Lot Sizing Games

Margarida Carvalho [1]

Mathieu Van Vyve [2]
Claudio Telha [2]

[1] Faculdade de Ciências da Universidade do Porto and INESC TEC

[2] CORE, Université catholique de Louvain

5th Porto Meeting on MATHEMATICS for INDUSTRY

April, 2014

1. Motivation
   - Game Theory and Operational Research
   - Integer Programming Games
   - State of the art

2. Lot Sizing Games
   - Formulation
   - Solution Concept: Nash equilibria
   - One Period Game
   - T Period Game
   - Future work

Game Theory

Game Theory Generalization of decision theory; an individual's success depends on the choices of others.

Game Theory

Game Theory Generalization of decision theory; an individual's success depends on the choices of others.

1838 Cournot Duopoly (simultaneous game): earliest examples of game analysis;

Game Theory

Game Theory Generalization of decision theory; an individual's success depends on the choices of others.

1838 Cournot Duopoly (simultaneous game): earliest examples of game analysis;

1952 Stackelberg Game (sequential game): a player, called the leader, takes his decision before decisions of other players, called the followers, are known;

Motivation: Integer Programming Games

Motivation: Integer Programming Games

Normal form games: explicit specification of the players' pure strategies.

		Player II			
		Cooperates		Defects	
Player I	Cooperates	1	1	3	0
	Defects	0	3	2	2

Motivation: Integer Programming Games

Normal form games: explicit specification of the players' pure strategies.

		Player II			
		Cooperates		Defects	
Player I	Cooperates	1	1	3	0
	Defects	0	3	2	2

Integer Programming Games: players' pure strategies are lattice points inside polytopes described by systems of linear inequalities.

Integer Programming games

Each player p solves a problem in the form of

$$\text{Maximize}_{x^p} \ \Pi^p\left(x^p, x^{-p}\right)$$

$$\text{subject to} \quad A_p x^p \leq b_p$$

$$x_i^p \text{ integer }, \qquad \forall i$$

State of Art

There are general methods to solve finite games:

1964 Lemke and Howson;

State of Art

There are general methods to solve finite games:

1964 Lemke and Howson;

1991 Elzen and Talman;

State of Art

There are general methods to solve finite games:

- 1964 Lemke and Howson;
- 1991 Elzen and Talman;
- 2003 Global Newton method by Govindan and Wilson ;

State of Art

There are general methods to solve finite games:

1964 Lemke and Howson;

1991 Elzen and Talman;

2003 Global Newton method by Govindan and Wilson ;

State of Art

There are general methods to solve finite games:

- 1964 Lemke and Howson;
- 1991 Elzen and Talman;
- 2003 Global Newton method by Govindan and Wilson ;

However an explicit description of the set of strategies is required.

Lot Sizing Game

Equilibria on a Game with Discrete Variables

João Pedro PEDROSO[1] and Yves SMEERS[2]

[1] INESC - Porto and
DCC - Faculdade de Ciências, Universidade do Porto, Portugal
jpp@fc.up.pt
[2] CORE, Université Catholique de Louvain
Yves.Smeers@uclouvain.be

Abstract. Equilibrium in Economics has been seldom addressed in a situation where some variables are discrete. This work introduces a problem related to lot-sizing with several players, and analyses some strategies which are likely to be found in real world games. An illustration with a simple example is presented, with concerns about the difficulty of the problem and computation possibilities.

