# Unit Commitment and Mixed Integer Programming

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# Outline

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# The Unit Commitment Problem

UC is the problem of scheduling a park of generators to satisfy the demand for electricity over a certain number of periods at minimum cost/maximum profit.

Classically there is a monopolistic producer.
Recently there are markets for electricity, and a Company owning Generators has to sell their generator capacity on one or more Power Exchange Markets. This leads to both the standard UC problem, as well as other variants.

Generators can be Thermal, Hydro or Renewable (Wind/Solar energy).
Markets lead to price uncertainty, and Renewable leads to production uncertainty.

Time Periods: 5-15-60 minutes, Time Horizon 1 day - 1 week.
Restrict attention to Thermal: Gas, Coal and Nuclear.

- commodity is nonstorable and instantly perishable
- demand is uncertain (highly correlated with weather)
- demand must be satisfied instantaneously
- very low price elasticity
- transmission (transportation) system is congestion prone
- flows in transmission lines cannot be directly controlled (Kirchoff's law)

# Basic UC Problem

Generators $i = 1, \ldots, I$ with lower and upper bounds $L^i, C^i$ on production per period

Aggregated demand $d_t$ in period $t = 1, \ldots, T$.

$f_t^i$ fixed cost if generator $i$ is active in period $t$

$q_t^i$ start-up cost if generator $i$ is turned on in period $t$

$c_t^i(x)$ cost of producing $x$ with generator $i$ in $t$

**Constraints**

Demand Satisfaction

Minimum Up/Down times: $\alpha^i, \beta^i$

Reserve Constraints to handle Uncertainty in demand - parameter $\rho$

# Basic Formulation

$x_t^i$ is production of $i$ in $t$,
$y_t^i = 1$ if generator $i$ is producing in $t$, $y_t^i = 0$ otherwise

$$\min \sum_{i,t} \left(c_t^i(x_t^i) + f_t^i y_t^i + q_t^i (y_t^i - y_{t-1}^i)^+\right)$$
$$L_t^i y_t^i \leq x_t^i \leq C_t^i y_t^i \quad \forall\, i,t$$
$$\sum_i x_t^i = d_t \quad \forall\, t$$
$$\sum_i C^i y_t^i \geq \rho d^t \quad \forall\, t$$
$$y_{t+j}^i \geq y_{t+1}^i - y_t^i \; j = 1,\ldots,\alpha^i, \; \forall\, i,t$$
$$y_{t+j}^i - y_{t+j-1}^i \leq 1 - y_t^i \; j = 2,\ldots,\beta^i, \; \forall\, i,t$$
$$x \geq 0, y \in \{0,1\}$$

The production cost function $c_t^i(x)$ is assumed to be
i) linear
ii) piecewise linear convex and non-decreasing
iii) convex quadratic and non-decreasing

Ramping Constraints
The changes in production level from one period to the next are limited
When on, one cannot increase by more than $\Delta^i$ and decrease by more than $\delta^i$.
When switching on, the production cannot exceed $\bar{u}^i$.
When switching off, the production cannot exceed $\bar{l}^i$.

Hot and Cold Starts

# Algorithms? Decomposition Approach

$X$ can be written as

$$X = (\prod_{i=1}^{I} Y^i) \cap (\prod_{t=1}^{n} Z^t)$$

where $Y^i =$

$$L_t^i y_t^i \leq x_t^i \leq C_t^i y_t^i \;\; \forall \, i, t$$
$$y_{t+j}^i \geq y_{t+1}^i - y_t^i \; j = 1, \ldots, \alpha^i, \; \forall \, i, t$$
$$y_{t+j}^i - y_{t+j-1}^i \leq 1 - y_{t+1}^i \; j = 1, \ldots, \alpha^i, \; \forall \, i, t$$
$$x^i \geq 0, y^i \in \{0, 1\}$$

Is it easy to optimize over $Y^i$, or to solve the separation problem for $Y^i$/describe conv$(Y^i)$?

# Algorithms

If OPT($Y^i$) is easy,

Lagrangean Relaxation or Column Generation(Dantzig-Wolfe Decomposition)

If SEP($Y^i$) is easy (or good approximation of convex hull),

MIP with tightened a priori formulation and/or cutting planes

# Dynamic Programming: OPT($Y^i$) is easy

Single Generator $i$. No ramping. Drop index $i$.

