Tree Search for the Recursive Circle Packing Problem

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Outline

Next up. . .

Origin of the problem: industrial setting

- A company produces hollow tubes of various radii
- Orders are sent to customers in containers
- All tubes have the length of a container
- How should the containers be loaded?

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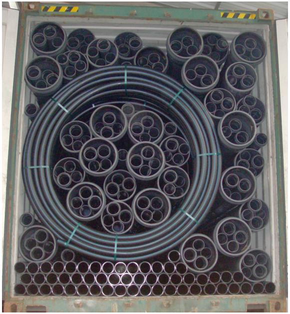

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- Currently: solution is constructed... MANUALLY (what?!? O_o)
- Very tedious and error prone
- Production engineers' time is expensive
- We can surely do better

Problem description (simplified)

Given:

- a set of tubes, where each tube is characterized by
  - external radius
  - internal radius
  - value
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Solution:

- a list of packed tubes and their positions

A mathematical model: parameters and variables

Parameters for describing an instance

- width W and height H of the rectangular container;
- set of tubes $\mathcal{N}$
- for each tube $i \in \mathcal{N}$:
  - external radius r_i^{ext}
  - internal radius r_i^{int}
  - value v_i

Variables

- $(x_i, y_i) \in \mathbb{R}^2$, position of the center of each tube i
- $p_i \in \{0, 1\}, p_i = 1$ if tube i is placed **directly** in the container
- $q_{ij} \in \{0, 1\}, q_{ij} = 1$ if tube i is placed **directly** inside tube j

A mathematical model: objective function

$$\text{maximize } V = \sum_{i \in \mathcal{N}} v_i \times \left(p_i + \sum_{j \in \mathcal{N}} q_{ij} \right)$$

A mathematical model: constraints

Tubes cannot be in multiple places

$$p_i + \sum_j q_{ij} \leq 1, \quad \forall i \in \mathcal{N}$$

Tubes must stay inside the container

$$r_i^{\text{ext}} \leq x_i \leq W - r_i^{\text{ext}}, \quad \forall i \in \mathcal{N}$$
$$r_i^{\text{ext}} \leq y_i \leq H - r_i^{\text{ext}}, \quad \forall i \in \mathcal{N}$$

A mathematical model: more constraints

Tubes i and j inside the container cannot overlap

$$\|xy_i - xy_j\|_2 \geq r_i^{\text{ext}} + r_j^{\text{ext}} - M \times (2 - p_i - p_j), \quad \forall i, j \in \mathcal{N}$$

Same goes for tubes i and j inside the same larger tube k

$$\|xy_i - xy_j\|_2 \geq r_i^{\text{ext}} + r_j^{\text{ext}} - M \times (2 - q_{ik} - q_{jk}), \quad \forall i, j, k \in \mathcal{N}$$

Tube i inside j must stay within j

$$\|xy_i - xy_j\|_2 \leq r_j^{\text{int}} - r_i^{\text{ext}} + M \times (1 - q_{ij}), \quad \forall i, j \in \mathcal{N}$$

(M disables constraints when any variable inside parenthesis is zero)

A mathematical model: limitations

- Previous model is exact
- Non-linear
- Very hard to solve
  - Quadratic number of variables
  - Cubic number of constraints
- Let's take a look at some practical solutions

Next up...

A possible greedy construction

```
def greedy_solution(C, T):
    O = {C}
    while O != {}:
        o = argmin(O, key=free_space)
        repeat:
            if o has no positions:
                O.remove(o)
                break

            t = argmax(T, key=(erad, value, irad))
            P = o.positions_for(t)
            p = argmin(P, key=(y_coord, x_coord))

            o.insert(t, p)
            O.add(t)
            T.remove(t)
            o = t

    return C
```

Generating positions

What is the set of positions for a tube?

In reality, this is an usually an infinite set.

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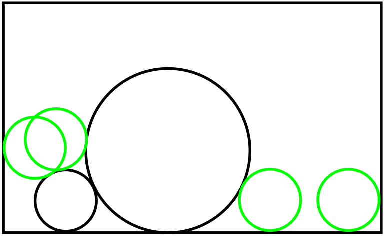
Reduce this to a finite set of possible positions

- the corners of the container
- positions touching another tube and a wall of the container
- positions touching (at least) two other tubes
- the bottom of the outer tube (telescoping; initial tube)