Lot Sizing Game: Model

①

Lot Sizing Game: Model

①　　②

Lot Sizing Game: Model

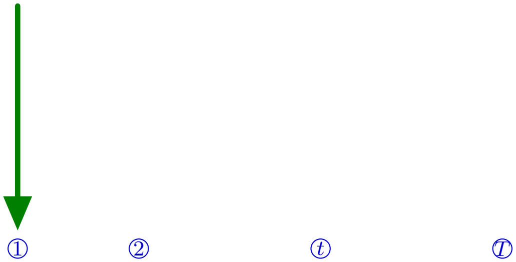
① ② ⓣ

Lot Sizing Game: Model

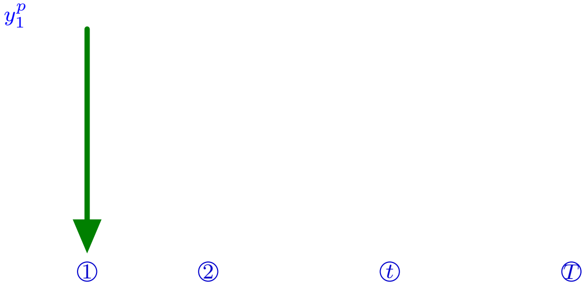
Lot Sizing Game: Model

① ② t T

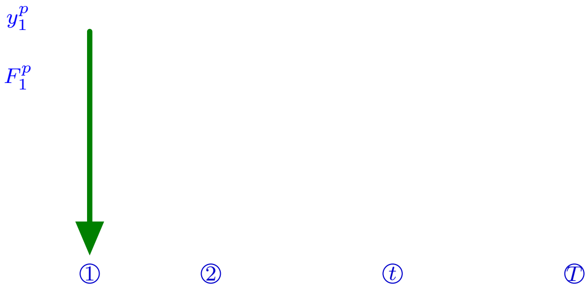
Lot Sizing Game: Model



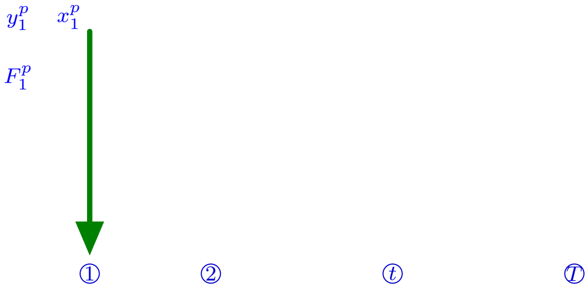
Lot Sizing Game: Model



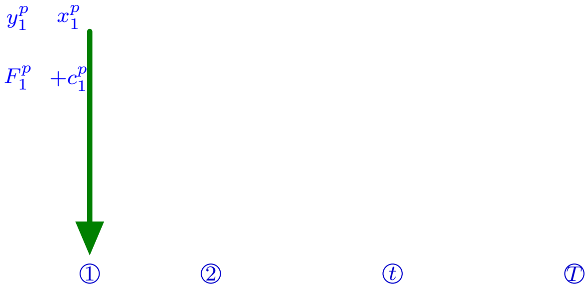
Lot Sizing Game: Model



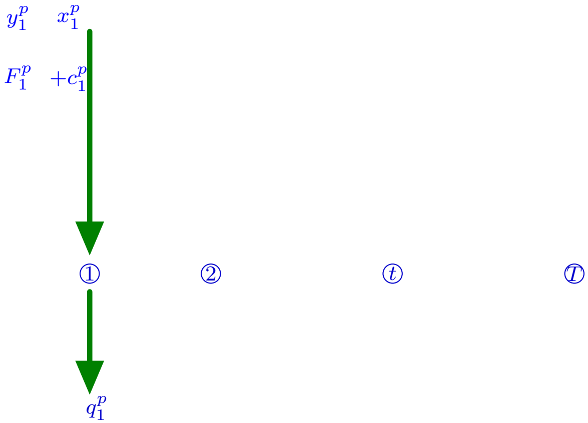
Lot Sizing Game: Model



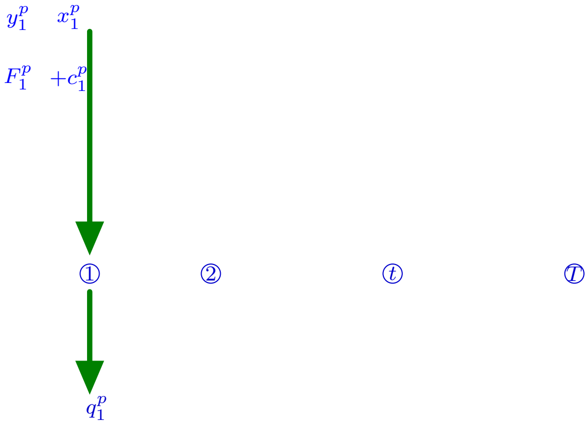
Lot Sizing Game: Model



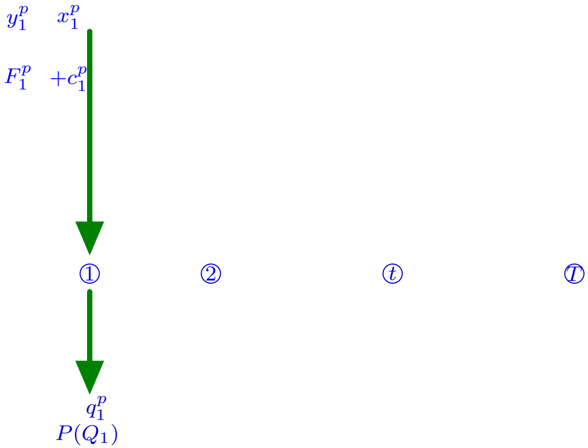
Lot Sizing Game: Model



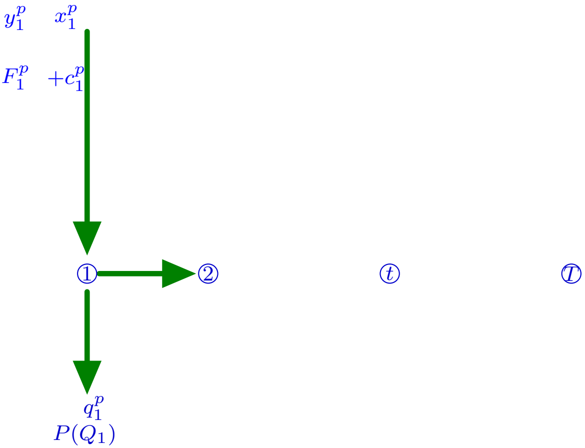
Lot Sizing Game: Model



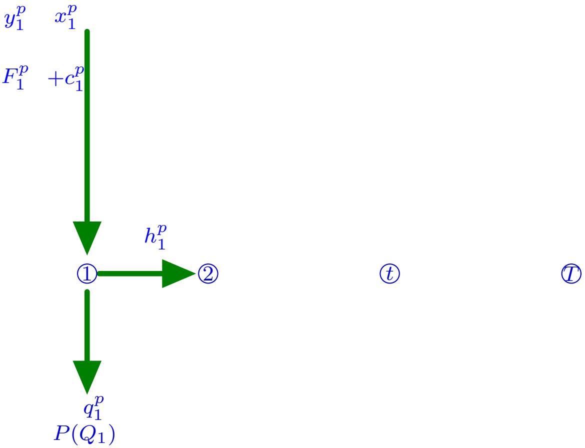
Lot Sizing Game: Model



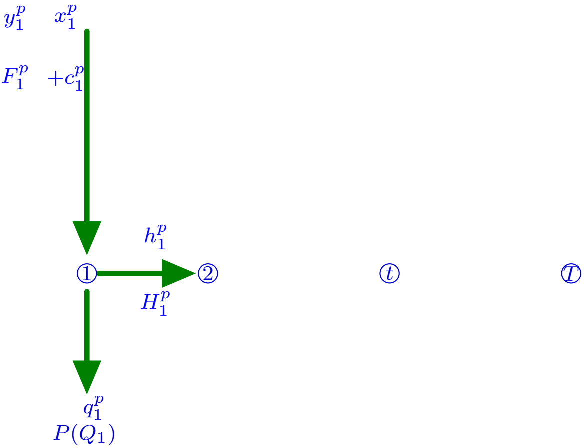
Lot Sizing Game: Model



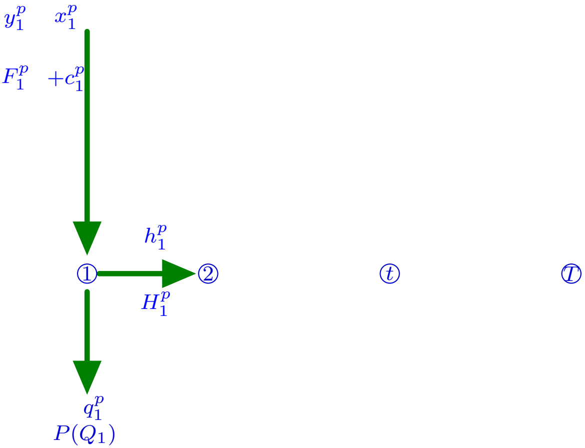
Lot Sizing Game: Model



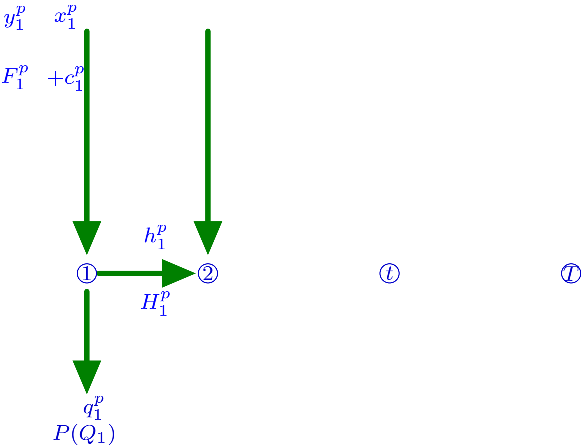
Lot Sizing Game: Model



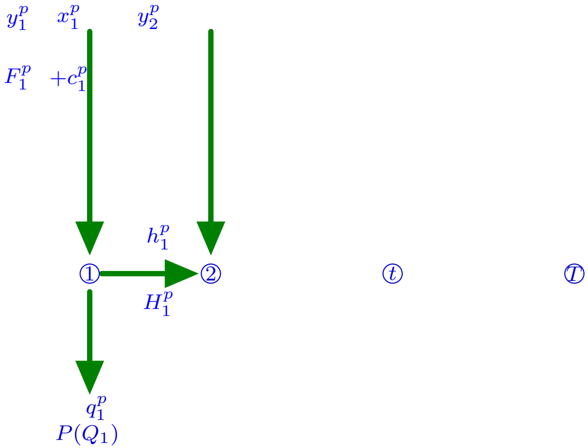
Lot Sizing Game: Model



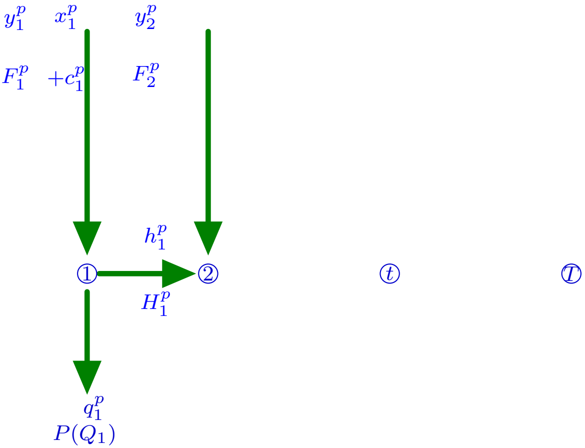
Lot Sizing Game: Model



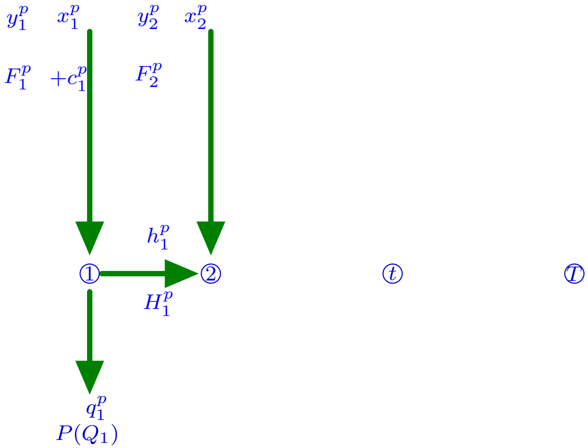
Lot Sizing Game: Model



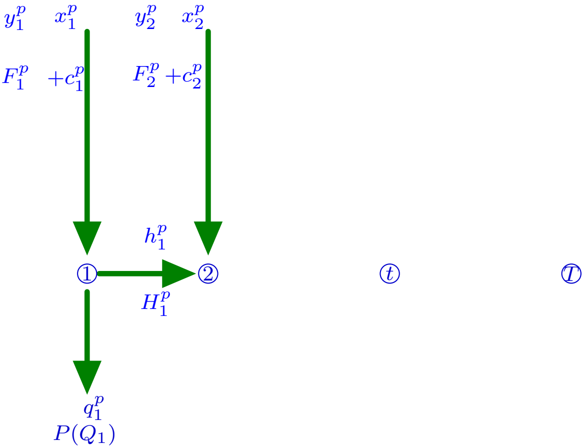
Lot Sizing Game: Model



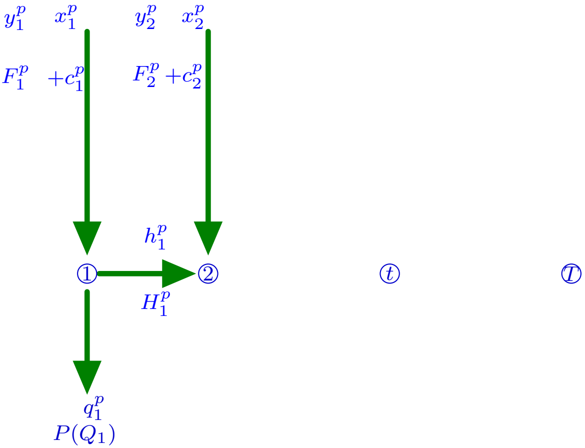
Lot Sizing Game: Model



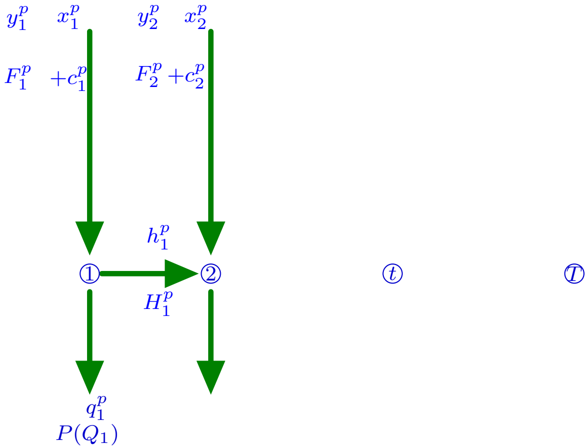
Lot Sizing Game: Model



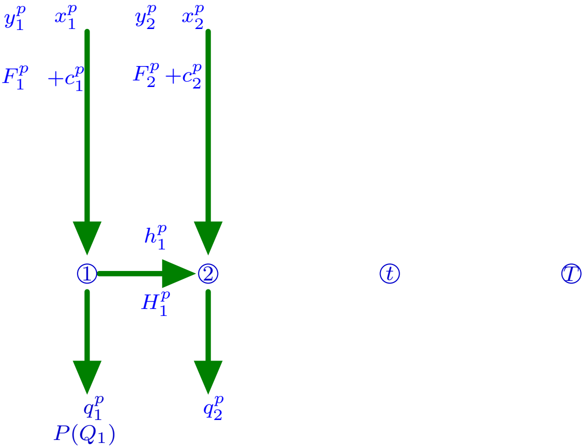
Lot Sizing Game: Model



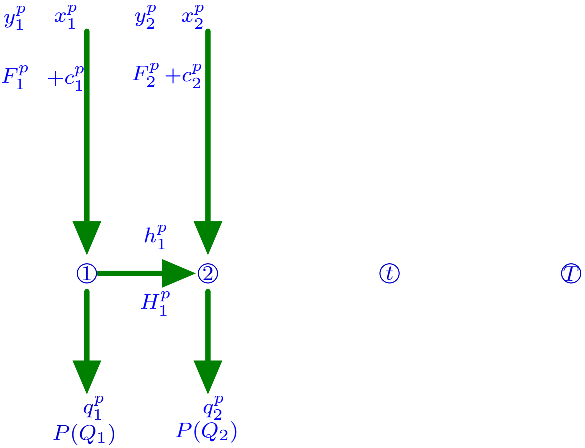
Lot Sizing Game: Model



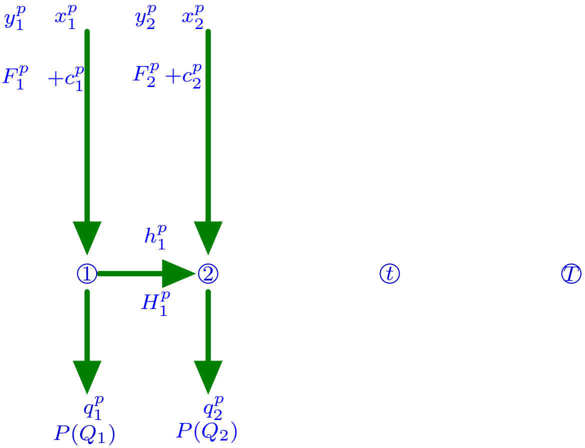
Lot Sizing Game: Model



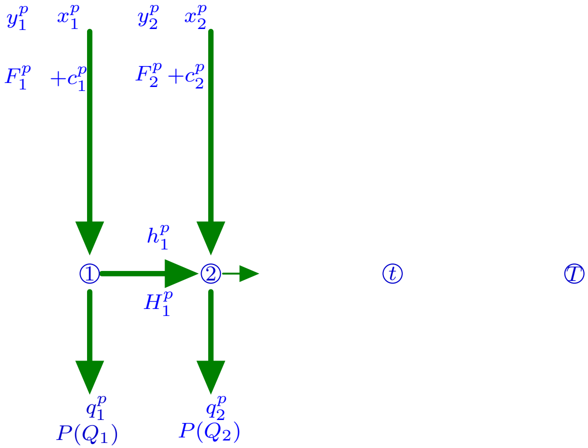
Lot Sizing Game: Model



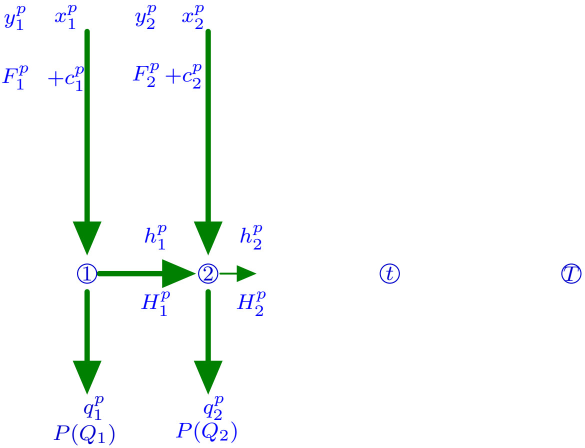
