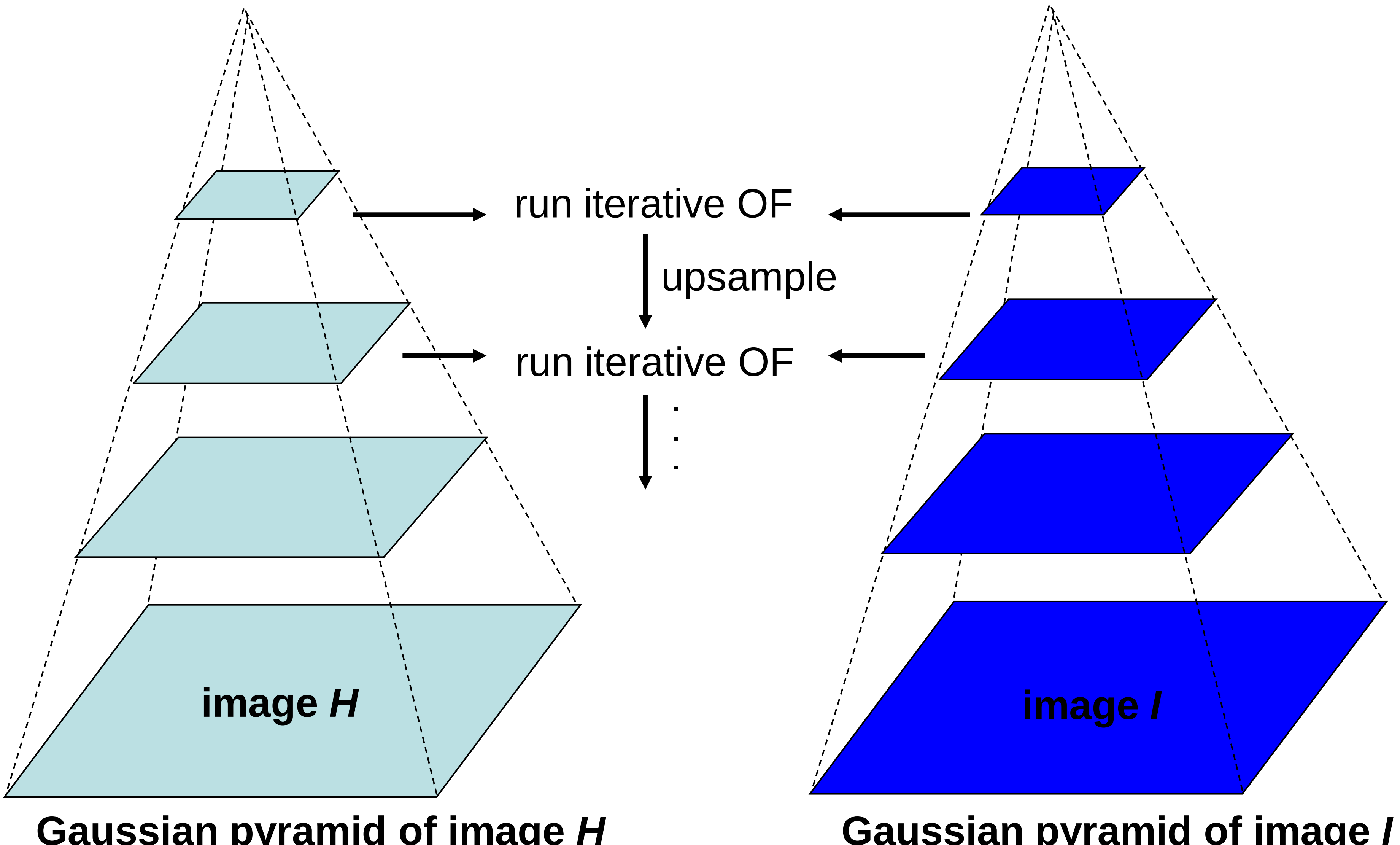
Coarse-to-fine Optical Flow Estimation



Gaussian pyramid of image *H*

Gaussian pyramid of image *I*

Coarse-to-fine Optical Flow Estimation



Types of OF methods

- Differential
  - Horn and Schunck [HS80], Lucas Kanade [LK81], Nagel [83].
- Region-based matching
  - Anandan [Anan87], Singh [Singh90], Digital video encoding standards.
- Energy-based
  - Heeger [Heeg87]
- Phase-based
  - Fleet and Jepson [FJ90]

Open problem!
Current solutions are not good enough!