Generating positions



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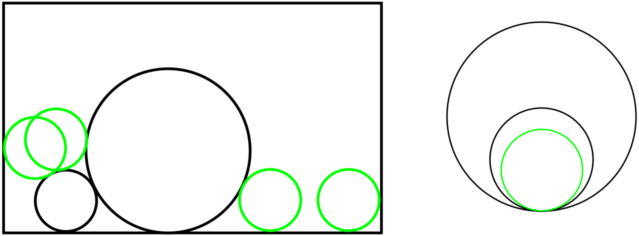

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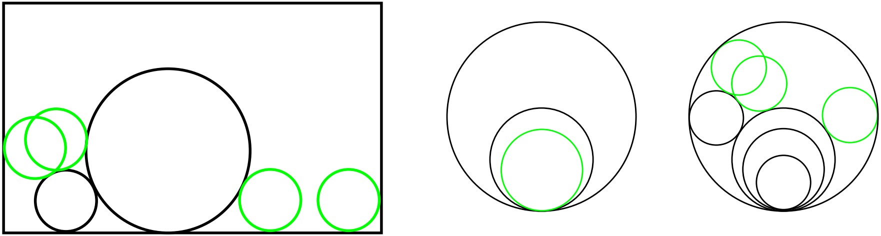

Does this lead to the optimal solution?

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NO

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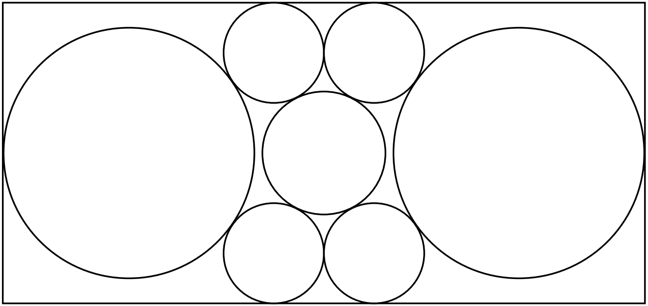

Semi-Greedy construction

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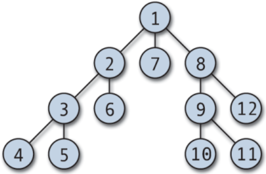
- SG = greedy construction + probabilistic choice + repetition
- Probabilistic choice: position of tube
- Optimization by repetition with different RNG seeds
- Could hardly be simpler

Local Search

- Not easy to define a proper (finite) neighborhood
- Moving one tube will likely cause overlaps
- May not be trivial to restore feasibility

Depth-First Tree Search



- Complete enumeration of the search space (if enough time is allowed)
- Avoids repeated solutions
- Very fast
- Low memory requirements

- First decisions are (in most cases) never changed
- Performance **extremely dependent** on a branch ordering heuristic

Monte-Carlo Tree Search

Some facts

- Tree search algorithm mostly employed in game playing
- Asymmetrically constructs a game tree
- Focuses on most promising* branches
- Uses Monte-Carlo simulations to estimate value of nodes
- Each node maintains basic statistics (# of sims and # of wins)
- Requires little/no domain-specific knowledge, but benefits from it
- An iteration consists of. . .

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  Simulation make a simulation from each new node

  Backpropagation using the results of the simulations, update the statistics on each node in the path up to the root

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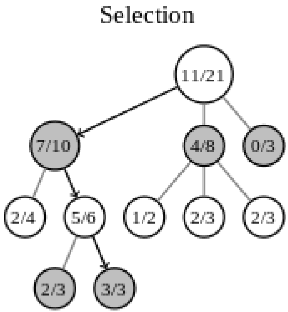

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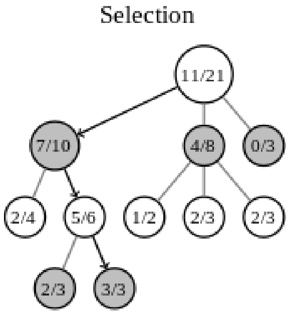

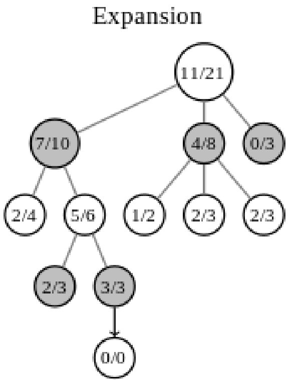

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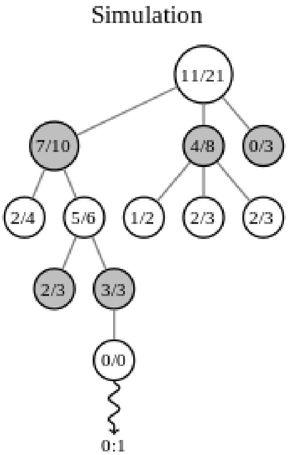