Lot Sizing Game: Model



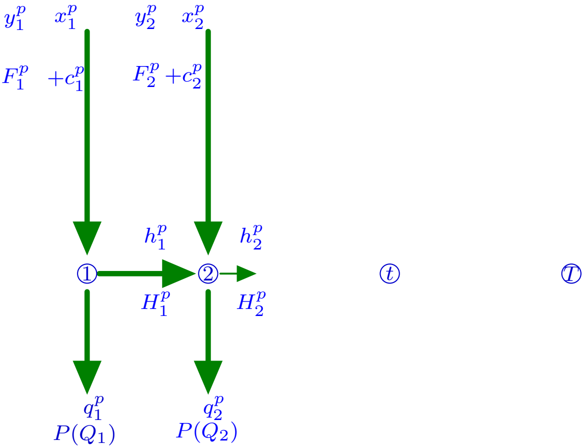
Lot Sizing Game: Model



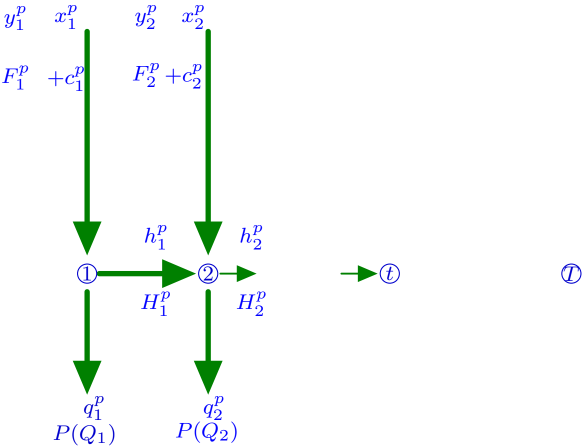
Lot Sizing Game: Model



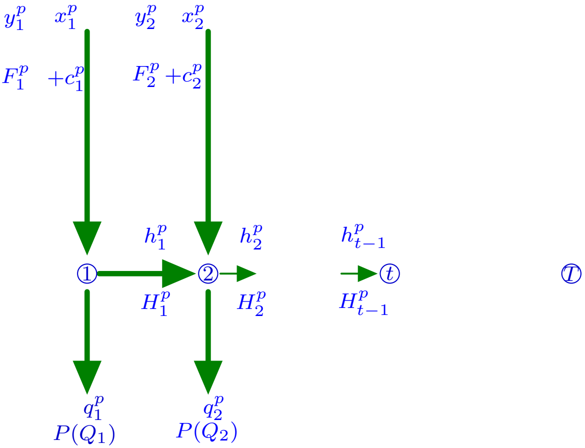
Lot Sizing Game: Model



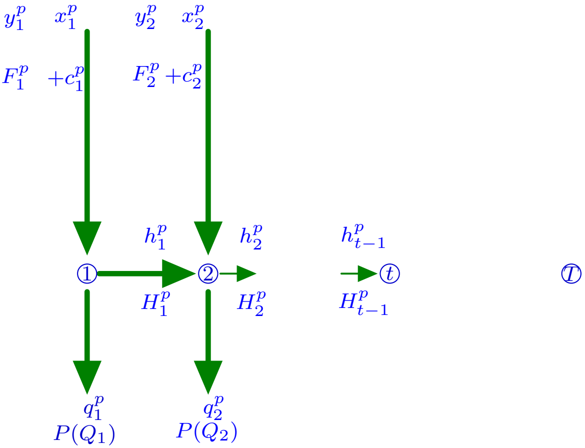
Lot Sizing Game: Model



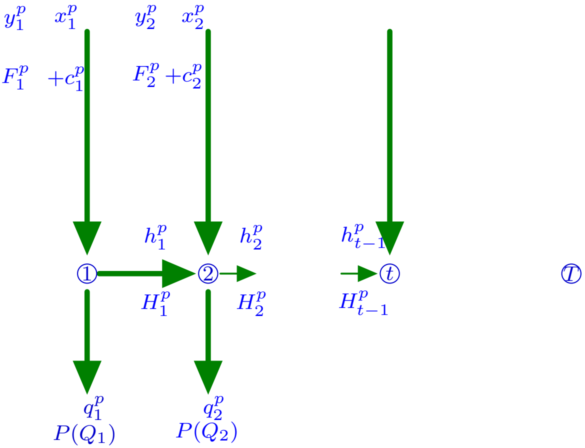
Lot Sizing Game: Model



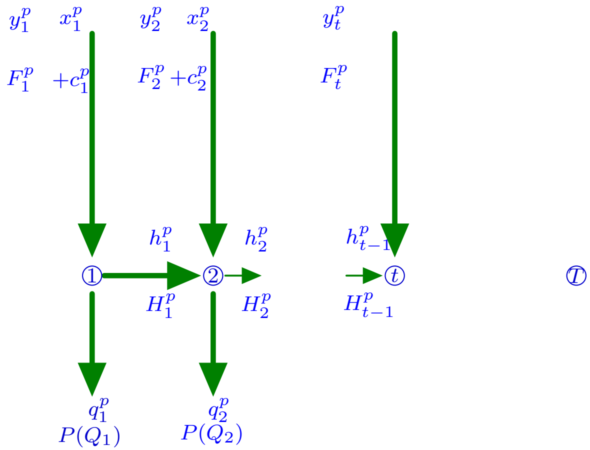
Lot Sizing Game: Model



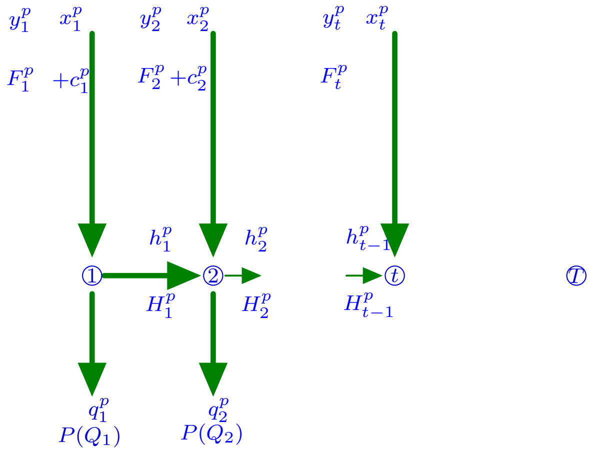
Lot Sizing Game: Model



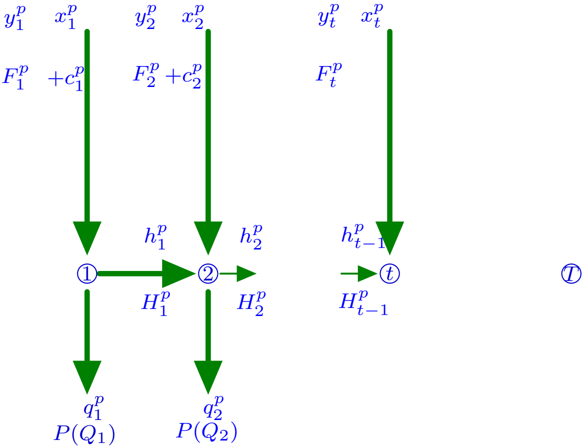
Lot Sizing Game: Model



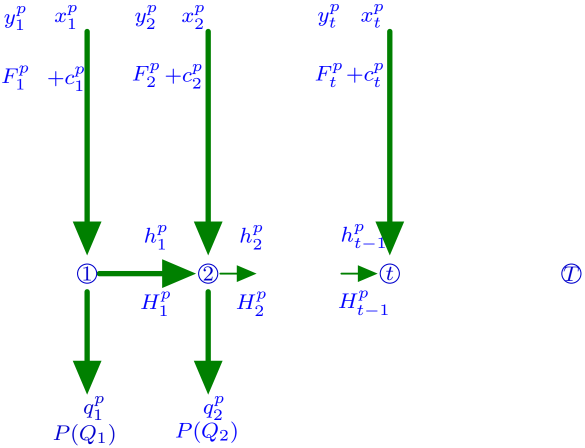
Lot Sizing Game: Model



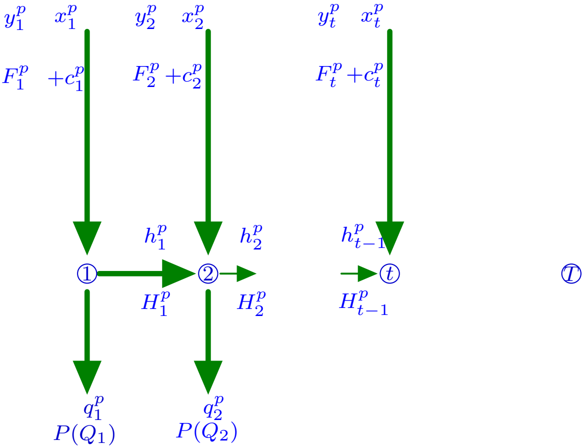
Lot Sizing Game: Model



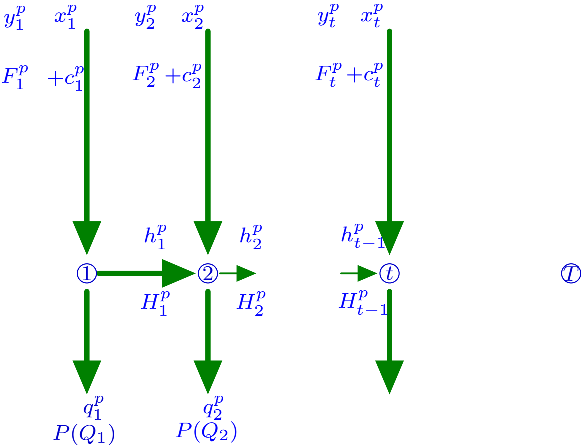
Lot Sizing Game: Model



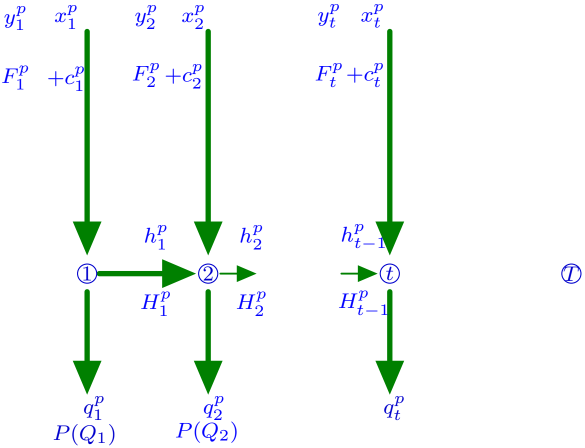
Lot Sizing Game: Model



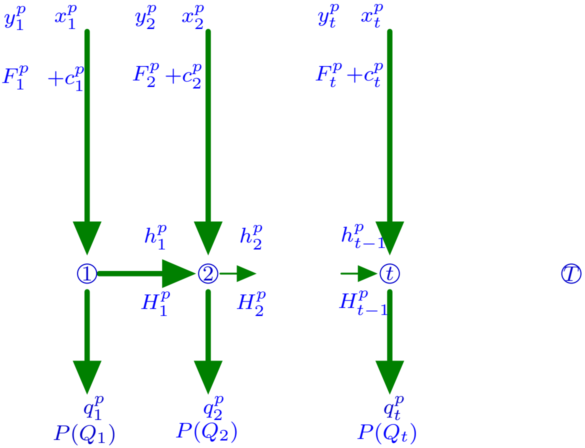
Lot Sizing Game: Model



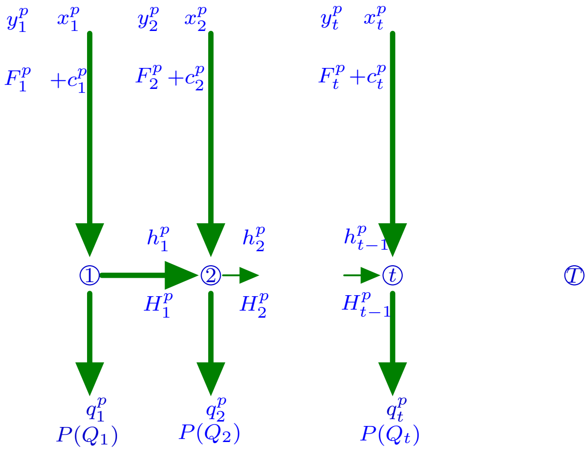
Lot Sizing Game: Model



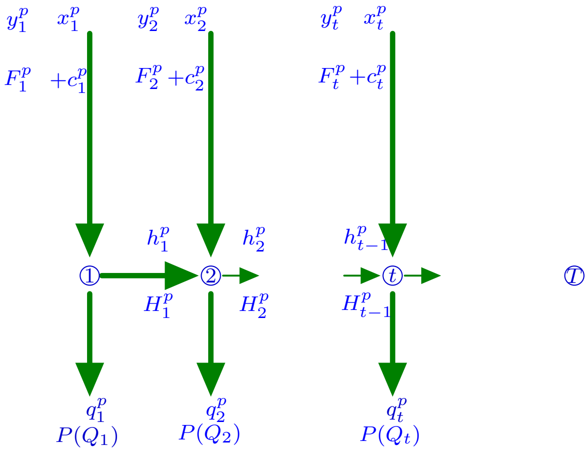
Lot Sizing Game: Model



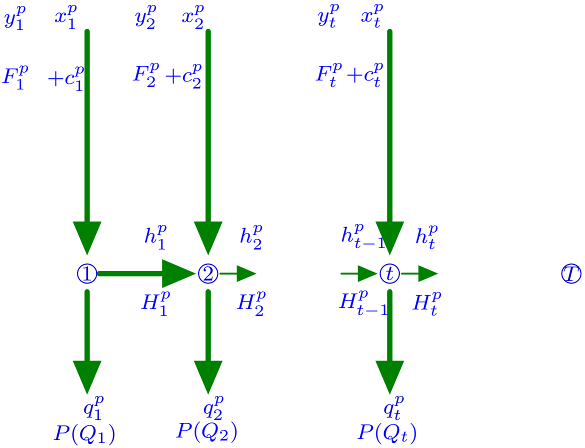
Lot Sizing Game: Model



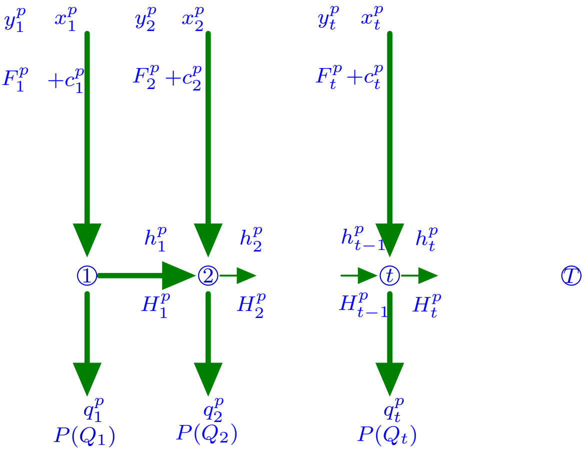
Lot Sizing Game: Model



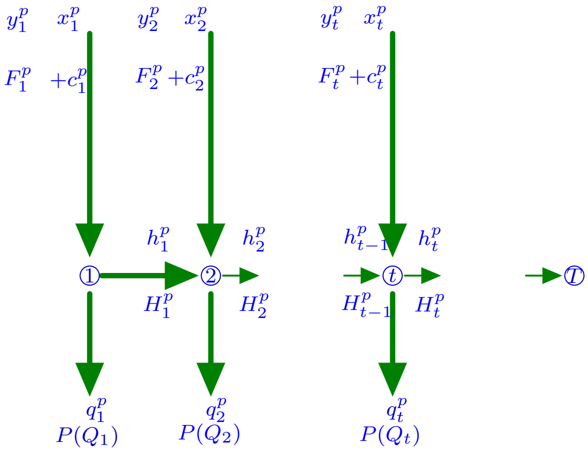
Lot Sizing Game: Model



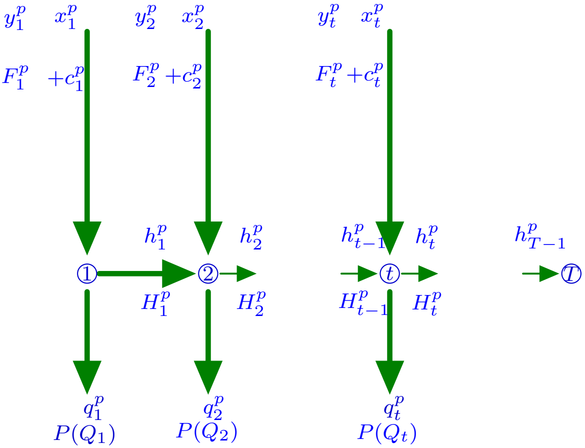
Lot Sizing Game: Model



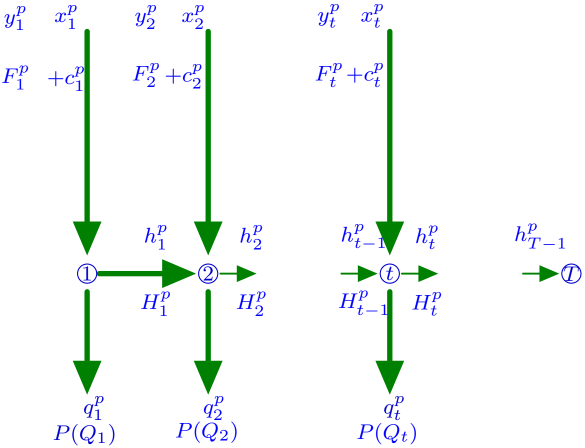
Lot Sizing Game: Model



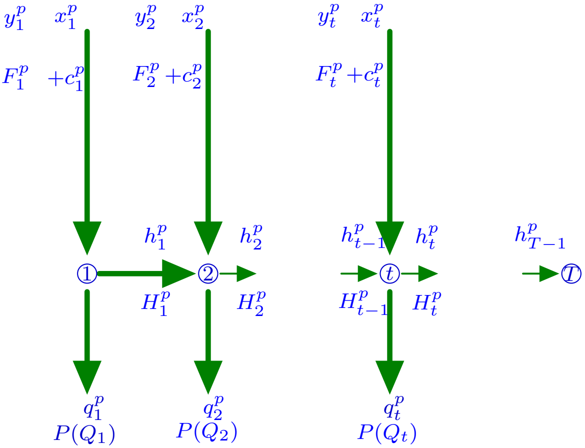
Lot Sizing Game: Model



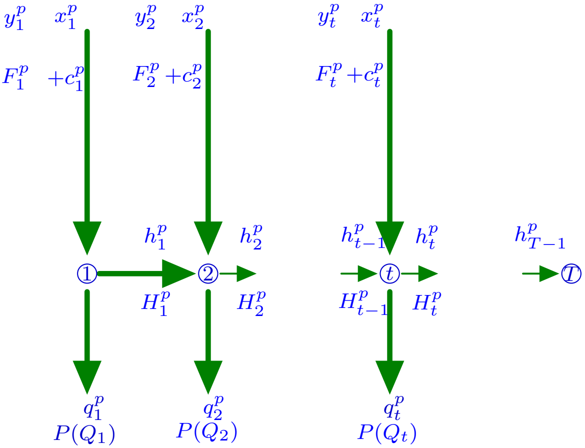
Lot Sizing Game: Model



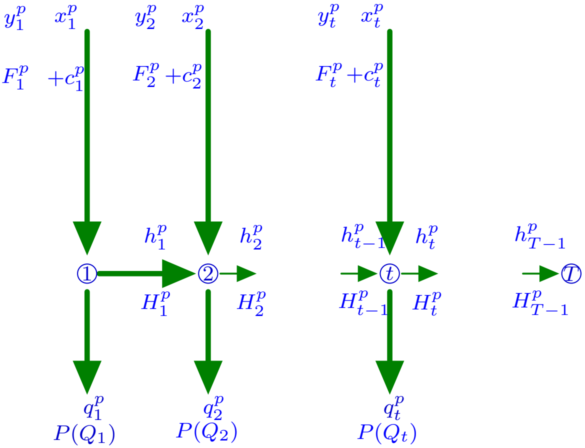
Lot Sizing Game: Model



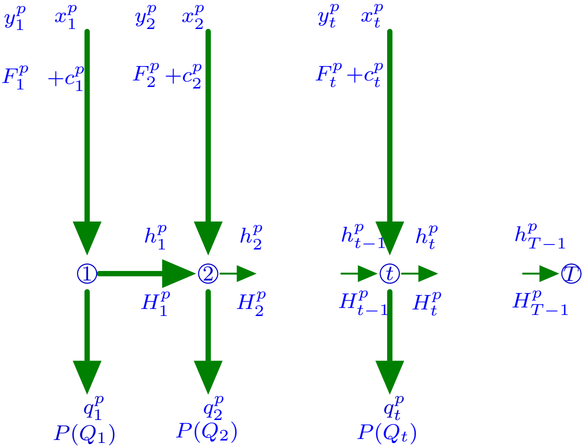
Lot Sizing Game: Model