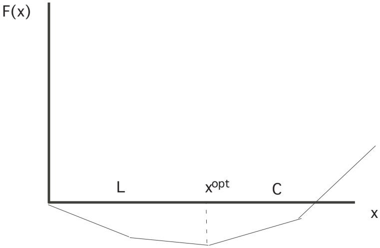
$$\min \sum_t \left(F_t(x_t) + f_t y_t + q_t(y_t - y_{t-1})^+)\right)$$
$$Ly_t \leq x \leq Cy_t \ \forall \ t$$
$$Min\ Up/Down\ Times$$
$$x \geq 0, y \in \{0,1\}$$

# Two Step Optimization: Step 1

Step 1: If $y_t = 1$, then

$$x_t^{opt} = arg\min\{F(x) : L \leq x \leq C\}$$

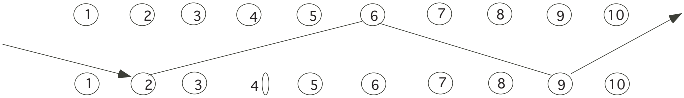
where $F(x) = c(x) - \pi x$ is of the form



# Two Step Optimization: Step 2

If the generator is on from periods $k$ to $t-1$ and off in $k-1$ and $t$ with $t-k \geq \alpha$, then upward arc $(k,t)$ and the associated cost is

$$q_k + \sum_{u=k}^{t-1} (F_u(x_u^{opt}) + f_u).$$




On from periods 2-5. Off in period 1 and periods 6-8.

Cost of upper arc 2-5: $F^2(x^2)$ $+f_2+$ ......$+F_5(x_5)+f_5+g_2$

Suppose that $\alpha = 2$ and $\beta = 3$.

# Lagrangean relaxation

$$\min \sum_{i,t} \left(c_t^i(x_t^i) + f_t^i y_t^i + q_t^i(y_t^i - y_{t-1}^i)^+\right)$$
$$\sum_i x_t^i = d_t \;\; \forall\, t$$
$$\sum_i C^i y_t^i \geq \rho d^t \;\; \forall\, t$$
$$(x^i, y^i) \in Y^i \; \forall\, i$$

Dualizing the demand and reserve constraints gives the Lagrangean dual:

$L(\pi, \mu) =$

$$\min \sum_{i,t} \left(c_t^i(x_t^i) + f_t^i y_t^i + q_t^i(y_t^i - y_{t-1}^i)^+\right) + \sum_t \pi_t(d_t - \sum_i x_t^i) + \sum_t \mu_t(\rho d_t - \sum_i C^i y_t^i)$$
$$(x^i, y^i) \in Y^i \; \forall\, i$$

$$L(\pi,\mu) = \min \sum_{i,t} \left((f_t^i - \mu_t C^i) y_t^i + q_t^i (y_t^i - y_{t-1}^i)^+ + c_t^i (x_t^i) - \pi^i x_t^i\right)$$

$$+ \sum_t (\pi_t d_t + \mu_t \rho d_t) : \qquad (x^i, y^i) \in Y^i \ \forall i.$$

$$L(\pi,\mu) = \sum_i L^i(\pi,\mu) + \sum_t (\pi_t d_t + \mu_t \rho d_t)$$

$$L^i(\pi,\mu) = \min \sum_t \left((f_t^i - \mu_t C^i) y_t^i + q_t^i (y_t^i - y_{t-1}^i)^+ + c_t^i (x_t^i) - \pi^i x_t^i\right)$$

$$(x^i, y^i) \in Y^i.$$

# Lower Bounds from Lagrangean Dual

$$Z \geq L(\pi, \mu) \; \forall \; \pi \in \mathbb{R}^T_+, \mu \in \mathbb{R}^t.$$

Lagrangean Dual

$$W = \max\{L(\pi, \mu) : \pi \in \mathbb{R}^T_+, \mu \in \mathbb{R}^T\}$$

Algorithms to solve Lagrangean Dual i.e. find optimal multipliers $(\pi, \mu)$
Subgradient Algorithms
Bundle Algorithms
Dantzig-Wolfe / Column Generation

# Feasible Solutions and Upper Bounds

Lagrangean relaxation provides solutions $(x, y)$ at each iteration that satisfy the generator constraints, but can violate the demand constraints.
The bundle or column generation algorithms provide convexified solution at each iteration that satisfies all the constraints

Combinatorial Heuristic:
Run at every step of the iterative algorithm

- Fix hydro power (if any) as in convexified solution
- Greedy-type heuristic selecting generators so that sufficient capacity to satisfy remaining demand. Interpret fractional $y$ as probabilities.
- Given $y$ fixed, solve the economic dispatch problem. Large-scale convex problem. Not guaranteed to be feasible.