Topic: The Lucas & Kanade Algorithm

- Optical Flow Constraint Equation
- Aperture problem.
- The Lucas & Kanade Algorithm

The Lucas & Kanade Method

- How to get more equations for a pixel?
  - Basic idea: impose additional constraints
    - most common is to assume that the flow field is smooth locally
    - one method: pretend the pixel's neighbors have the same (u,v)
      - If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \ \ v]$$

$$\underset{25\times 2}{\underbrace{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}}_{A}} \underset{2\times 1}{\underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{d}} = - \underset{25\times 1}{\underbrace{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}_{b}}$$

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Lukas-Kanade flow

- Prob: we have more equations than unknowns

$$\underset{25\times2}{A}\;\underset{2\times1}{d} = \underset{25\times1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

- Solution: solve least squares problem
  - minimum least squares solution given by solution (in d) of:

$$\underset{2\times2}{(A^T A)}\;\underset{2\times1}{d} = \underset{2\times1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

  - The summations are over all pixels in the K x K window
  - This technique was first proposed by Lukas & Kanade (1981)
    - described in Trucco & Verri reading

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Conditions for solvability

– Optimal (u, v) satisfies Lucas-Kanade equation

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$A^T A \qquad\qquad\qquad\qquad A^T b$$

When is This Solvable?

- $\mathbf{A^T A}$ should be invertible
- $\mathbf{A^T A}$ should not be too small due to noise
  - eigenvalues λ_1 and λ_2 of $\mathbf{A^T A}$ should not be too small
- $\mathbf{A^T A}$ should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

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Eigenvectors of A^TA

$$A^TA = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

- Suppose (x,y) is on an edge. What is A^TA?
  - gradients along edge all point the same direction
  - gradients away from edge have small magnitude

$$\left(\sum \nabla I (\nabla I)^T\right) \approx k \nabla I \nabla I^T$$

$$\left(\sum \nabla I (\nabla I)^T\right) \nabla I = k \|\nabla I\| \nabla I$$

  - ∇I is an eigenvector with eigenvalue $k\|\nabla I\|$
  - What's the other eigenvector of A^TA?
    - let N be perpendicular to ∇I

$$\left(\sum \nabla I (\nabla I)^T\right) N = 0$$

    - N is the second eigenvector with eigenvalue 0
- The eigenvectors of A^TA relate to edge direction and magnitude

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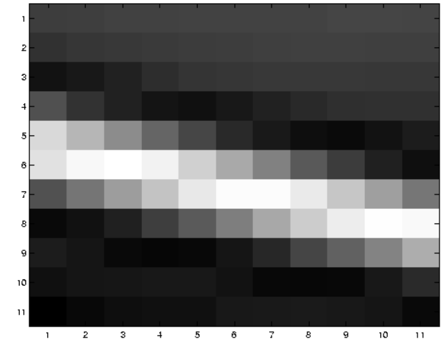
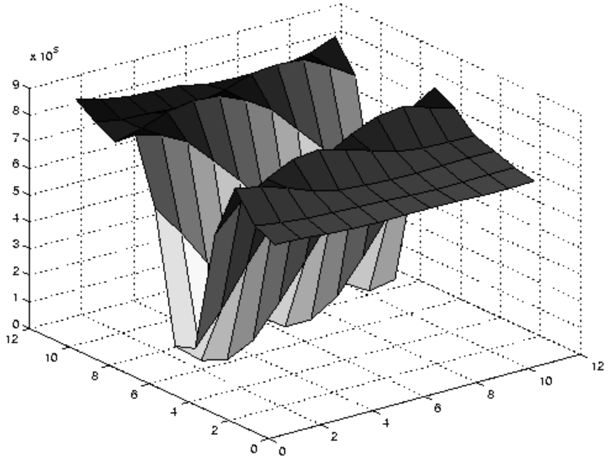
Edge