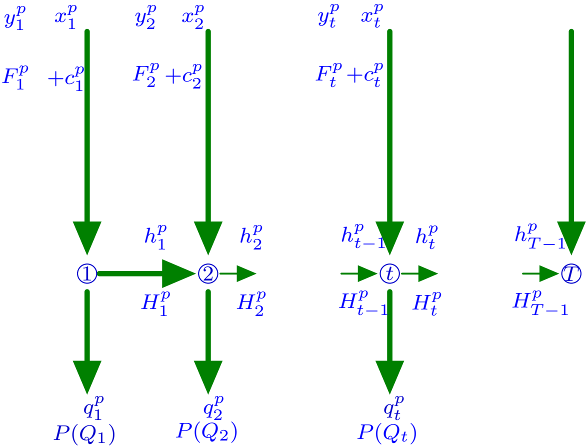
Lot Sizing Game: Model



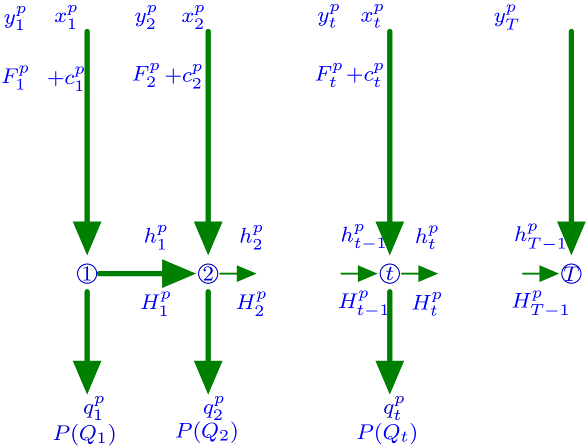
Lot Sizing Game: Model



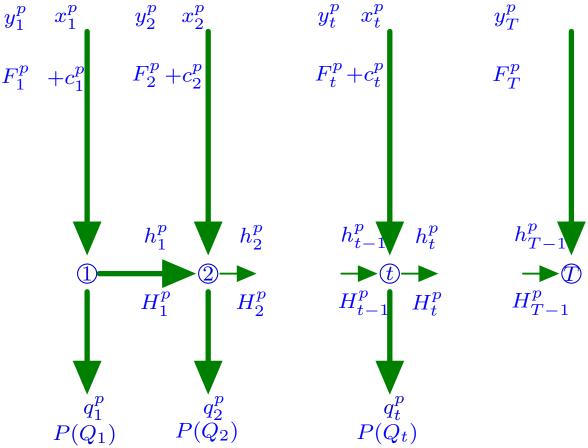
Lot Sizing Game: Model



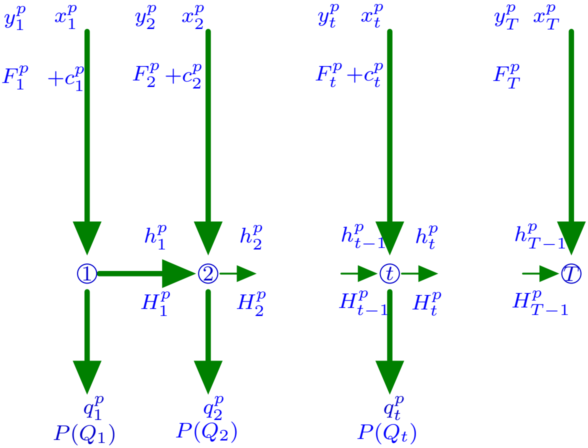
Lot Sizing Game: Model



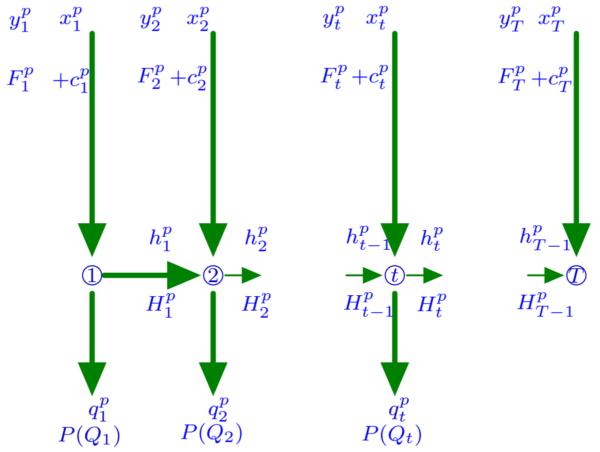
Lot Sizing Game: Model



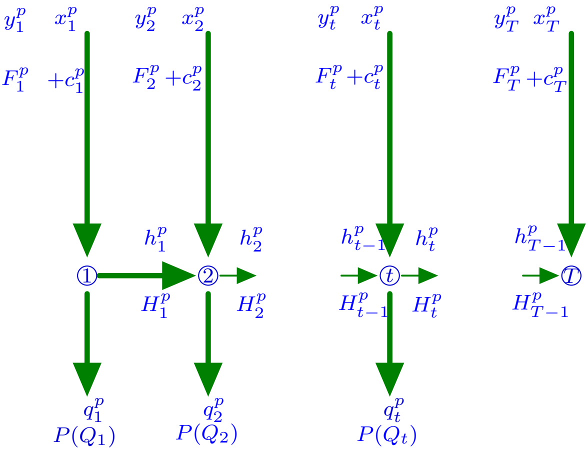
Lot Sizing Game: Model



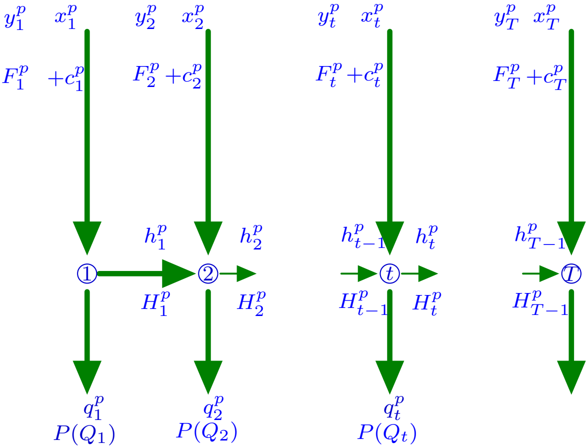
Lot Sizing Game: Model



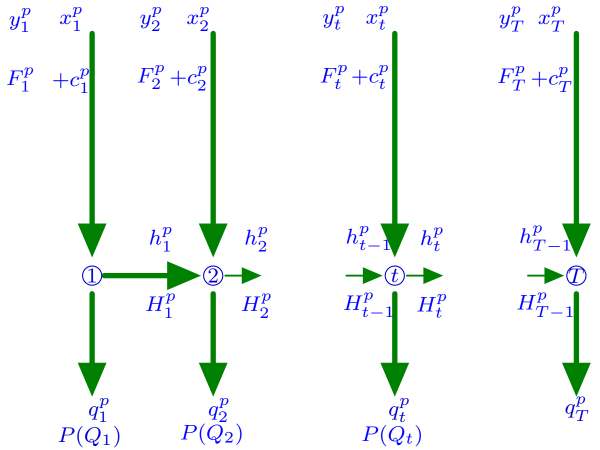
Lot Sizing Game: Model



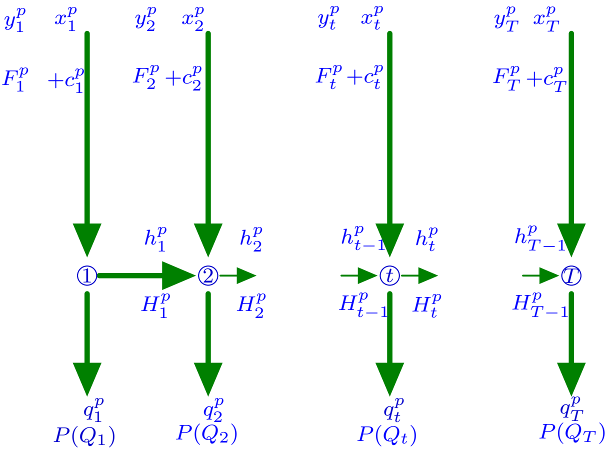
Lot Sizing Game: Model



Lot Sizing Game: Model

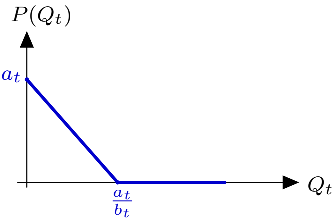


Lot Sizing Game: Model



Lot Sizing Game: Model

$P(Q_t) = \max(a_t - b_t Q_t, 0)$ with $Q_t = \sum_{t=1}^{m} q_t^p$



Lot Sizing Game: Formulation

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \quad \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i$$

$$\begin{aligned}
\text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i && \text{for } t = 1, \ldots, T \\
& 0 \leq x_t^i \leq M y_t^i && \text{for } t = 1, \ldots, T \\
& h_0^i = h_T^i = 0 && \\
& y_t^i \in \{0, 1\} && \text{for } t = 1, \ldots, T
\end{aligned}$$

Lot Sizing Game: Formulation

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\begin{aligned}
\max_{y^i, x^i, q^i, h^i} \quad & \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i \\
\text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i \qquad \text{for } t = 1, \ldots, T \\
& 0 \leq x_t^i \leq M y_t^i \qquad \text{for } t = 1, \ldots, T \\
& h_0^i = h_T^i = 0 \\
& y_t^i \in \{0, 1\} \qquad \text{for } t = 1, \ldots, T
\end{aligned}$$

