

Problem A - Quebrar Um Protocolo

The ONI (National Internet Organization, or ONI in Portuguese) recruited you to test a new cryptographic protocol created by them. The protocol consists of:

- a *public key* \boldsymbol{A} , which is a sequence of \boldsymbol{N} integers,
- and two private keys \boldsymbol{B} and \boldsymbol{C} , which are two sequences of \boldsymbol{N} integers such that $\boldsymbol{A}[i] = \boldsymbol{B}[i] + \boldsymbol{C}[i]$ for all $1 \leq i \leq N$.



Every one has access to $\boldsymbol{A},$ which is used to encrypt messages.

To decrypt messages, access to both sequences \boldsymbol{B} and \boldsymbol{C} is

required. In addition to the element-wise sums forming the key A, the keys B and C must also adhere to a certain set of properties. The ONI wants you to analyze the security of some possibilities for these properties. That is, given A, is it easy to find keys B and C that allow breaking the cryptographic protocol?

Part I

The first property that ONI would like you to try to satisfy is the following: for every element B[i] of B and C[j] of C, we have B[i] > C[j]. That is, all elements of B must be greater than all elements of C.

Given a sequence of N integers A, find two sequences of N integers B and C such that the element-wise sums coincide with the elements of A and such that all elements of B are greater than all elements of C.

Example

If N = 6 and $A = \{0, 7, -3, 2, 2, -100\}$, one possibility for the keys B and C could be, for example, $B = \{10, 6, 3, 2, 8, 15\}$ and $C = \{-10, 1, -6, 0, -6, -115\}$.

Constraints

The following limits are guaranteed in all test cases of this part that will be placed to the program:

 $1 \leq N \leq 10^5$ Length of the key

 $-10^4 \le \mathbf{A}[i] \le 10^4$ Size of the integers in the public key

Additionally, the following constraint is required for the private keys found:

 $-10^9 \leq \boldsymbol{B}[\boldsymbol{i}] \leq 10^9$ Size of the integers in the private key \boldsymbol{B} $-10^9 \leq \boldsymbol{C}[\boldsymbol{i}] \leq 10^9$ Size of the integers in the private key \boldsymbol{C}

Note: If the output of your program contains values that do not respect these intervals, the program will obtain a *Wrong Answer* result.

The test cases of this part of the problem are organized into a single group without any additional restrictions:

SubtaskPointsAdditional Constraints135No additional restrictions

Part II

After several vulnerabilities were found in the previous protocol, ONI asked you to try to respect a new property, which they believe is much safer: for every i > 1 we have B[i] > B[i-1] and C[i] < C[i-1]. In other words, the elements of the private key B must be in strictly increasing order and the elements of the private key C must be in strictly decreasing order.

Given a sequence of integers A, find two sequences B and C such that the element-wise sums coincide with the elements of A and such that the elements of B are in strictly increasing order and the elements of C are in strictly decreasing order.

Example

If N = 6 and $A = \{0, 7, 3, 2, 2, -100\}$, one possibility for the keys B and C could be, for example, $B = \{-5, 3, 4, 6, 22, 50\}$ and $C = \{5, 4, -1, -4, -20, -150\}$.

Constraints

The following limits are guaranteed in all test cases of this part that will be placed to the program:

 $\begin{array}{ll} 1 \leq \pmb{N} \leq 10^5 & \mbox{Length of the key} \\ -10^4 \leq \pmb{A}[i] \leq 10^4 & \mbox{Size of the integers in the public key} \end{array}$

Additionally, the following constraint is required for the private keys found:

 $-10^9 \leq \boldsymbol{B}[i] \leq 10^9$ Size of the integers in the private key \boldsymbol{B} $-10^9 \leq \boldsymbol{C}[i] \leq 10^9$ Size of the integers in the private key \boldsymbol{C}

The test cases of this part of the problem are organized into a single group without any additional restrictions:

$\mathbf{Subtask}$	Points	Additional Constraints	
2	15	A is a constant sequence (all elements are the same)	
3	15	\boldsymbol{A} is a strictly increasing sequence	
4	35	No additional restrictions	

Summary of Subtasks

The test cases of the problem are organized into four groups with different additional restrictions:

$\mathbf{Subtask}$	Points	Part	Additional Constraints
1	35	Part I	No additional restrictions
2	15	Part II	A is a constant sequence (all elements are the same)
3	15	Part II	$oldsymbol{A}$ is a strictly increasing sequence
4	35	Part II	No additional restrictions

Input Format

The first line contains an integer P, corresponding to the part that the test case represents, 1 for part I and 2 for part II.

The second line contains an integer N, which indicates the length of the public key A. Finally, there is one last line, containing N integers separated by spaces, which corresponds to the values of $A[1], A[2], \ldots, A[N]$.

Output Format

For each test case, print two lines of N integers, the first containing the N elements, in order, of the found private key B, and the second containing the N elements, in order, of the found private key C.

Example 1 Input

1 6 0 7 -3 2 2 -100

Example 1 Output

10 6 3 2 8 15 -10 1 -6 0 -6 -115

Example 1 Description

This example corresponds to the example from Part I mentioned in the statement.

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Example 2 Input

2 6 0 7 3 2 2 -100

Example 2 Output

-5 3 4 6 22 50 5 4 -1 -4 -20 -150

Example 2 Description

This example corresponds to the example from Part II mentioned in the statement.

Example 3 Input

2 8 0 0 0 0 0 0 0 0 0

Example 3 Output

-4 -3 -1 0 1 2 3 8 4 3 1 0 -1 -2 -3 -8

Example 3 Description

This example corresponds to the first group of Part II.

Example 4 Input

2 8 1 2 3 4 5 6 7 8

Example 4 Output

-4 -1 1 4 6 8 11 15 5 3 2 0 -1 -2 -4 -7

Example 4 Description

This example corresponds to the second group of Part II.



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