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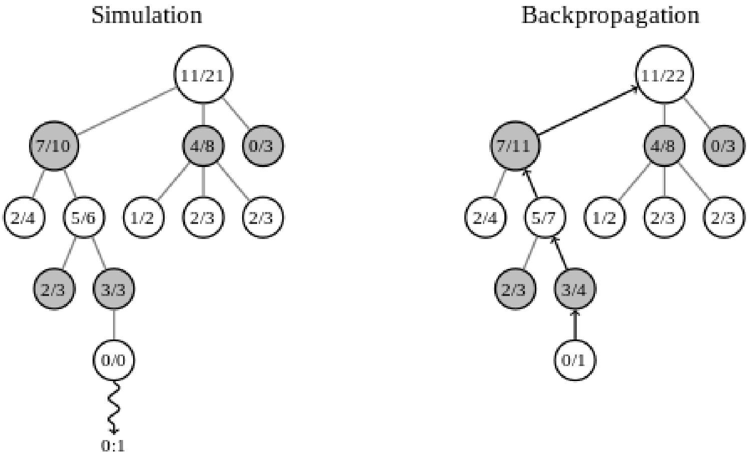

UCT: Upper Confidence Bound 1 applied to Trees

$$UCT_n = X_n + c.\sqrt{\frac{\ln N_{P_n}}{N_n}}$$

- Formula for selecting the "best" child (selection step)
- Most popular variant of MCTS
- UCT formula consists of two components:

  Exploitation prefers nodes with best known values
  Exploration prefers nodes that have few simulations
- X_n is assumed to be in $[0, 1]$

Adapting UCT for optimization

Normalization

$$X_n = \frac{e^{1-\frac{z_n^*-z^*}{w^*-z^*}} - 1}{e-1}$$

- $X_n \in [0, 1]$ ✓ UCT-approved
- uses both z^* and w^* to assess how good a value is
- avoids scale issues with objective function values

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Weighting exploration with $\overline{X_n}$

$$E_n = \overline{X_n}.c.\sqrt{\frac{\ln N_{P_n}}{N_n}}$$

- use mean to help guide the search
- assign less time to branches with worse mean score

Next up. . .

Experiment

- 6 instances
  - 3, 5, and 16 different types of tubes
  - 2 container sizes: large and small (= large/2)
- Competing algorithms

  DFS Depth-First Tree Search
  SG repeated Semi-Greedy construction
  MCTS Monte Carlo Tree Search
- Software implemented in Python 2.X, run in PyPy

Results: large instances

		large03	large05	large16
DFS*		3570033	3720124	18492283
SG**	min	3660028	3810114	22381890
	avg	3660029.3	3822105.7	22548874.4
	max	3660030	3840093	22851844
MCTS**	min	3660029	4050053	24241737
	avg	**3660031.3**	**4098052.3**	**24842685.8**
	max	3660034	4140048	25451624

* result of 1 run of 600s

** results of 10 independent runs of 600s

Results: small instances

		small03	small05	small16
DFS*		900000	1090000	9540056
SG**	min	940000	1090000	9790035
	avg	940000.0	1090000.0	9820032.6
	max	940000	1090000	9840031
MCTS**	min	940000	1120000	10470034
	avg	**956000.1**	**1120000.0**	**10643039.3**
	max	960000	1120000	10700041

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Next up...

Conclusion

Contributions

- Definition of the RCPP
- Non-linear formulation: not usable in practice
- Adaptation of MCTS/UCT
  - make X_n independent of problem scale
  - use mean performance to guide exploration
- Interesting results

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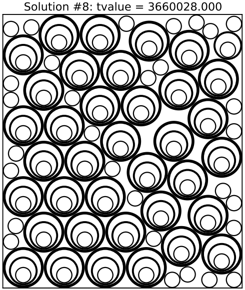
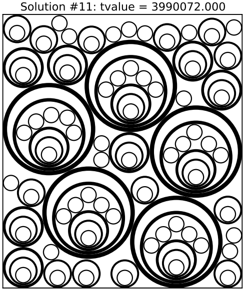
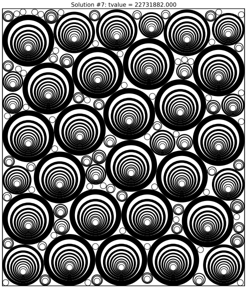
Future work

- Compare with more challenging opponents
- Apply MCTS to other problems, *e.g.*, MIP
- Add some mechanism to discard nodes when the tree grows too large (beam search)

Thank you!


Solution #8: tvalue = 3660028.000


Solution #11: tvalue = 3990072.000


Solution #7: tvalue = 22731882.000