Lot Sizing Game: Formulation

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i$$

$$\begin{aligned}
\text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i && \text{for } t = 1, \ldots, T \\
& 0 \leq x_t^i \leq M y_t^i && \text{for } t = 1, \ldots, T \\
& h_0^i = h_T^i = 0 && \\
& y_t^i \in \{0, 1\} && \text{for } t = 1, \ldots, T
\end{aligned}$$

Lot Sizing Game: Formulation

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\begin{aligned}
\max_{y^i, x^i, q^i, h^i} \quad & \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i \\
\text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i \qquad \text{for } t = 1, \ldots, T \\
& 0 \leq x_t^i \leq M y_t^i \qquad \text{for } t = 1, \ldots, T \\
& h_0^i = h_T^i = 0 \\
& y_t^i \in \{0, 1\} \qquad \text{for } t = 1, \ldots, T
\end{aligned}$$

Lot Sizing Game: Formulation

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \quad \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i$$

$$\begin{aligned} \text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i && \text{for } t = 1, \ldots, T \\ & 0 \leq x_t^i \leq M y_t^i && \text{for } t = 1, \ldots, T \\ & h_0^i = h_T^i = 0 \\ & y_t^i \in \{0, 1\} && \text{for } t = 1, \ldots, T \end{aligned}$$

Nash Equilibrium

Definition

A Nash equilibrium (in pure strategies) is a vector of feasible strategies $\left(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m\right)$, such that for $i = 1, 2 \ldots, m$:

$$\Pi^i\left(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, \overline{y}^i, \overline{x}^i, \overline{q}^i, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m\right) \geq \Pi^i\left(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, y^i, x^i, q^i, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m\right)$$

$\forall (y^i, x^i, q^i)$ feasible

Nash Equilibrium

Definition

A Nash equilibrium (in pure strategies) is a vector of feasible strategies $(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m)$, such that for $i = 1, 2 \ldots, m$:

$$\Pi^i \left(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, \overline{y}^i, \overline{x}^i, \overline{q}^i, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m\right) \geq \Pi^i \left(\overline{y}^1, \overline{x}^1, \overline{q}^1, \ldots, y^i, x^i, q^i, \ldots, \overline{y}^m, \overline{x}^m, \overline{q}^m\right)$$

$\forall (y^i, x^i, q^i)$ feasible

In a Nash equilibrium no player has incentive to unilaterally deviate.

Lot Sizing Game: should it be reformulated?

Each player $i = 1, 2, \dots, m$ solves the following parametric programming optimization problem

$$\begin{aligned}
\max_{y^i, x^i, q^i, h^i} \quad & \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i \\
\text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i \qquad \text{for } t = 1, \dots, T \\
& 0 \leq x_t^i \leq M y_t^i \qquad \text{for } t = 1, \dots, T \\
& h_0^i = h_T^i = 0 \\
& y_t^i \in \{0, 1\} \qquad \text{for } t = 1, \dots, T
\end{aligned}$$

Lot Sizing Game: should it be reformulated?

Each player $i = 1, 2, \dots, m$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i$$

subject to $(y_1^i, x_1^i, q_1^i, h_1^i) \in X_1$

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=2}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=2}^{T} F_t^i y_t^i - \sum_{t=2}^{T} H_t^i h_t^i - \sum_{t=2}^{T} C_t^i x_t^i$$

subject to $(y_2^i, x_2^i, q_2^i, h_2^i) \in X_2$

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=3}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=3}^{T} F_t^i y_t^i - \sum_{t=3}^{T} H_t^i h_t^i - \sum_{t=3}^{T} C_t^i x_t^i$$

subject to $(y_3^i, x_3^i, q_3^i, h_3^i) \in X_3$

$$\ddots$$

$$\max_{y^i, x^i, q^i, h^i} \max(a_T - b_T \sum_{j=1}^{m} q_T^j, 0) q_T^i - F_T^i y_T^i - H_T^i h_T^i - C_T^i x_T^i$$

subject to $(y_T^i, x_T^i, q_T^i, h_T^i) \in X_T$

Lot Sizing Game: should it be reformulated?

Each player $i = 1, 2, \ldots, m$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=1}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i - \sum_{t=1}^{T} H_t^i h_t^i - \sum_{t=1}^{T} C_t^i x_t^i$$

subject to $(y_1^i, x_1^i, q_1^i, h_1^i) \in X_1$

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=2}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=2}^{T} F_t^i y_t^i - \sum_{t=2}^{T} H_t^i h_t^i - \sum_{t=2}^{T} C_t^i x_t^i$$

subject to $(y_2^i, x_2^i, q_2^i, h_2^i) \in X_2$

$$\max_{y^i, x^i, q^i, h^i} \sum_{t=3}^{T} \max(a_t - b_t \sum_{j=1}^{m} q_t^j, 0) q_t^i - \sum_{t=3}^{T} F_t^i y_t^i - \sum_{t=3}^{T} H_t^i h_t^i - \sum_{t=3}^{T} C_t^i x_t^i$$

subject to $(y_3^i, x_3^i, q_3^i, h_3^i) \in X_3$

$$\ddots$$

$$\max_{y^i, x^i, q^i, h^i} \max(a_T - b_T \sum_{j=1}^{m} q_T^j, 0) q_T^i - F_T^i y_T^i - H_T^i h_T^i - C_T^i x_T^i$$

subject to $(y_T^i, x_T^i, q_T^i, h_T^i) \in X_T$

In order to compute Nash equilibria the multilevel optimization problem can be relaxed leading to a one level optimization programming one.

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Each player i solves the following parametric programming optimization problem

$$\max_{x^i} \quad \Pi^i(x^i, \sum_{j=1}^{m} x^j) = \max(a - b\sum_{j=1}^{m} x^j, 0)x^i - x^i c^i \tag{4a}$$

$$\text{subject to} \quad x^i \geq 0 \qquad \text{for } i = 1, \ldots, m \tag{4b}$$

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Each player i solves the following parametric programming optimization problem

$$\max_{x^i} \quad \Pi^i(x^i, \sum_{j=1}^{m} x^j) = \max(a - b\sum_{j=1}^{m} x^j, 0)x^i - x^i c^i \tag{4a}$$

$$\text{subject to} \quad x^i \geq 0 \qquad \text{for } i = 1, \ldots, m \tag{4b}$$

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

Optimal quantity to be placed in the market by player $i \in S$ is

$$\frac{\partial \Pi^i}{\partial x^i} = a - 2bx^i - b \sum_{j \in S - \{i\}} x^j - c^i = 0 \Leftrightarrow x^i = \frac{a - b \sum_{j \in S - \{i\}} x^j - c^i}{2b}.$$

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

$$x^i = \frac{p(S) - c^i}{b} \quad \forall i \in S \tag{5a}$$

$$x^i = 0 \quad \forall i \notin S. \tag{5b}$$

where $p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S+1|}$ is the average of the numbers $a, \{c^j\}_{j \in S}$.

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Let $S \subseteq \{1, 2, \dots, m\}$ be a subset of players producing a strictly positive quantity.

$$x^i = \frac{p(S) - c^i}{b} \qquad \forall i \in S \tag{5a}$$

$$x^i = 0 \qquad \forall i \notin S. \tag{5b}$$

where $p(S) \equiv \frac{a+\sum_{j \in S} c^j}{|S+1|}$ is the average of the numbers $a, \{c^j\}_{j \in S}$.

$p(S)$ is the resulting **market price** and the total quantity placed in the market is $\sum_i x_i = \frac{a-p(S)}{b}$.

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Using the Nash equilibrium conditions we get

m-Player Lot Sizing Game

INSTANCE Positive integers a, b, c^1, c^2, ..., c^{m-1} and c^m.

QUESTION Is there a subset S of $\{1, 2, \ldots, m\}$ such that

$$p(S) > c^k \qquad \forall k \in S \tag{6a}$$

$$p(S) \leq c^k \qquad \forall k \notin S. \tag{6b}$$

where $p(S) \equiv \frac{a+\sum_{j \in S} c^j}{|S|+1}$.

Uncapacitated One Period Lot Sizing Game: m-Players and No Fixed Cost

Using the Nash equilibrium conditions we get

m-Player Lot Sizing Game

INSTANCE Positive integers a, b, c^1, c^2, ..., c^{m-1} and c^m.

QUESTION Is there a subset S of $\{1, 2, \ldots, m\}$ such that

$$p(S) > c^k \qquad \forall k \in S \tag{6a}$$

$$p(S) \leq c^k \qquad \forall k \notin S. \tag{6b}$$

where $p(S) \equiv \frac{a+\sum_{j \in S} c^j}{|S|+1}$.

There is always exactly one NE and we can find it in $O(m)$ time (assuming c^i are sorted).

m-Players and Fixed and Production Costs

Each player i solves the following parametric programming optimization problem

$$\max_{y^i, x^i} \quad \Pi^i(x^i, \sum_{j=1}^{m} x^j) = \quad \max(a - b\sum_{j=1}^{m} x^j, 0)x^i - F^i y^i - c^i x^i$$

$$\text{subject to} \quad 0 \leq x^i \leq M y^i \qquad \text{for } i = 1, \ldots, m$$

$$y^i \in \{0, 1\} \qquad \text{for } i = 1, \ldots, m$$

m-Players and Fixed and Production Costs

Each player i solves the following parametric programming optimization problem

$$\max_{y^i, x^i} \quad \Pi^i(x^i, \sum_{j=1}^{m} x^j) = \quad \max(a - b\sum_{j=1}^{m} x^j, 0)x^i - F^i y^i - c^i x^i$$

$$\text{subject to} \quad 0 \leq x^i \leq M y^i \qquad \text{for } i = 1, \ldots, m$$

$$y^i \in \{0, 1\} \qquad \text{for } i = 1, \ldots, m$$

m-Players and Fixed and Production Costs

Each player i solves the following parametric programming optimization problem

$$\max_{y^i, x^i} \quad \Pi^i(x^i, \sum_{j=1}^{m} x^j) = \quad \max(a - b\sum_{j=1}^{m} x^j, 0)x^i - F^i y^i - c^i x^i$$

$$\text{subject to} \quad 0 \leq x^i \leq My^i \qquad \text{for } i = 1, \ldots, m$$

$$y^i \in \{0, 1\} \qquad \text{for } i = 1, \ldots, m$$

m-Players and Fixed and Production Costs

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

m-Players and Fixed and Production Costs

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

Optimal quantity to be placed in the market by player $i \in S$ is

$$x^i = \frac{(p(S) - c^i)^+}{b}$$

m-Players and Fixed and Production Costs

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

Optimal quantity to be placed in the market by player $i \in S$ is

$$x^i = \frac{(p(S) - c^i)^+}{b}$$

Player $k \in S$ - A player k does not have incentive to stop producing if

$$\frac{(p(S) - c^k)^+}{b}(p(S) - c^k) \geq F^k \Leftrightarrow c^k + \sqrt{F^k b} \leq p(S)$$

m-Players and Fixed and Production Costs

Let $S \subseteq \{1, 2, \ldots, m\}$ be a subset of players producing a strictly positive quantity.

Optimal quantity to be placed in the market by player $i \in S$ is

$$x^i = \frac{(p(S) - c^i)^+}{b}$$

Player $k \in S$ - A player k does not have incentive to stop producing if

$$\frac{(p(S) - c^k)^+}{b}(p(S) - c^k) \geq F^k \Leftrightarrow c^k + \sqrt{F^k b} \leq p(S)$$

Player $k \notin S$ - A player k does not have incentive to start producing if

$$\frac{(p(S) - c^k)}{2b}\frac{(p(S) - c^k)}{2} \leq F^k \Leftrightarrow c^k + 2\sqrt{F^k b} \geq p(S)$$

m-Players and Fixed and Production Costs

Using the Nash equilibrium conditions we get

m-Player Lot Sizing Game with fixed and production costs

INSTANCE Positive integers a, b, c^1, c^2, ..., c^m, F^1, F^2, ..., F^m.

