VC 10/11 – T3
Digital Images

Mestrado em Ciência de Computadores

Mestrado Integrado em Engenharia de Redes e Sistemas Informáticos

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U. PORTO FC

Outline

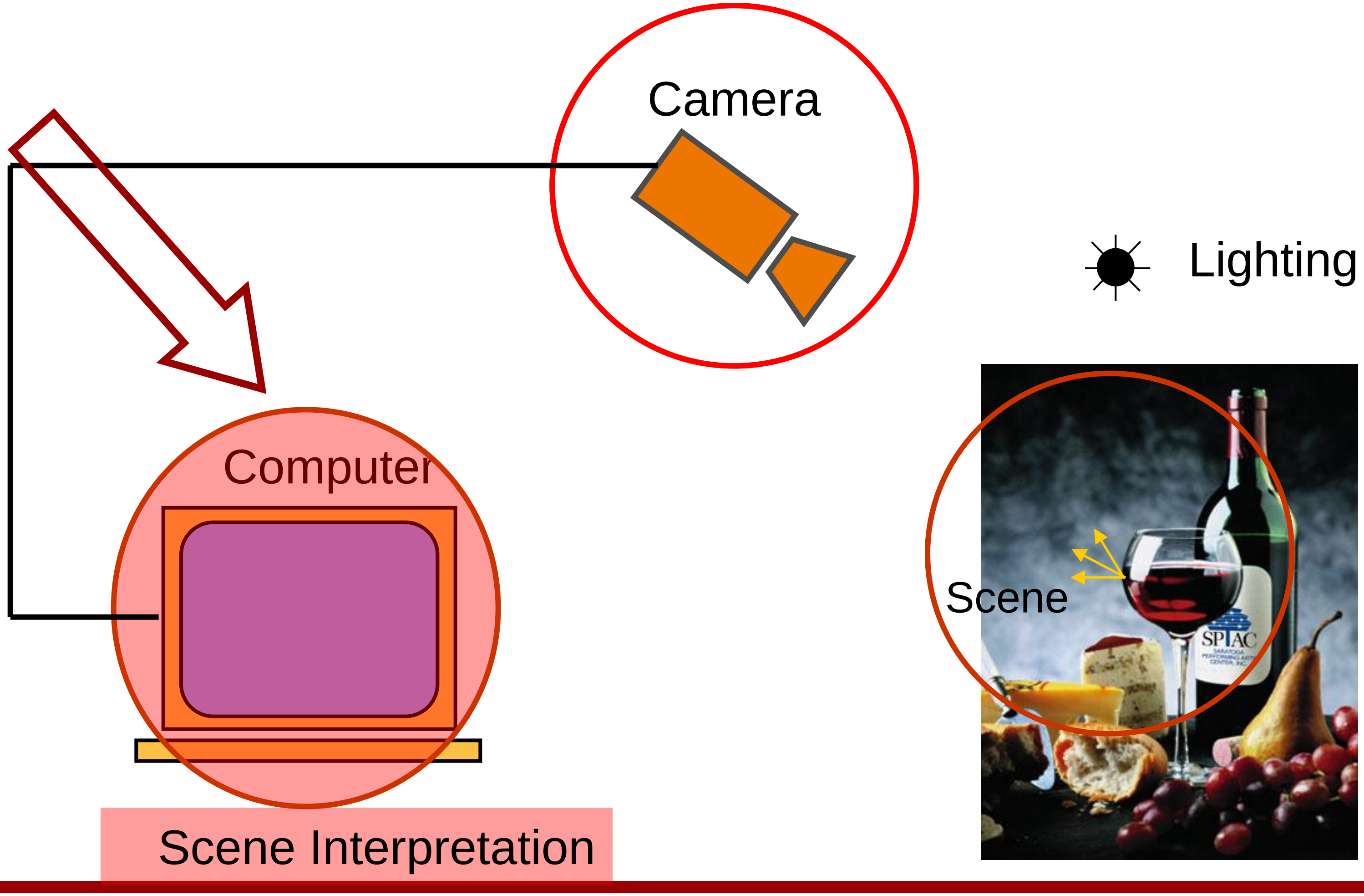
- Sampling and quantization
- Data structures for digital images
- Histograms

Acknowledgements: Most of this course is based on the excellent courses offered by Prof. Shree Nayar at Columbia University, USA and by Prof. Srinivasa Narasimhan at CMU, USA. Please acknowledge the original source when reusing these slides for academic purposes.