$$\sum \nabla I (\nabla I)^T$$

– large gradients, all the same

– large λ_1, small λ_2

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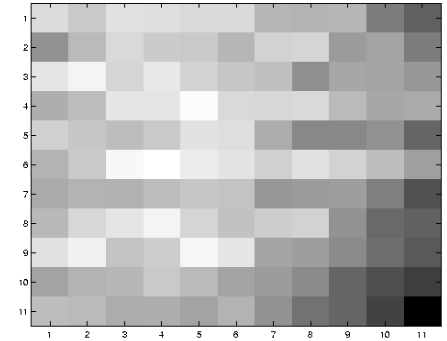
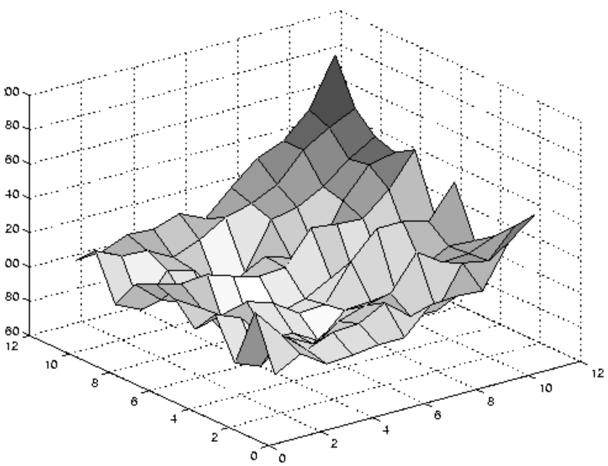
Low texture region





$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

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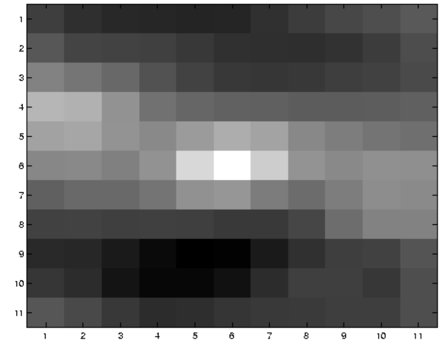
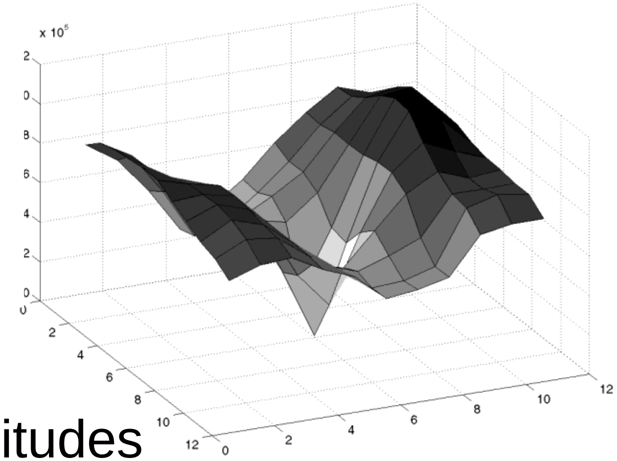
High textured region





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

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Sparse Motion Field

- We are only confident in motion vectors of areas with two strong eigenvectors.
  - Optical flow.
- Not so confident when we have one or zero strong eigenvectors.
  - Normal flow (apperture problem).
  - Unknown flow (blank-wall problem).

Summing all up

- Optical flow:
  - Algorithms try to approximate the true motion field of the image plane.
  - The Optical Flow Constraint Equation needs an additional constraint (e.g. smoothness, constant local flow).
  - The Lucas Kanade method is the most popular Optical Flow Algorithm.
- What applications is this useful for?
- What about block matching?

Resources

- Barron, “Tutorial: Computing 2D and 3D Optical Flow.”, http://www.tina-vision.net/docs/memos/2004-012.pdf
- CVonline: Optical Flow - http://homepages.inf.ed.ac.uk/cgi/rbf/CVONLINE/entries.pl?TAG518
- Fast Image Motion Estimation Demo http://extra.cmis.csiro.au/IA/changs/motion/