QUESTION Is there a subset S of $\{1, 2, \dots, m\}$ such that

$$c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \tag{8a}$$

$$c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S. \tag{8b}$$

where $p(S) \equiv \frac{a+\sum_{j \in S} c^j}{|S|+1}$

m-Players and Fixed and Production Costs

$$c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S$$
$$c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S.$$

m-Players and Fixed and Production Costs

$$c^k + \sqrt{F^k b} \leq p(S) \ \ \forall k \in S$$
$$c^k + 2\sqrt{F^k b} \geq p(S) \ \ \forall k \notin S.$$

Computation of one Nash equilibrium

Assume that the players are ordered according with $\sqrt{F^1 b} + c^1 \leq \sqrt{F^2 b} + c^2 \leq \ldots \leq \sqrt{F^m b} + c^m$.
Initialize $S \leftarrow \emptyset$
for $1 \leq k \leq m$ **do**
    if $c^k + 2\sqrt{F^k b} < p(S)$ **then**
        $S = S \cup \{k\}$
    else
        if $p(S \cup \{k\}) \geq \sqrt{F^k b} + c^k$ **then**
            Arbitrarily decide to set k in S.
        end if
    end if
end for
return S

m-Players and Fixed and Production Costs

$$c^k + \sqrt{F^k b} \leq p(S) \;\; \forall k \in S$$
$$c^k + 2\sqrt{F^k b} \geq p(S) \;\; \forall k \notin S.$$

Computation of one Nash equilibrium

Assume that the players are ordered according with $\sqrt{F^1 b} + c^1 \leq \sqrt{F^2 b} + c^2 \leq \ldots \leq \sqrt{F^m b} + c^m$.
Initialize $S \leftarrow \emptyset$
for $1 \leq k \leq m$ **do**
    if $c^k + 2\sqrt{F^k b} < p(S)$ **then**
        $S = S \cup \{k\}$
    else
        if $p(S \cup \{k\}) \geq \sqrt{F^k b} + c^k$ **then**
            Arbitrarily decide to set k in S.
        end if
    end if
end for
return S

- The algorithm implies that there is always (at least) one NE.

m-Players and Fixed and Production Costs

$$c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S$$
$$c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S.$$

Computation of one Nash equilibrium

```
Assume that the players are ordered according with √(F^1 b) + c^1 ≤ √(F^2 b) + c^2 ≤ ... ≤ √(F^m b) + c^m.
Initialize S ← ∅
for 1 ≤ k ≤ m do
    if c^k + 2√(F^k b) < p(S) then
        S = S ∪ {k}
    else
        if p(S ∪ {k}) ≥ √(F^k b) + c^k then
            Arbitrarily decide to set k in S.
        end if
    end if
end for
return S
```

- The algorithm implies that there is always (at least) one NE.
- Consider ans instance with $c^i = 0$ and $F^i = F$ for $i = 1, \ldots, m$. Any set S of cardinality $\lceil a/(2\sqrt{Fb}) \rceil - 1$ is a NE.

m-Players and Fixed and Production Costs: Nash equilibria refinements

m-Player Lot Sizing Game with fixed and production costs: Optimization

INSTANCE Positive integers a, b, and integer vectors $c, F, p \in \mathbb{Z}^m$.

QUESTION Find a subset S of $\{1, 2, \ldots, m\}$ maximizing $\sum_{i \in S} p_i$ such that

$$c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \tag{9a}$$

$$c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S. \tag{9b}$$

where $p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1}$

Example of a refinement: Compute a NE with the minimum or the maximum market price, largest number of players producing,...

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Idea: dynamic programming

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \le p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \ge p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$H(k, l, r, s, C)-$ optimal cost of the problem limited to $\{1, 2, \ldots, k\}$

$$\begin{aligned} & |S| = l \\ & L_r - \text{ the tightest lower bound} \\ & U_s - \text{ the tightest upper bound} \\ & \sum_{i \in S} c^i = C. \end{aligned}$$

Nash equilibria refinements

Goal

$$
\begin{aligned}
\max \quad & \sum_{i \in S} p_i \\
\text{s. t.} \quad & c^k + \sqrt{F^k b} \le p(S) \quad \forall k \in S \\
& c^k + 2\sqrt{F^k b} \ge p(S) \quad \forall k \notin S \\
& p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1}
\end{aligned}
$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$H(k, l, r, s, C)-$ optimal cost of the problem limited to $\{1, 2, \ldots, k\}$

$|S| = l$

$L_r -$ the tightest lower bound

$U_s -$ the tightest upper bound

$$\sum_{i \in S} c^i = C.$$

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$H(k, l, r, s, C)-$ optimal cost of the problem limited to $\{1, 2, \ldots, k\}$

$|S| = l$

$L_r -$ the tightest lower bound

$U_s -$ the tightest upper bound

$$\sum_{i \in S} c^i = C.$$

Nash equilibria refinements

Goal

$$
\begin{aligned}
\max \quad & \sum_{i \in S} p_i \\
\text{s. t.} \quad & c^k + \sqrt{F^k b} \le p(S) \quad \forall k \in S \\
& c^k + 2\sqrt{F^k b} \ge p(S) \quad \forall k \notin S \\
& p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1}
\end{aligned}
$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$H(k, l, r, s, C)-$ optimal cost of the problem limited to $\{1, 2, \ldots, k\}$

$|S| = l$

$L_r -$ the tightest lower bound

$U_s -$ the tightest upper bound

$\sum_{i \in S} c^i = C.$

Nash equilibria refinements

Goal

$$
\begin{aligned}
\max \quad & \sum_{i\in S} p_i \\
\text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\
& c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\
& p(S) \equiv \frac{a + \sum_{j\in S} c^j}{|S|+1}
\end{aligned}
$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \dots, m$$

$$
\begin{aligned}
H(k, l, r, s, C) - & \text{ optimal cost of the problem limited to } \{1, 2, \dots, k\} \\
& |S| = l \\
& L_r - \text{ the tightest lower bound} \\
& U_s - \text{ the tightest upper bound} \\
& \sum_{i\in S} c^i = C.
\end{aligned}
$$

Nash equilibria refinements

Goal

$$\begin{aligned} \max \quad & \sum_{i \in S} p_i \\ \text{s. t.} \quad & c^k + \sqrt{F^k b} \leq p(S) \quad \forall k \in S \\ & c^k + 2\sqrt{F^k b} \geq p(S) \quad \forall k \notin S \\ & p(S) \equiv \frac{a + \sum_{j \in S} c^j}{|S| + 1} \end{aligned}$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$$\begin{aligned} H(k, l, r, s, C) - & \text{ optimal cost of the problem limited to } \{1, 2, \ldots, k\} \\ & |S| = l \\ & L_r - \text{ the tightest lower bound} \\ & U_s - \text{ the tightest upper bound} \\ & \sum_{i \in S} c^i = C. \end{aligned}$$

1: Initialize $H(\cdot) \leftarrow -\infty$ but $H(0, 0, 0, 0, 0) = 0$.
2: **for** $k = 0 \to m - 1;\ l, r, s = 0 \to k; C = 0 \to \sum_i c^i$ **do**
3: $\quad H(k + 1, l + 1, \arg\max_{i=k+1,r} L_i, s, C + c^{k+1}) = H(k, l, r, s, C) + p^{k+1}$
4: $\quad H(k + 1, l, r, \arg\min_{i=k+1,s} U_i, C) = H(k, l, r, s, C)$
5: **end for**
6: **return** $\arg\max_{l,r,s,C}\{H(m, l, r, s, C) | L_r \leq \frac{a+C}{l+1} \leq U_s\}$.

Nash equilibria refinements

Goal

$$
\begin{array}{lll}
\max & \sum_{i \in S} p_i & \\
\text{s. t.} & c^k + \sqrt{F^k b} \leq p(S) & \forall k \in S \\
& c^k + 2\sqrt{F^k b} \geq p(S) & \forall k \notin S \\
& p(S) \equiv \dfrac{a + \sum_{j \in S} c^j}{|S|+1} &
\end{array}
$$

Idea: dynamic programming

$$L_k = \sqrt{F^k b} + c^k \text{ and } U_k = 2\sqrt{F^k b} + c^k \text{ for } k = 1, 2, \ldots, m$$

$$
\begin{aligned}
H(k,l,r,s,C) - \ & \text{optimal cost of the problem limited to } \{1,2,\ldots,k\} \\
& |S| = l \\
& L_r - \text{ the tightest lower bound} \\
& U_s - \text{ the tightest upper bound} \\
& \sum_{i \in S} c^i = C.
\end{aligned}
$$

Initialize $H(\cdot) \leftarrow -\infty$ but $H(0,0,0,0,0) = 0$.
for $k = 0 \rightarrow m-1$; $l, r, s = 0 \rightarrow k$; $C = 0 \rightarrow \sum_i c^i$ **do**
    $H(k+1, l+1, \arg\max_{i=k+1,r} L_i, s, C + c^{k+1}) = H(k,l,r,s,C) + p^{k+1}$
    $H(k+1, l, r, \arg\min_{i=k+1,s} U_i, C) = H(k,l,r,s,C)$
end for
return $\arg\max_{l,r,s,C}\{H(m,l,r,s,C) | L_r \leq \frac{a+C}{l+1} \leq U_s\}$.

We can solve this problem in $\mathcal{O}(m^4 \lceil \sum_i c^i \rceil)$ time by dynamic programming.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Each player $i = 1, 2$ solves the following parametric programming optimization problem

$$\max_{y^i, x^i, q^i, h^i} \quad \Pi^i(y^i, x^i, q^i, h^i) = \sum_{t=1}^{T} \max(a_t - b_t(q_t^1 + q_t^2), 0) q_t^i - \sum_{t=1}^{T} F_t^i y_t^i$$

$$\begin{aligned} \text{subject to} \quad & x_t^i + h_{t-1}^i = h_t^i + q_t^i && \text{for } t = 1, \ldots, T \\ & 0 \leq x_t^i \leq M y_t^i && \text{for } t = 1, \ldots, T \\ & h_0^i = h_T^i = 0 && \\ & y_t^i \in \{0, 1\} && \text{for } t = 1, \ldots, T \end{aligned}$$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma

There is always a Player 1's best reaction to a Player 2's strategy q^2 in which production takes place only once.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma

There is always a Player 1's best reaction to a Player 2's strategy q^2 in which production takes place only once.

Proof.

Assume that given Player 2's strategy q^2 the best reaction of Player 1 involves producing in periods $1 \leq t_1 < t_2 < \ldots < t_k \leq T$ with $k \geq 2$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma

There is always a Player 1's best reaction to a Player 2's strategy q^2 in which production takes place only once.

Proof.

Assume that given Player 2's strategy q^2 the best reaction of Player 1 involves producing in periods $1 \leq t_1 < t_2 < \ldots < t_k \leq T$ with $k \geq 2$.
Let (q^1, h^1, x^1, y^1) be the associated Player 1's strategy. Then, Player 1's profit is

$$\sum_{t=t_1}^{T} \max(a_t - b_t(q_t^2 + q_t^1, 0)q_t^1 - F_{t_1} - F_{t_2} - \ldots - F_{t_k}.$$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma

There is always a Player 1's best reaction to a Player 2's strategy q^2 in which production takes place only once.

Proof.

Assume that given Player 2's strategy q^2 the best reaction of Player 1 involves producing in periods $1 \leq t_1 < t_2 < \ldots < t_k \leq T$ with $k \geq 2$.
Let (q^1, h^1, x^1, y^1) be the associated Player 1's strategy. Then, Player 1's profit is

$$\sum_{t=t_1}^{T} \max(a_t - b_t(q_t^2 + q_t^1, 0)q_t^1 - F_{t_1} - F_{t_2} - \ldots - F_{t_k}.$$

However, Player 1 can maintain or increase her profit by producing only at t_1 the quantity $x_{t_1}^1 + x_{t_1}^1 + \ldots + x_{t_k}^1$. □

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma

Consider that Player 1 only produces at $1 \leq t_1 \leq T$ *and Player 2 only at* $1 \leq t_2 \leq T$. *Then, Player 1 optimal strategy is*

$$
\begin{aligned}
q_t^1 &= 0 && \textit{for } t \in 1, 2, \ldots, t_1 - 1 \\
q_t^1 &= \frac{a_t}{2b_t} && \textit{for } t \in t_1, \ldots, t_2 - 1, \quad \textit{if } \min(t_1, t_2) = t_1 \\
q_t^1 &= \frac{a_t}{3b_t} && \textit{for } t \in \max(t_1, t_2), \ldots, T \\
x_t^1 &= 0 && \textit{for } t \neq t_1 \\
x_{t_1}^1 &= \sum_{t=t_1}^{T} q_t^1
\end{aligned}
$$

Analogous for Player 2.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

All pure Nash equilibria can be computed in $O(T^2)$ *time.*

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

All pure Nash equilibria can be computed in $O(T^2)$ time.

Proof.

Each player has $T+1$ strategies to consider. There are $(T+1)^2$ combinations of strategies to check the Nash equilibria conditions. □

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

All pure Nash equilibria can be computed in $O(T^2)$ *time.*

Proof.

Each player has $T+1$ strategies to consider. There are $(T+1)^2$ combinations of strategies to check the Nash equilibria conditions. □

The computational time can be improved!.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

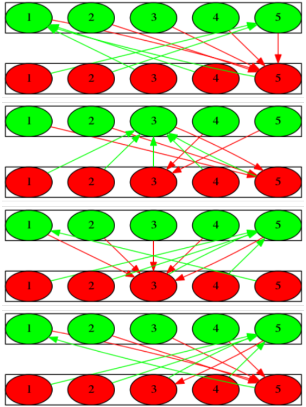
Definition

$t^{R_p}(t)$ is Player p's best time to produce when her rival produces at time t.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Definition

$t^{R_p}(t)$ is Player p's best time to produce when her rival produces at time t.



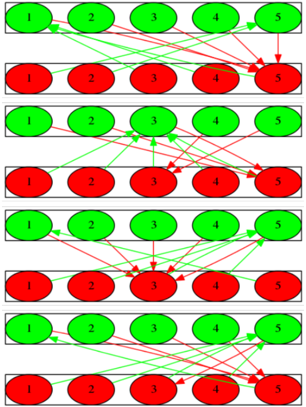
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Definition

$t^{R_p}(t)$ is Player p's best time to produce when her rival produces at time t.