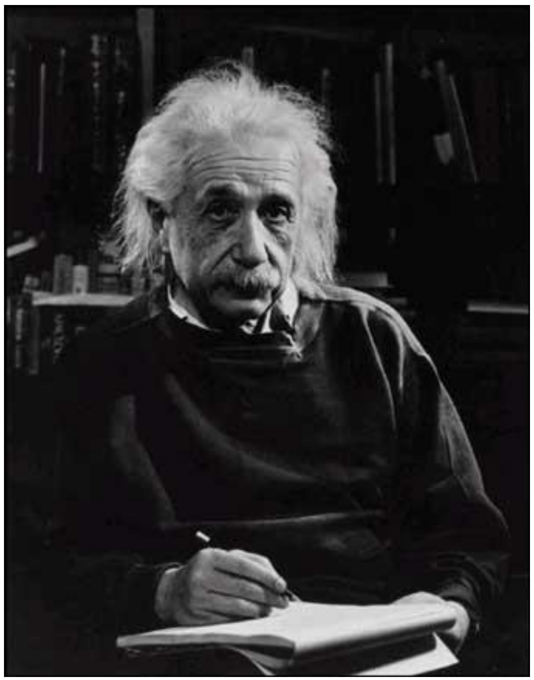
Topic: Sampling and quantization

- Sampling and quantization
- Data structures for digital images
- Histograms

Components of a Computer Vision System



Digital Images

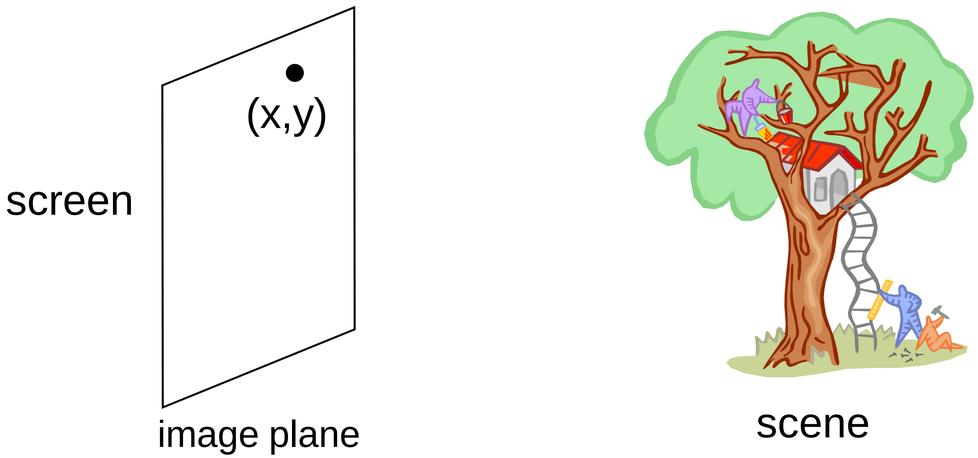


What we see

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

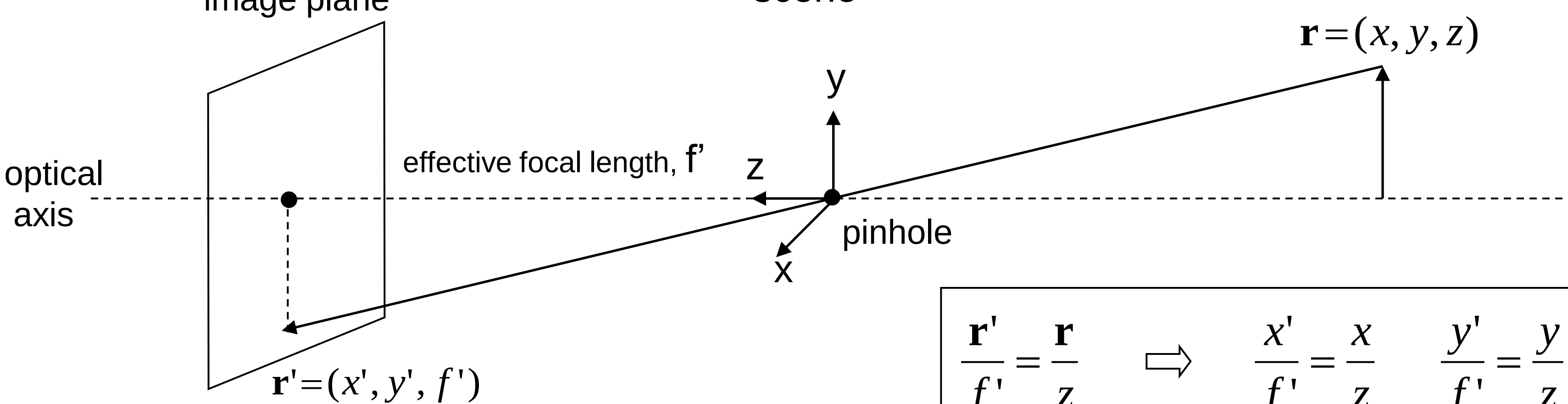
What a computer sees

Pinhole and the Perspective Projection



Is an image being formed on the screen?

YES! But, not a “clear” one.



$$\frac{\mathbf{r}'}{f'}=\frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'}=\frac{x}{z} \qquad \frac{y'}{f'}=\frac{y}{z}$$

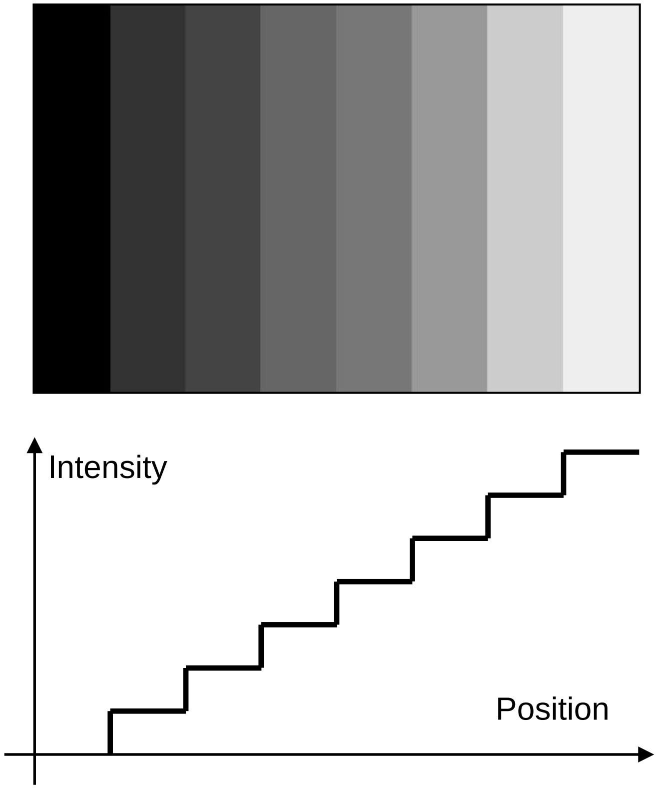
Simple Image Model

- Image as a 2D light-intensity function

$$f(x, y)$$

- Continuous
- Non-zero, finite value

$$0 < f(x, y) < \infty$$

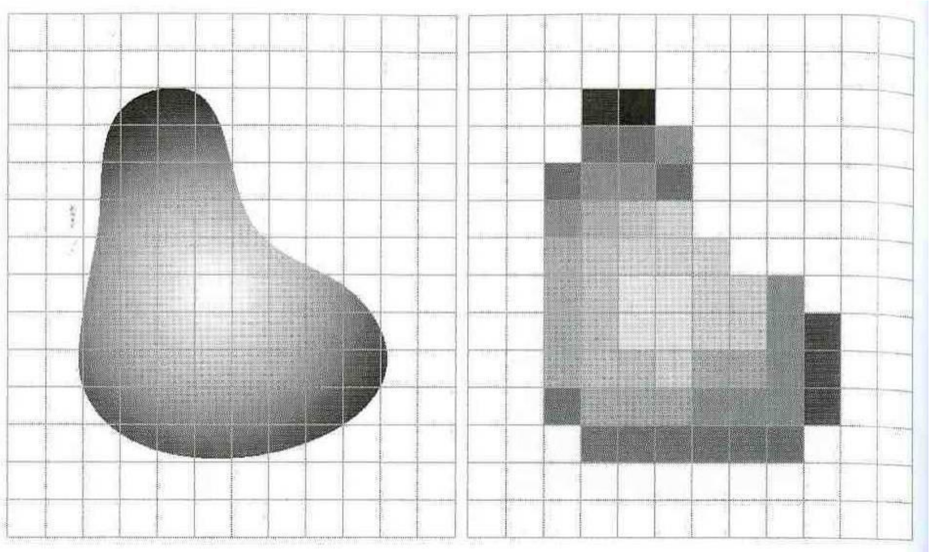


[Gonzalez & Woods]

Analog to Digital

The scene is:

- **projected** on a 2D plane,
- **sampled** on a regular grid, and each sample is
- **quantized** (rounded to the nearest integer)