# Extensions: Ramping and Convex Production Costs

Construct a digraph with source $s$, sink $d$ and nodes $(h,k)$ representing switch-on in $h$ and generator off on $k$
Arc $(h,k)$ to $(r,q)$ if it is feasible to turn on in $r$ after turning off in $k$. Cost that of a start-up in $r$.
Arc $s$ to $(h,k)$ if possible initial state.
Node costs: Solve Economic Dispatch for $(h,k)$

$$z_{hk} = \min \sum_i c^i(x^i_t) : (x^i, y^i, z^i) \in Y^i, y^i_t = 1 \;\; t = h, \ldots, k-1, \text{ else } y^i_t =$$

Then solve min cost path problem. $O(n^2)$ nodes and $O(n^4)$ arcs.
With care, can calculate all $z_{hk}$ and solve shortest path problem in $O(T^3)$.

Table: Solving the Lagrangean Dual: no hydro units

| I | secs | iter | sol | % gap |
|---|---|---|---|---|
| 20 | 8 | 189 | 34 | 0.44 |
| 50 | 17 | 195 | 33 | 0.26 |
| 100 | 46 | 213 | 21 | 0.48 |
| 200 | 134 | 317 | 67 | 0.06 |

# MIP: Tightening Formulations

Min Up-Down Times

$$y^i_{t+j} \geq y^i_{t+1} - y^i_t \; j = 1, \ldots, \alpha^i, \; \forall \, i, t$$
$$y^i_{t+j} - y^i_{t+j-1} \leq 1 - y^i_{t+1} \; j = 1, \ldots, \alpha^i, \; \forall \, i, t$$
$$y^i \in \{0, 1\}$$

With start-up variables:

$$z_t \geq y_t - y_{t-1}, z_t \leq y_t, z_t \leq 1 - y_{t-1} \quad \forall \, t$$
$$\textstyle\sum_{j=1}^{\alpha} z_{t+j} \leq y_{t+\alpha} \quad \forall \, t$$
$$\textstyle\sum_{j=1}^{\beta} z_{t+j} \leq 1 - y_t \quad \forall \, t$$

Without start-up variables: using $y_{j+1} - y_j \leq z_{j+1}$ and $0 \leq z_{j+1}$, we obtain

$$\sum_{j \in S \cap [t+1, t+\alpha]} (y_j - y_{j-1}) \leq y_{t+\alpha}.$$

# Extensions: MIP

Up-Ramping.

$$Ly_t \le x_t \le Cy_t \ \forall\, t$$
$$x_{t+1} - x_t \le \Delta_t y_t + u_{t+1}(1 - y_{t+1}) \ \forall\, t$$
$$y \in \{0,1\}^T$$

Valid inequalities approximating convex hull.

$$x_{t+1} - x_t \le (\bar{u} - L - \Delta) z_{t+1} + (L + \Delta) y_{t+1} - L y_t)$$

$$x_t \le C y_t - (C - \bar{u}) z_t$$

In any extreme point, each $x_t$ takes one of $K$ values $p_1, \ldots, p_K$ s where $K$ is O($T$).
OPT as a shortest path problem with nodes $(t, p_k)$,
Arc variable $\alpha_{j,k,t}$ from $(j, t-1)$ to $(k, t)$ with cost $f_t + c_t(k)$
Arc variable $\beta_{k,t}$ from $(0, t-1)$ to $(k, t)$ with cost $g_t + f_t + c_t(k)$
Extended formulation:

$$x_t = \sum_j k\alpha_{jkt} + \sum_k k\beta_{kt}$$

$$y_t = \sum_j \sum_{k \neq 0} \alpha_{jkt} + \sum_{k \neq 0} \beta_{kt}$$

$$z_t = \sum_{k \neq 0} \beta_{kt}$$

# MIP: Heuristics

Relax and Fix
Local Re-optimization
LP Fixing

# Uncertain Unit Commitment

- Scenario Tree Approaches
  Select a certain number of demand/price scenarios $k = 1, \ldots, K$. Decide generator variables $y, z$, observe outcomes and then decide production levels $x^k, k = 1, \ldots, K$.
  Very large MIPs. Possible too use Benders' decomposition
- Robust Optimization
  Demand/price scenarios lie in a “small" polytope. Solve for all possible “ worst-case" outcomes in the polytope.
- Chance-Constrained Optimization Satisfy “demand constraints" with 99% probability.

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