Lemma

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \textit{for } p = 1, 2.$$



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T + 1\}$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \dots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \dots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t_2', t_1') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. Then, these arcs cross. The symmetric result also holds.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t'_2, t'_1) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t'_1 = t^{R_1}(t'_2)$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \dots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \dots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t_2', t_1') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t_1' = t^{R_1}(t_2')$.
Assume $t_2 < t_2'$,

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t'_2, t'_1) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t'_1 = t^{R_1}(t'_2)$.
Assume $t_2 < t'_2$,
then $t^{R_1}(t_2) > t^{R_1}(t'_2)$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

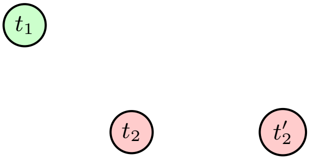
$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t_2', t_1') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t_1' = t^{R_1}(t_2')$.
Assume $t_2 < t_2'$,
then $t^{R_1}(t_2) > t^{R_1}(t_2')$.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

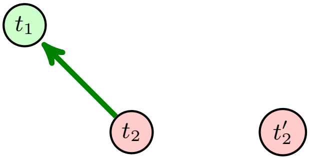
$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t_2', t_1') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t_1' = t^{R_1}(t_2')$.
Assume $t_2 < t_2'$,
then $t^{R_1}(t_2) > t^{R_1}(t_2')$.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

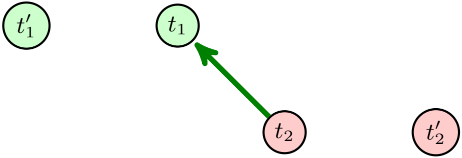
$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t_2', t_1') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t_1' = t^{R_1}(t_2')$.
Assume $t_2 < t_2'$,
then $t^{R_1}(t_2) > t^{R_1}(t_2')$.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Consider the *time reaction graph* G^R:

- Bipartite graph: $R_2 = R_1 = \{1, 2, \ldots, T+1\}$.
- (i, j) is an arc of G^R if $t^{R_1}(i) = j$ or $t^{R_2}(i) = j$.

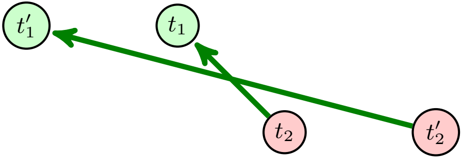
$$t^{R_p}(T+1) \leq t^{R_p}(T) \leq \ldots t^{R_p}(1) \quad \text{for } p = 1, 2.$$

Traduces in

Lemma (Property 1)

Let (t_2, t_1) and (t'_2, t'_1) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. Then, these arcs cross. The symmetric result also holds.

Idea: $t_1 = t^{R_1}(t_2)$ and $t'_1 = t^{R_1}(t'_2)$.
Assume $t_2 < t'_2$,
then $t^{R_1}(t_2) > t^{R_1}(t'_2)$.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 2)

A cycle of length two in G^R *represents a Nash Equilibrium.*

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 2)

A cycle of length two in G^R represents a Nash Equilibrium.

Cycle of length two (t_1, t_2, t_1).

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 2)

A cycle of length two in G^R represents a Nash Equilibrium.

Cycle of length two (t_1, t_2, t_1).

Then $t^{R_2}(t_1) = t_2$,

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 2)

A cycle of length two in G^R represents a Nash Equilibrium.

Cycle of length two (t_1, t_2, t_1).

Then $t^{R_2}(t_1) = t_2$, $t^{R_1}(t_2) = t_1$.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 2)

A cycle of length two in G^R represents a Nash Equilibrium.

Cycle of length two (t_1, t_2, t_1).

Then $t^{R_2}(t_1) = t_2$, $t^{R_1}(t_2) = t_1$.

No player has incentive to unilaterally deviate from the profile of strategies (t_1, t_2).

T-Periods Lot Sizing Game with Fixed Costs: duopoly

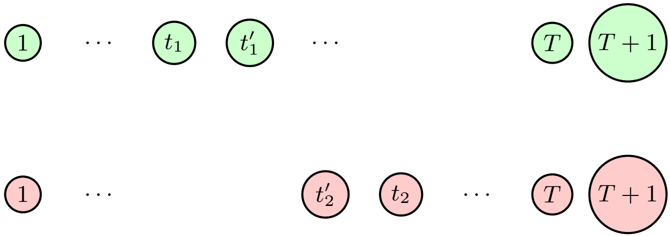
Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

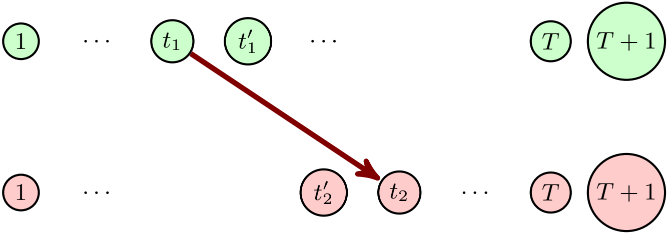
Let (t_1, t_2), (t_2, t'_1) *and* (t'_1, t'_2) *be arcs of* G^R *with* $t_2, t'_2 \in R_2$ *and* $t_1, t'_1 \in R_1$. *If* $t_1 \leq t'_1 \leq t'_2 \leq t_2$ *then,* (t'_1, t_2) *is a NE.*



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

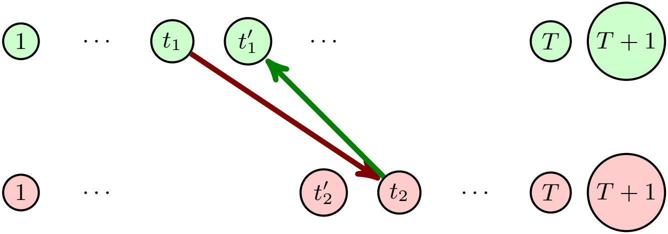


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

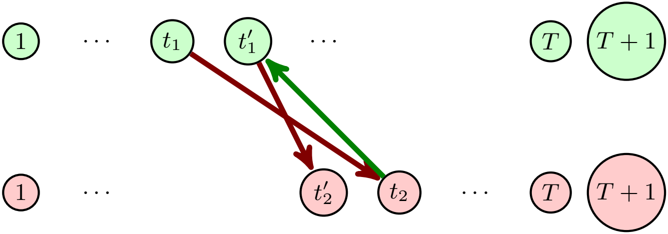


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t_1') and (t_1', t_2') be arcs of G^R with $t_2, t_2' \in R_2$ and $t_1, t_1' \in R_1$. If $t_1 \leq t_1' \leq t_2' \leq t_2$ then, (t_1', t_2) is a NE.

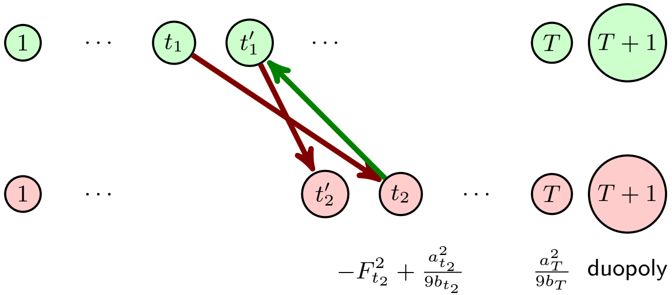


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t_2')$ and $\Pi^2(t_1', t_2') \geq \Pi^2(t_1', t_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

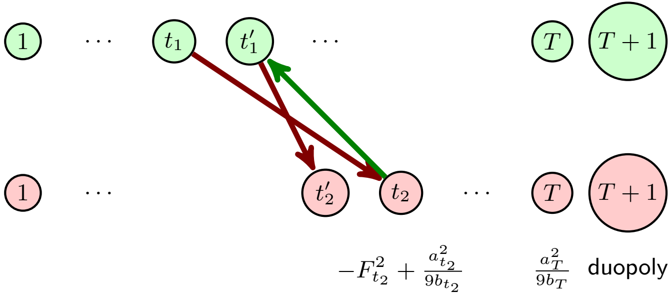


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

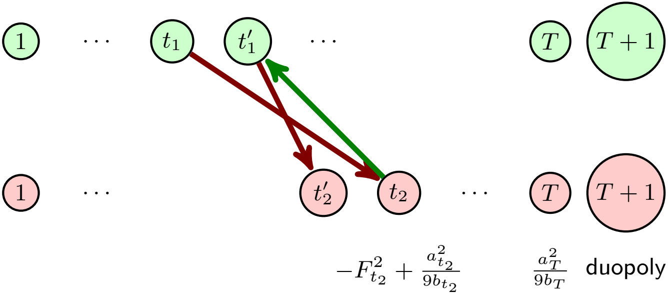


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$
$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

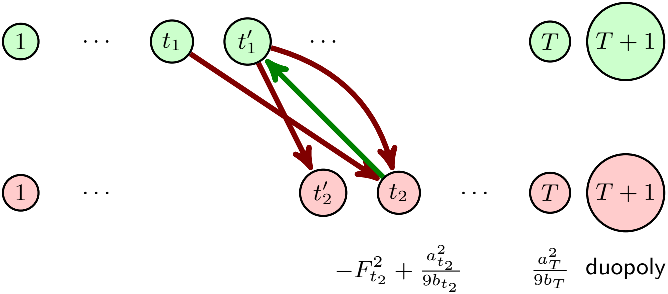


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$
$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$
$\Rightarrow \Pi^2(t_1, t_2) = \Pi^2(t_1, t'_2) = \Pi^2(t'_1, t'_2) = \Pi^2(t'_1, t_2) \Rightarrow t^R_2(t_1) = t^{R_2}(t'_1) = t_2$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) *and* (t'_1, t'_2) *be arcs of* G^R *with* $t_2, t'_2 \in R_2$ *and* $t_1, t'_1 \in R_1$. *If* $t_1 \leq t'_1 \leq t'_2 \leq t_2$ *then,* (t'_1, t_2) *is a NE.*

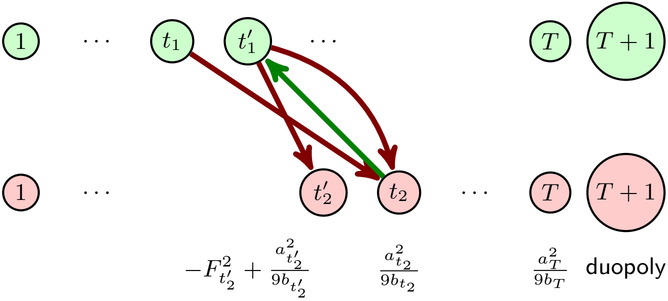


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$
$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$
$\Rightarrow \Pi^2(t_1, t_2) = \Pi^2(t_1, t'_2) = \Pi^2(t'_1, t'_2) = \Pi^2(t'_1, t_2) \Rightarrow t^R_2(t_1) = t^{R_2}(t'_1) = t_2 \Rightarrow$ NE: (t'_1, t_2)

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

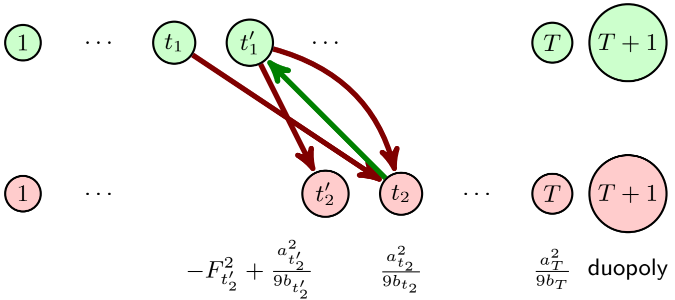


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$
$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$
$\Rightarrow \Pi^2(t_1, t_2) = \Pi^2(t_1, t'_2) = \Pi^2(t'_1, t'_2) = \Pi^2(t'_1, t_2) \Rightarrow t_2^R(t_1) = t^{R_2}(t'_1) = t_2 \Rightarrow$ NE: (t'_1, t_2)
$\Pi^2(t'_1, t'_2) = \Pi^2(t_1, t'_2)$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) and (t'_1, t'_2) be arcs of G^R with $t_2, t'_2 \in R_2$ and $t_1, t'_1 \in R_1$. If $t_1 \leq t'_1 \leq t'_2 \leq t_2$ then, (t'_1, t_2) is a NE.