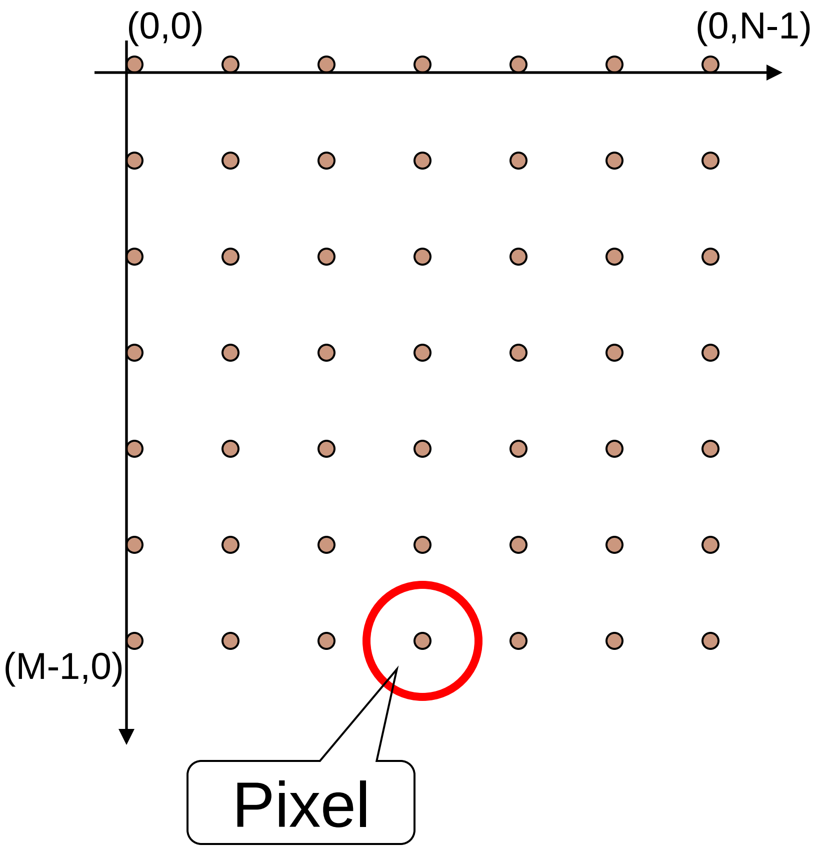


$$f(i, j) = \text{Quantize}\{ f(i\Delta, j\Delta)\}$$

Images as Matrices

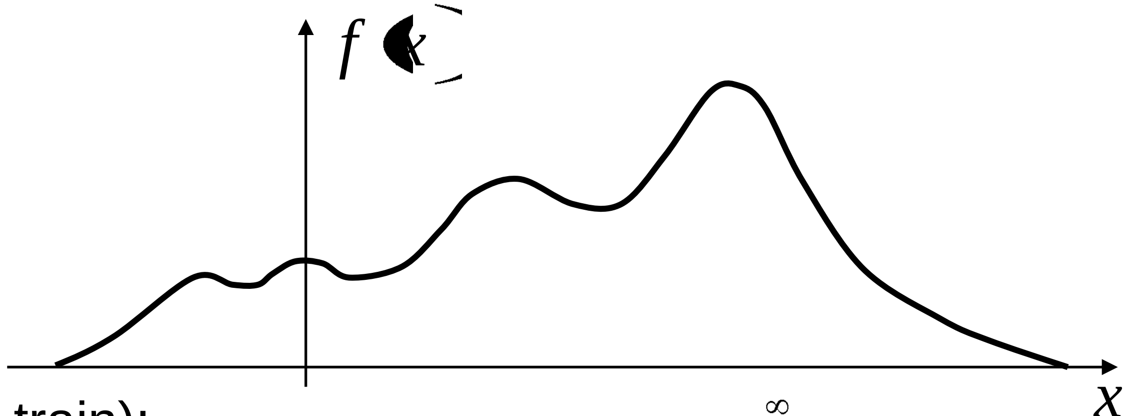
- Each point is a pixel with amplitude:
  - f(x,y)
- An image is a matrix with size N x M

M = [(0,0) (0,1) …
[(1,0) (1,1) …
…



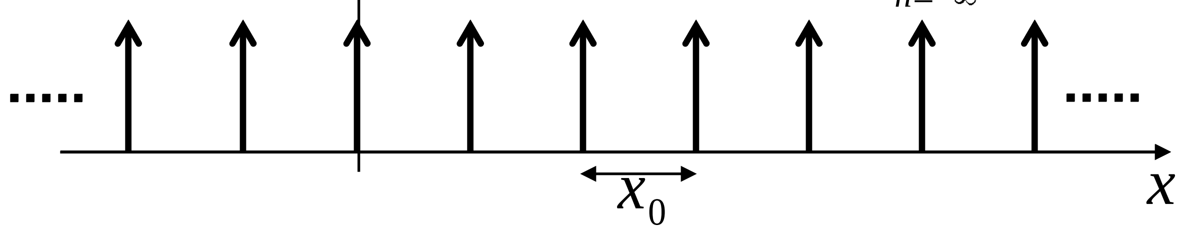
Sampling Theorem

Continuous signal:



Shah function (Impulse train):

$$s(x) = \sum_{n=-\infty}^{\infty} \delta(x - nx_0)$$



Sampled function:

$$f_s(x) = f(x)s(x) = f(x)\sum_{n=-\infty}^{\infty} \delta(x - nx_0)$$

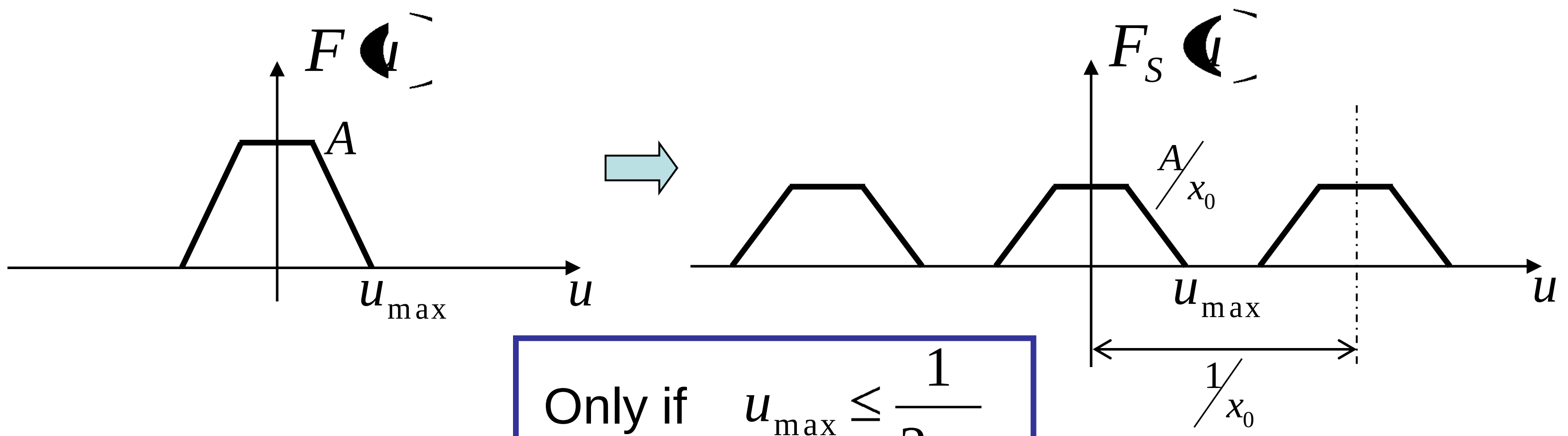
Sampling Theorem

Sampled function:

$$f_s(x) = f(x)s(x) = f(x)\sum_{n=-\infty}^{\infty}\delta(x - nx_0)$$

Sampling frequency $\frac{1}{x_0}$

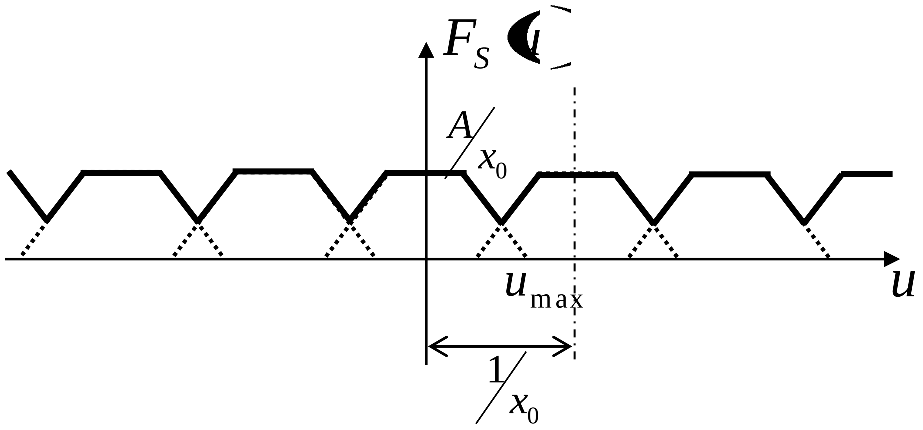
$$F_S(u) = F(u) * S(u) = F(u) * \frac{1}{x_0}\sum_{n=-\infty}^{\infty}\delta\left(u - \frac{n}{x_0}\right)$$



Only if $u_{max} \leq \frac{1}{2x_0}$

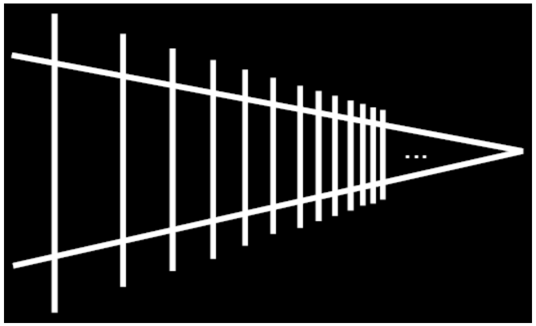
Nyquist Theorem

If $u_{max} > \frac{1}{2x_0}$



Sampling frequency must be greater than $2u_{max}$

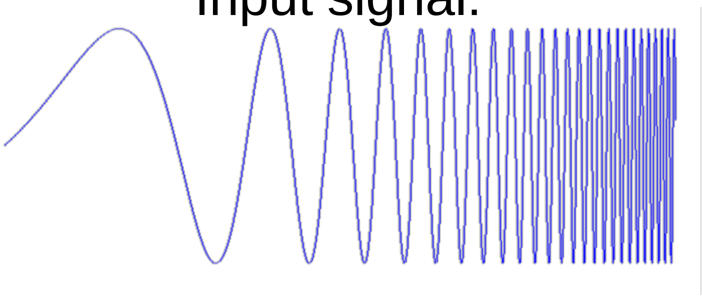
Aliasing



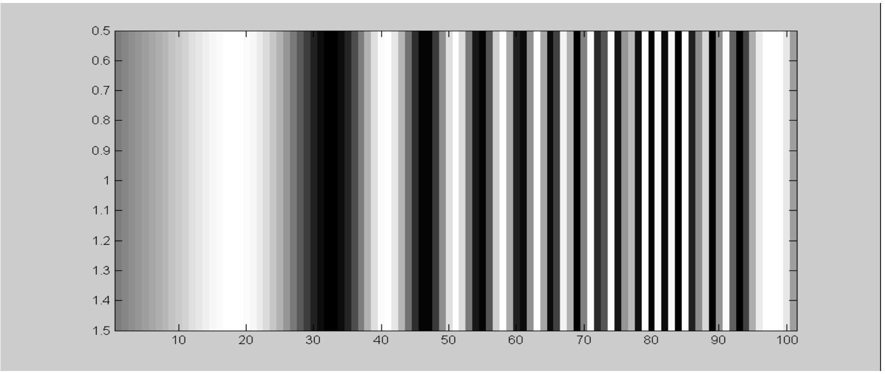
Picket fence receding into the distance will produce aliasing…

WHY?

Input signal:



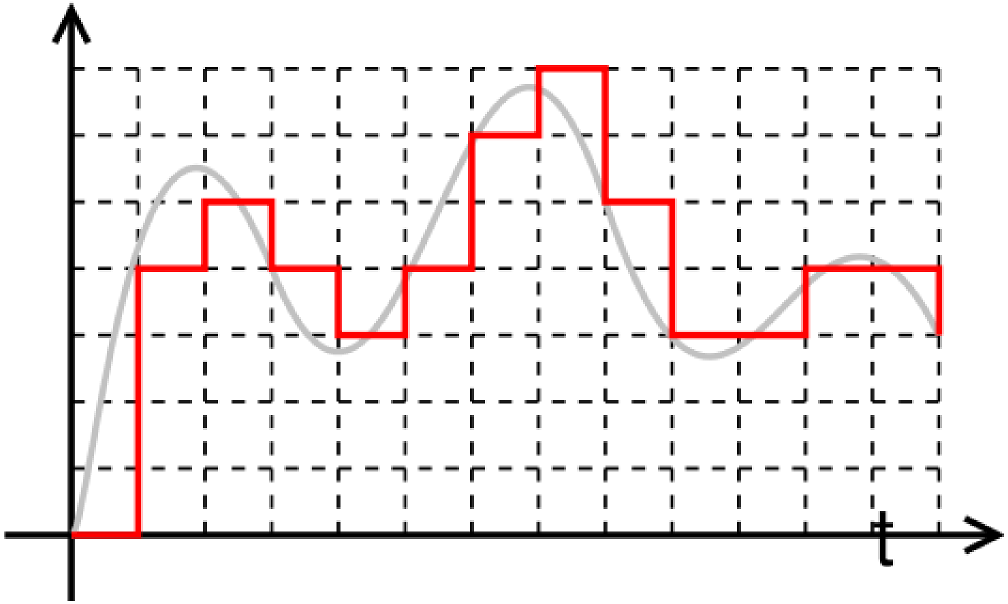
Matlab output:



x = 0:.05:5; imagesc(sin((2.^x).*x))