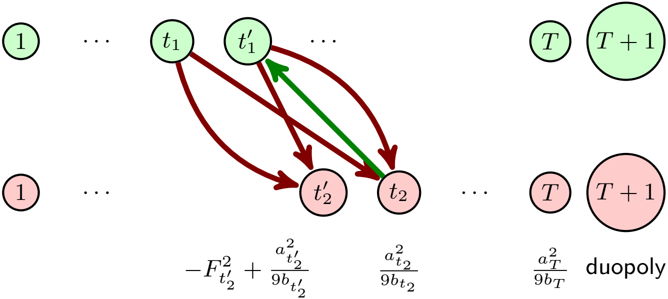


$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$
$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$
$\Rightarrow \Pi^2(t_1, t_2) = \Pi^2(t_1, t'_2) = \Pi^2(t'_1, t'_2) = \Pi^2(t'_1, t_2) \Rightarrow t_2^R(t_1) = t^{R_2}(t'_1) = t_2 \Rightarrow$ NE: (t'_1, t_2)
$\Pi^2(t'_1, t'_2) = \Pi^2(t_1, t'_2) \Rightarrow t_2^R(t'_1) = t^{R_2}(t_1) = t'_2$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Lemma (Property 3)

Let (t_1, t_2), (t_2, t'_1) *and* (t'_1, t'_2) *be arcs of* G^R *with* $t_2, t'_2 \in R_2$ *and* $t_1, t'_1 \in R_1$. *If* $t_1 \leq t'_1 \leq t'_2 \leq t_2$ *then,* (t'_1, t_2) *is a NE.*



$\Pi^2(t_1, t_2) \geq \Pi^2(t_1, t'_2)$ and $\Pi^2(t'_1, t'_2) \geq \Pi^2(t'_1, t_2)$

$\Pi^2(t_1, t_2) = \Pi^2(t'_1, t_2)$

$\Rightarrow \Pi^2(t_1, t_2) = \Pi^2(t_1, t'_2) = \Pi^2(t'_1, t'_2) = \Pi^2(t'_1, t_2) \Rightarrow t_2^R(t_1) = t^{R_2}(t'_1) = t_2 \Rightarrow$ NE: (t'_1, t_2)

$\Pi^2(t'_1, t'_2) = \Pi^2(t_1, t'_2) \Rightarrow t_2^R(t'_1) = t^{R_2}(t_1) = t'_2$

T-Periods Lot Sizing Game with Fixed Costs: duopoly

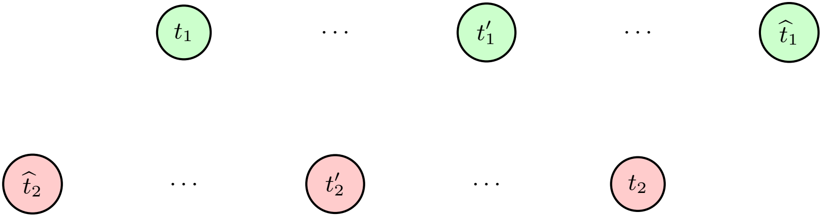
Corollary

A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

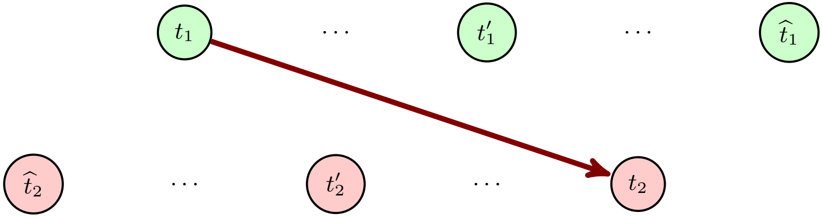
A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

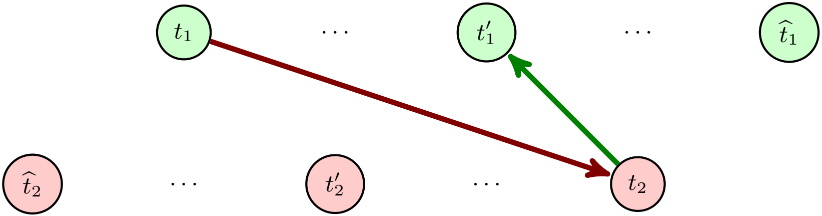
A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

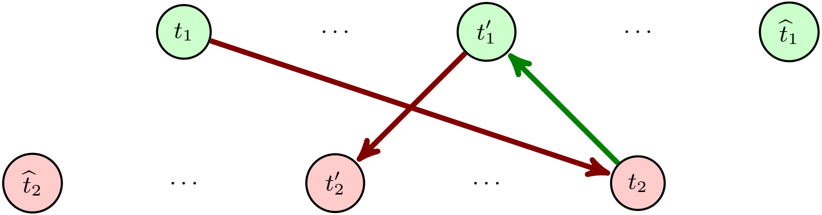
A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

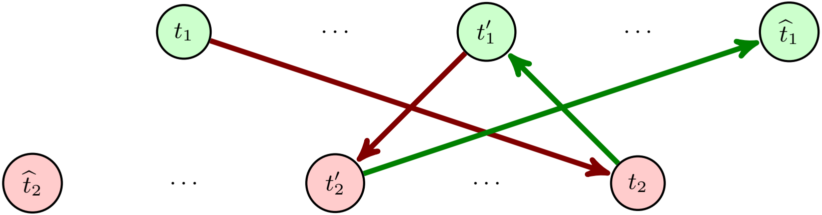
A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

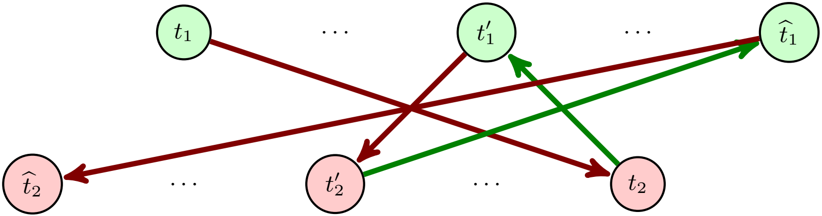
A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

Corollary

A Nash equilibrium is found after following at most a path of length 5 in G^R. In particular, there is always a Nash equilibrium.



T-Periods Lot Sizing Game with Fixed Costs: duopoly

```
Terminal
In [51]: T= 10; RandomInstance(T)
t_R1 =  [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]
t_R2 =  [7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1]
Nash equilibria:  [(3, 7)]

In [52]: T= 10; RandomInstance(T)
t_R1 =  [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
t_R2 =  [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]
Nash equilibria:  [(1, 2)]

In [53]: T= 10; RandomInstance(T)
t_R1 =  [4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4]
t_R2 =  [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]
Nash equilibria:  [(4, 3)]

In [54]: T= 10; RandomInstance(T)
t_R1 =  [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
t_R2 =  [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]
Nash equilibria:  [(1, 2)]

In [55]: T= 10; RandomInstance(T)
t_R1 =  [7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1]
t_R2 =  [4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
Nash equilibria:  [(1, 4), (7, 1)]

In [56]: T= 10; RandomInstance(T)
t_R1 =  [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]
t_R2 =  [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]
Nash equilibria:  [(2, 2)]

In [57]: T= 10; RandomInstance(T)
t_R1 =  [3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3]
t_R2 =  [2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2]
Nash equilibria:  [(3, 2)]
```

T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \ldots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*

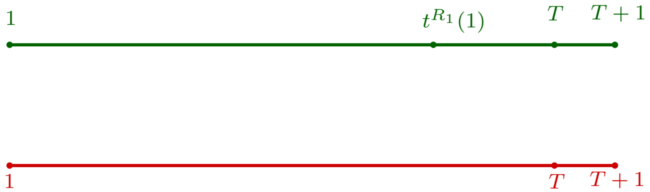
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \ldots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*



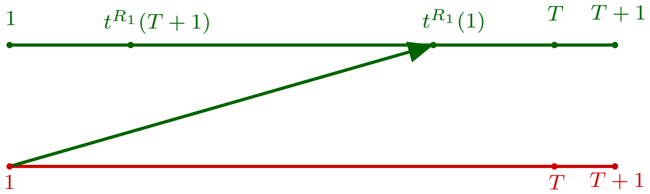
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \dots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*



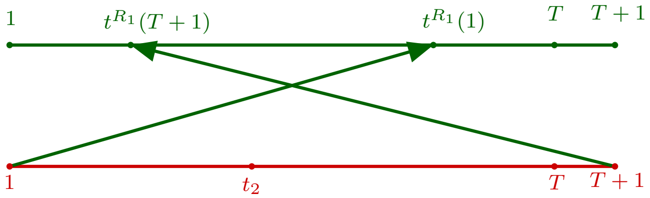
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \ldots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*



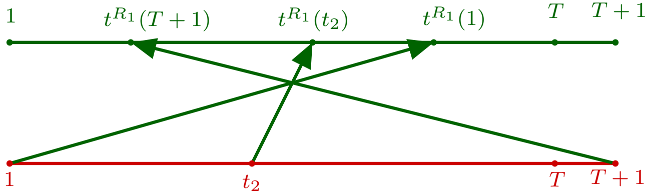
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \dots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*



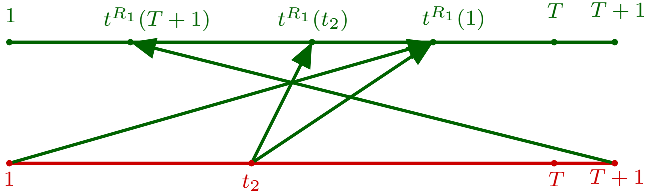
T-Periods Lot Sizing Game with Fixed Costs: duopoly

Theorem

For $p = 1, 2$

$$t^{R_p}(t) \in \{t^{R_p}(T+1), t^{R_p}(1)\} \qquad \forall t \in \{1, 2, \ldots, T, T+1\}.$$

Moreover, $(t^{R_1}(1), t^{R_2}(T+1))$ *and* $(t^{R_1}(T+1), t^{R_2}(1))$ *are the only candidates to be Nash equilibria.*



T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

- All pure Nash equilibria can be computed in polynomial time for a fixed number of players, more precisely, in $O(T^m)$ time. *Idea: Each player only has to decide one period to produce.*

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

- All pure Nash equilibria can be computed in polynomial time for a fixed number of players, more precisely, in $O(T^m)$ time. *Idea: Each player only has to decide one period to produce.*
- All pure Nash equilibria can be computed in polynomial time for a fixed number of periods, more precisely, in $O(m^T)$ time. *Idea:*
  - *Define S_i as the set of players producing in period i.*

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

- All pure Nash equilibria can be computed in polynomial time for a fixed number of players, more precisely, in $O(T^m)$ time.
  Idea: Each player only has to decide one period to produce.
- All pure Nash equilibria can be computed in polynomial time for a fixed number of periods, more precisely, in $O(m^T)$ time.
  Idea:
  - *Define S_i as the set of players producing in period i.*
  - *For the point of view of player k, only the fixed costs of k and the sizes of the sets S_i matter.*

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

- All pure Nash equilibria can be computed in polynomial time for a fixed number of players, more precisely, in $O(T^m)$ time.
  Idea: Each player only has to decide one period to produce.
- All pure Nash equilibria can be computed in polynomial time for a fixed number of periods, more precisely, in $O(m^T)$ time.
  Idea:
  - *Define S_i as the set of players producing in period i.*
  - *For the point of view of player k, only the fixed costs of k and the sizes of the sets S_i matter.*
  - *We can enumerate all possible sizes for these partitions: $O(m^T)$ time.*

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

It is easy to generalize the previous ideas for $m > 2$.

- All pure Nash equilibria can be computed in polynomial time for a fixed number of players, more precisely, in $O(T^m)$ time. *Idea: Each player only has to decide one period to produce.*
- All pure Nash equilibria can be computed in polynomial time for a fixed number of periods, more precisely, in $O(m^T)$ time. *Idea:*
  - *Define S_i as the set of players producing in period i.*
  - *For the point of view of player k, only the fixed costs of k and the sizes of the sets S_i matter.*
  - *We can enumerate all possible sizes for these partitions: $O(m^T)$ time.*
  - *Once these sizes are fixed, assigning the players to the sets S_i is easy - a transportation problem.*

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

Theorem

For $p = 1, 2, \ldots, m$ *and for all feasible partitions* $S_{-p} = (|S_1|, |S_2|, \ldots, |S_T|)$ *of the set of all players except* p*:*

$$t^{R_p}(S_{-p}) \in \{t^{R_p}(0,0,\ldots,0), t^{R_p}(1,0,\ldots,0), \ldots, t^{R_p}(m-1,0.\ldots,0)\}.$$

T-Periods Lot Sizing Game with Fixed Costs: oligopoly

Theorem

For $p = 1, 2, \ldots, m$ and for all feasible partitions $S_{-p} = (|S_1|, |S_2|, \ldots, |S_T|)$ of the set of all players except p:

$$t^{R_p}(S_{-p}) \in \{t^{R_p}(0, 0, \ldots, 0), t^{R_p}(1, 0, \ldots, 0), \ldots, t^{R_p}(m-1, 0. \ldots, 0)\}.$$

There are m^m candidates to be Nash equilibria...

Conclusion and Future work

1-Period Lot sizing game:

Conclusion and Future work

1-Period Lot sizing game:

★ Existence of a pure Nash equilibrium.

Conclusion and Future work

1-Period Lot sizing game:

★ Existence of a pure Nash equilibrium.

★ A pure Nash equilibrium can be computed in polynomial time.

Conclusion and Future work

1-Period Lot sizing game:

★ Existence of a pure Nash equilibrium.

★ A pure Nash equilibrium can be computed in polynomial time.

★ Current work: Is it "NP-complete" to compute a Nash equilibrium optimizing a certain function? What is the economic meaning of $c^k + \sqrt{F^k b}$?

Conclusion and Future work

1-Period Lot sizing game:

★ Existence of a pure Nash equilibrium.

★ A pure Nash equilibrium can be computed in polynomial time.

★ Current work: Is it "NP-complete" to compute a Nash equilibrium optimizing a certain function? What is the economic meaning of $c^k + \sqrt{F^k b}$?

T-Period Lot sizing game with fixed costs:

★ Computation in polynomial time of all equilibria for the 2-players game.

Conclusion and Future work

1-Period Lot sizing game:

★ Existence of a pure Nash equilibrium.

★ A pure Nash equilibrium can be computed in polynomial time.

★ Current work: Is it "NP-complete" to compute a Nash equilibrium optimizing a certain function? What is the economic meaning of $c^k + \sqrt{F^k b}$?

T-Period Lot sizing game with fixed costs:

★ Computation in polynomial time of all equilibria for the 2-players game.

★ Current work: Can we compute in polynomial time (on the number of players and number of periods) a Nash equilibrium?

Acknowledgments

I am grateful to GEMAC - Gabinete de Estatística, Modelação e Aplicações Computacionais - for supporting my participation in 5th Porto Meeting on MATHEMATICS for INDUSTRY.