Quantization

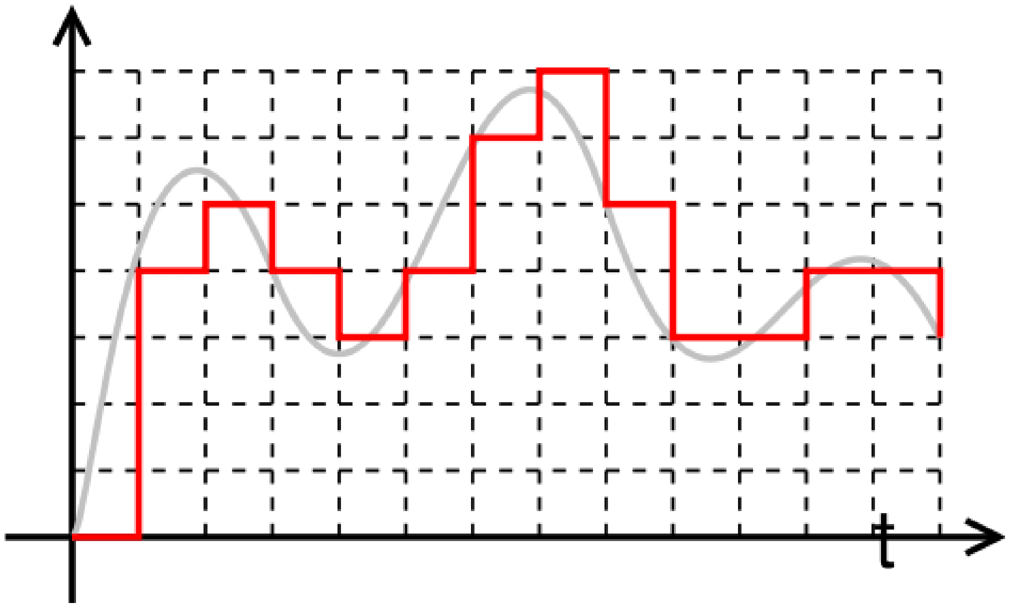
- Analog: $0 < f(x, y) < \infty$
- Digital: Infinite storage space per pixel!
- Quantization



Quantization Levels

- G - number of levels
- m – storage bits
- Round each value to its nearest level

$$G = 2^m$$



Effect of quantization

Effect of quantization

Image Size

- Storage space
  - Spatial resolution: N x M
  - Quantization: m bits per pixel
  - Required bits b:

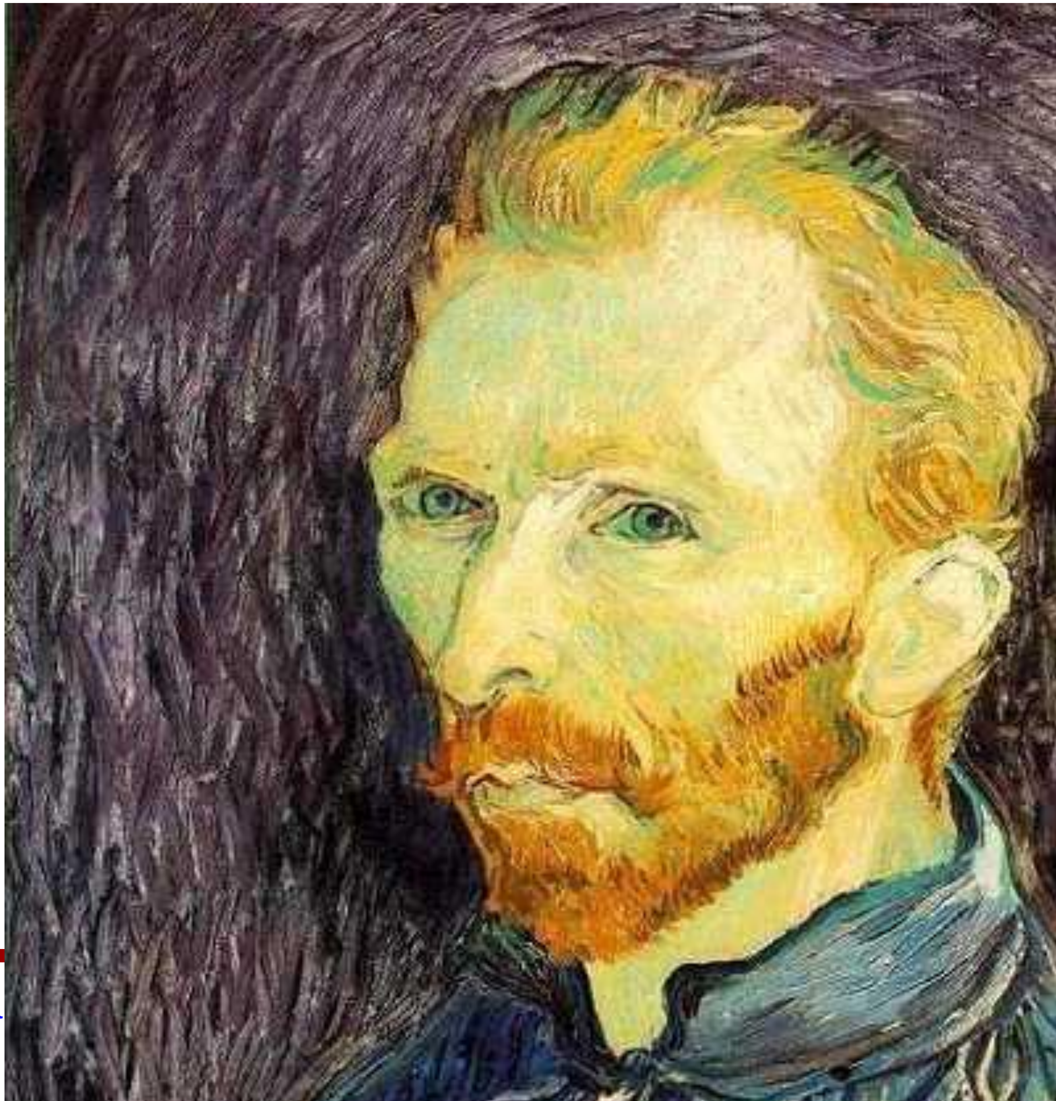
$$b = N \times M \times m$$

- Rule of thumb:
  - More storage space means more image quality

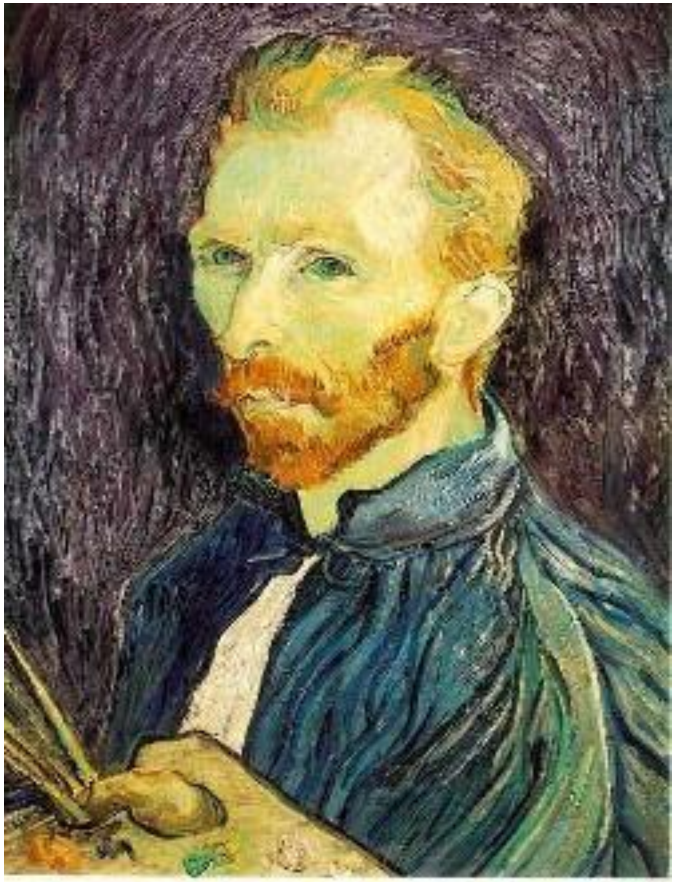
Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?



Sub-sampling

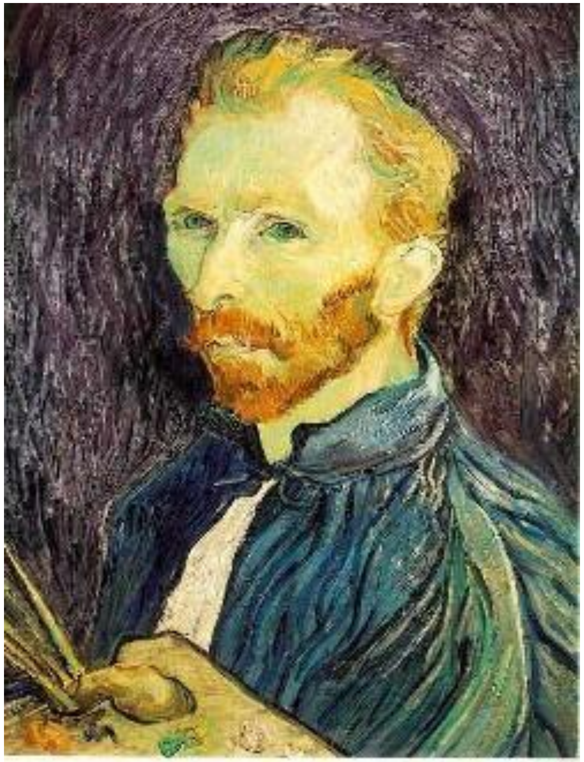


1/4

1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

Sub-sampling

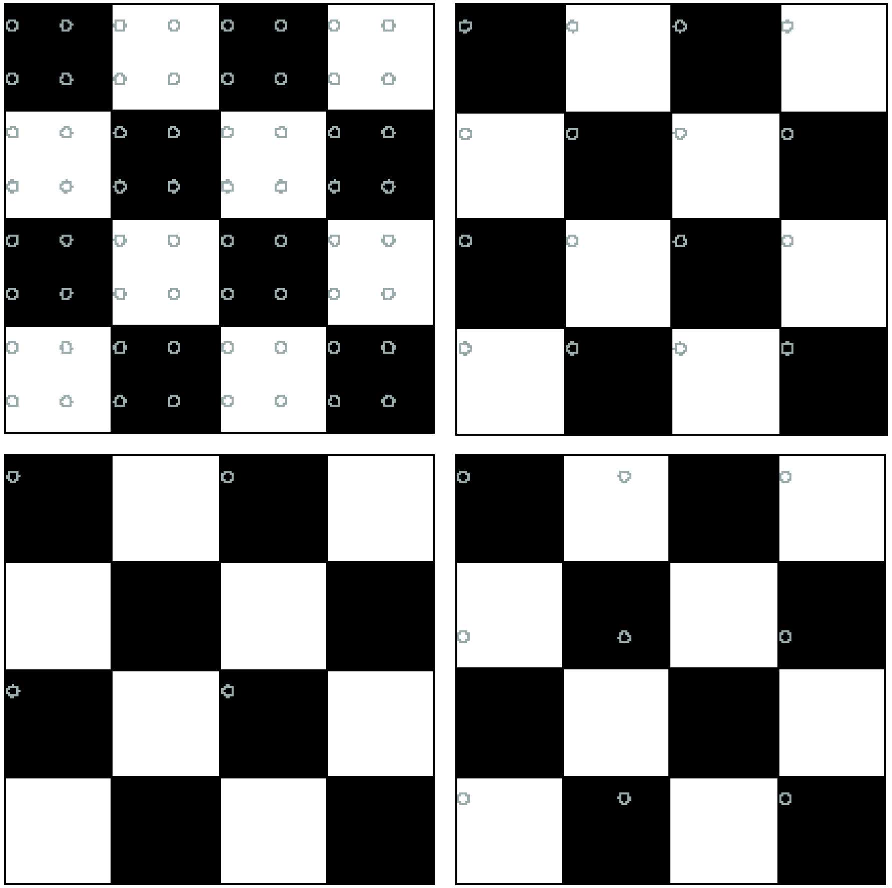


1/2

1/4 (2x zoom)

1/8 (4x zoom)

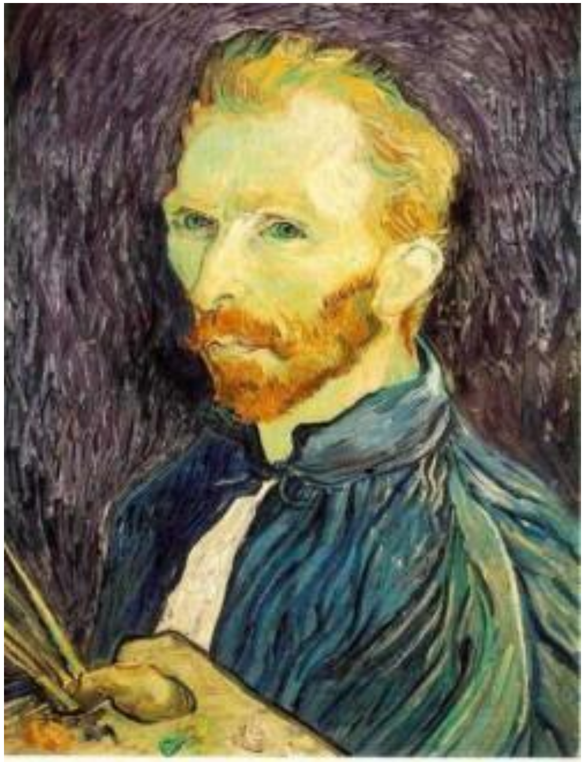
Good and Bad Sampling



Good sampling:
- Sample often or,
- Sample wisely

Bad sampling:
- see aliasing in action!

Sub-Sampling with Gaussian Pre-Filtering

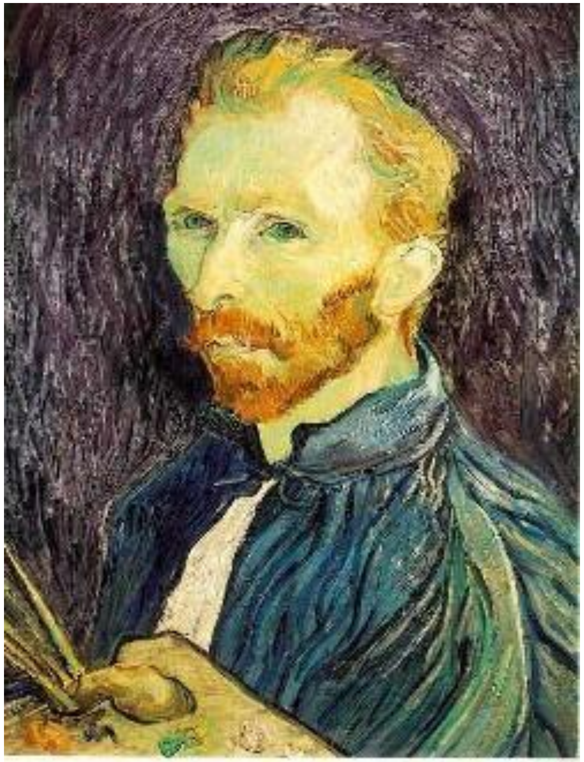


Gaussian 1/2

G 1/4

G 1/8

Compare with...



1/2

1/4 (2x zoom)

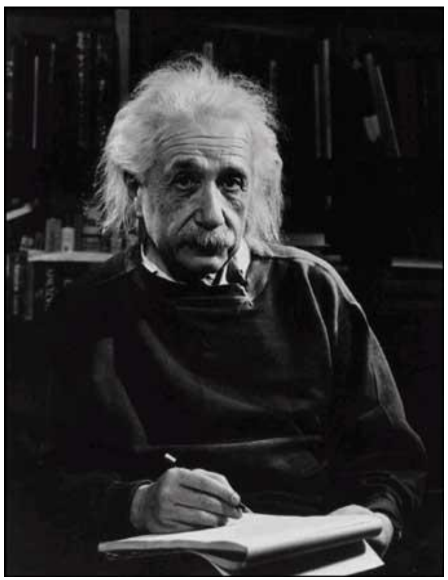
1/8 (4x zoom)

Topic: Data structures for digital images

- Sampling and quantization
- Data structures for digital images
- Histograms

Data Structures for Digital Images

- Are there other ways to represent digital images?



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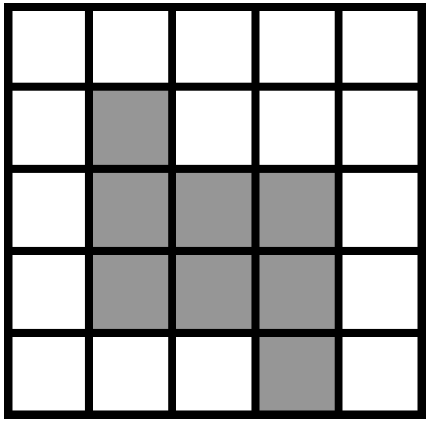
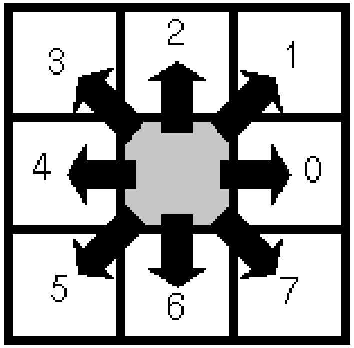
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

What a computer sees

Chain codes

- Chains represent the borders of objects.
- Coding with *chain codes*.
  - Relative.
  - Assume an initial starting point for each object.
- Needs segmentation!



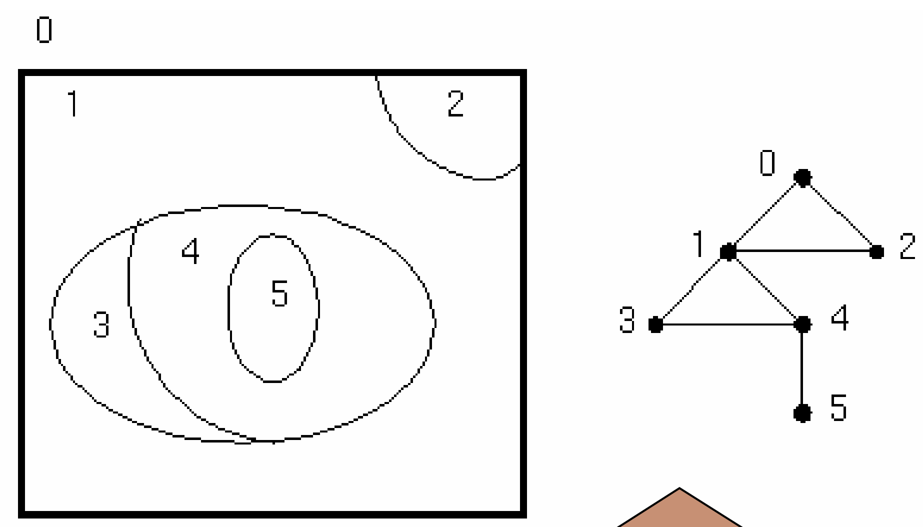


Freeman Chain Code

Using a Freeman Chain Code and considering the top-left pixel as the starting point:
70663422

Topological Data Structures

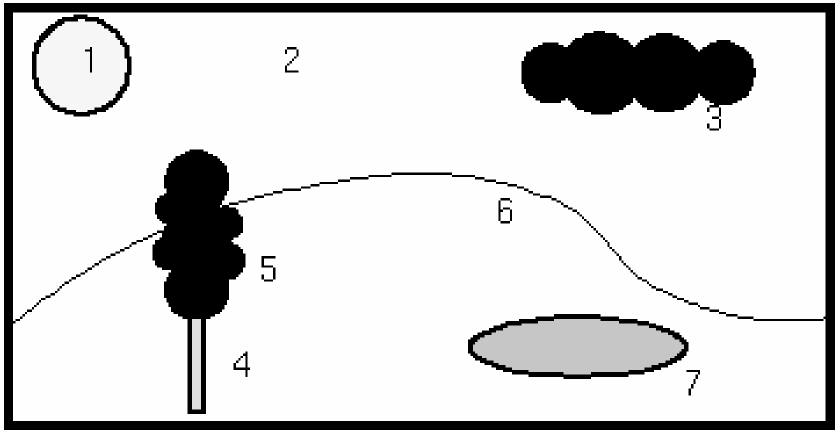
- *Region Adjacency Graph*
  - **Nodes** - Regions
  - **Arcs** – Relationships
- Describes the elements of an image and their spatial relationships.
- Needs segmentation!



Region Adjacency Graph

Relational Structures

- Stores relations between objects.
- Important semantic information of an image.
- Needs segmentation and an image description (features)!



No.	Object name	Colour	Min. row	Min. col.	Inside
1	sun	white	5	40	2
2	sky	blue	0	0	-
3	cloud	grey	20	180	2
4	tree trunk	brown	95	75	6
5	tree crown	green	53	63	-
6	hill	light green	97	0	-
7	pond	blue	100	160	6

Relational Table

Topic: Histograms

- Sampling and quantization
- Data structures for digital images
- Histograms

Histograms

- "In statistics, a histogram is a graphical display of tabulated frequencies."

  [Wikipedia]

- Typically represented as a bar chart:

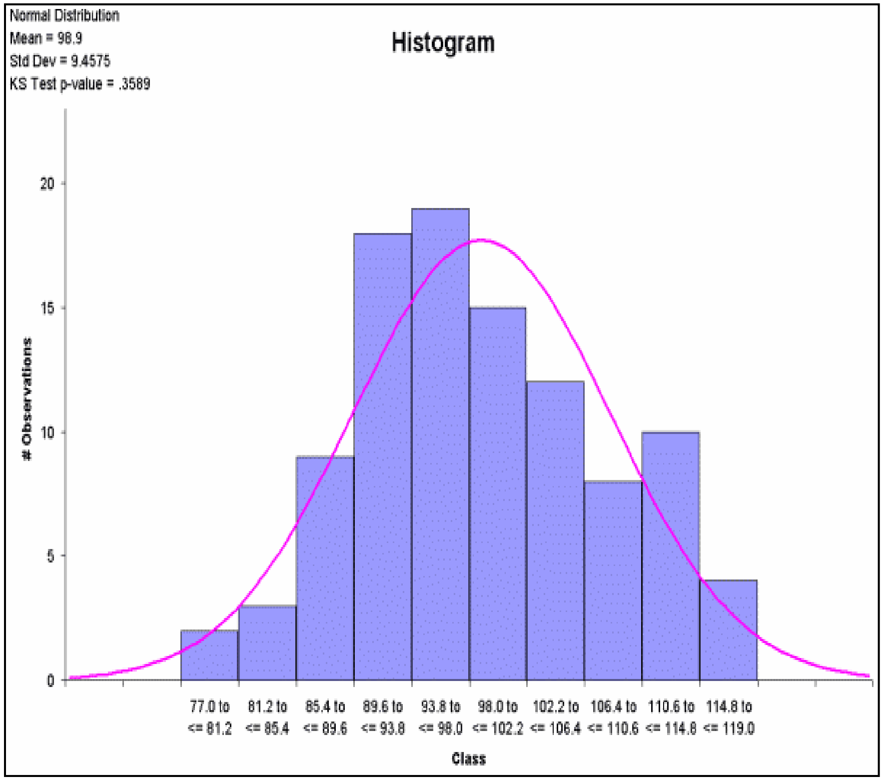
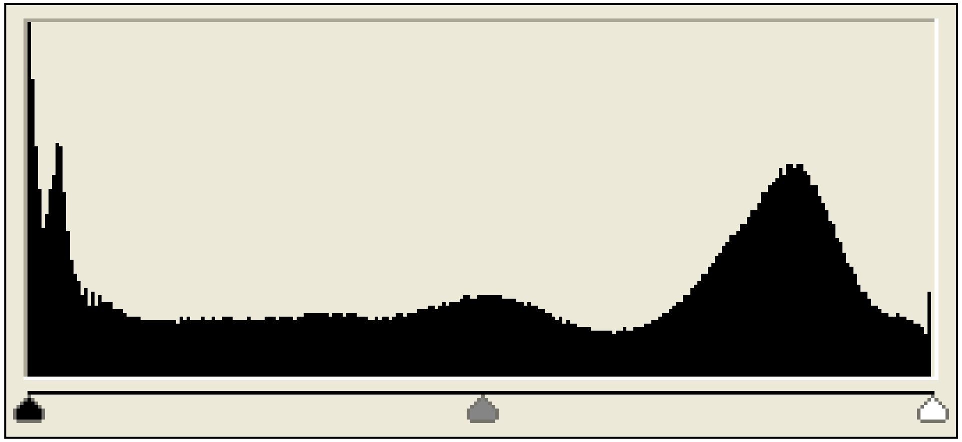


Image Histograms

- Colour or Intensity distribution.
- Typically:
  - Reduced number of *bins*.
  - Normalization.
- Compressed representation of an image.
  - No spatial information whatsoever!



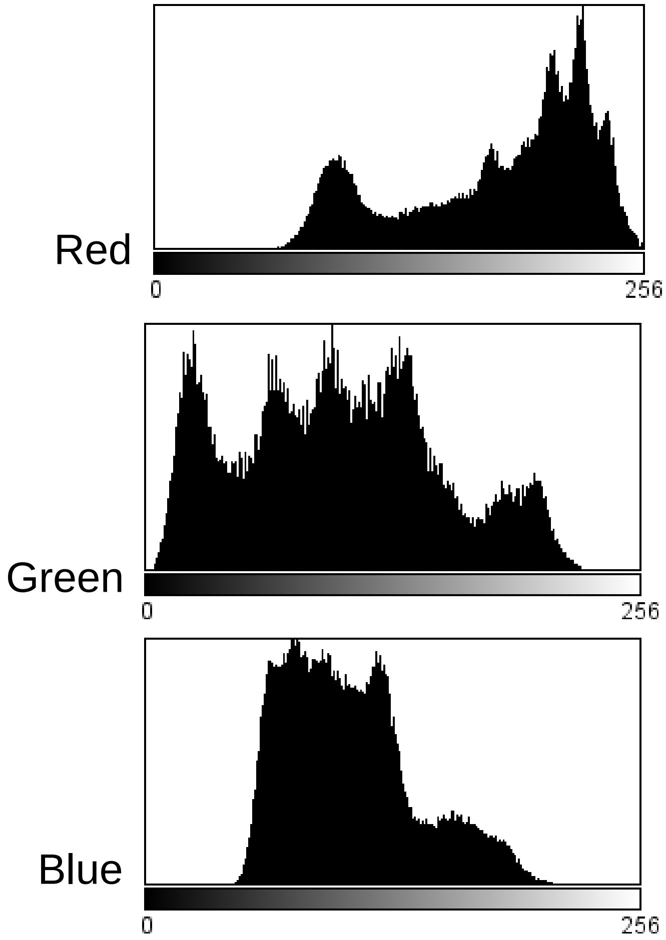
Colour Histogram

- As many histograms as axis of the colour space.

  Ex: RGB Colour space

  - Red Histogram
  - Green Histogram
  - Blue Histogram

- Combined histogram.



Resources

- R. Gonzalez, and R. Woods – Chapter 2