Vehicle Routing and MIP

Laurence A. Wolsey

CORE, Université Catholique de Louvain

5th Porto Meeting on Mathematics for Industry,
11th April 2014
The Capacitated Vehicle Routing Problem
Subproblems: Trees and the TSP
CVRP
  Cutting Planes
  Column Generation
  Robust Branch-and-Price-and Cut
An Inventory Routing Problem
The Capacitated Vehicle Routing Problem

Laurence A. Wolsey
Vehicle Routing and MIP
Single Depot. Single Period

- $K$ vehicles can make tours beginning and ending at the depot
- Client $i$ has a demand $q_i$ and is served by exactly one vehicle
- Each vehicle has capacity $Q$.
- The objective is to find subtours for the $K$ vehicles such that the amount delivered by each vehicle does not exceed $Q$, the demand of each client is satisfied and the total travel cost/distance is minimized.
Graph $G = (V, E)$ with edge costs $c_e$ for $e \in E$. Let $y_e = 1$ if $e$ is in the tree.

$$\min \sum_{e \in E} c_e y_e$$

$$\sum_{e \in E(S)} y_e \leq |S| - 1 \quad \forall \ S \subset V$$

$$\sum_{e \in E} y_e = |V| - 1$$

$y \in \mathbb{R}^E_+$

where $E(S) = \{e = (i, j) \in E : i, j \in S\}$.

This linear program is tight. The subtour elimination constraints (SECs) describe the convex hull of the spanning trees, but there is an exponential number of constraints.
Given a point $\hat{y} \in \mathbb{R}_+^E$, when does it violate one of the SECs? We formulate this question as an IP.

Let $\alpha_i = 1$ if $i \in S$. We have to check whether there exists $\alpha \in \{0, 1\}^V$ such that

$$\sum_{e=(i,j) \in E} \hat{y}_e \alpha_i \alpha_j - \sum_{i \in V} \alpha_i > -1.$$ 

This is a quadratic 0-1 integer program of the form:

$$\max_{\alpha \in \{0, 1\}^V} \alpha^T Q \alpha - p \alpha.$$ 

It is well known that such IPs can be solved as max flow/min cut problems when $Q \geq 0$. 

Laurence A. Wolsey  
Vehicle Routing and MIP
An Extended “Multicommodity” Formulation

How else can one ensure connectivity?
One way: Construct a graph in which there is a path (one can send a flow) from one node \( (r = 1) \) to all the others. 
\( f_{ij}^k \) is the flow (commodity \( k \)) in arc \( (i, j) \) on the path from \( r \) to \( k \).

\[
\sum_j f_{ij}^k - \sum_j f_{ji}^k = 0 \quad i \neq k, r \\
\sum_j f_{ij}^k - \sum_j f_{ji}^k = 1 \quad i = k \\
f_{ij}^k + f_{ji}^k \leq y_e \quad \forall \ e \in E \\
f_{ij}^k \geq 0 \quad \forall \ i, j, k
\]

This formulation is as strong as that with subtours, but it has \( O(V^3) \) variables.
Symmetric Version $c_{ij} = c_{ji} = c_e$

$y_e = 1$ if edge $e$ in the tour

$$\min \sum_{e \in E} c_e y_e$$

$$\sum_{e \in \delta(v)} y_e = 2 \ \forall \ i \in V$$

$$\sum_{e \in E(S)} y_e \leq |S| - 1 \ \forall \ S \subset V$$

$y \in [0, 1]^E$

SECS can be replaced by “cut” inequalities

$$\sum_{e \in \delta(S)} y_e \geq 2 \ \forall \emptyset \subset S \subset V.$$
Weak Extended Formulation (Directed):

\( u_i \) is position of node \( i \) in the tour starting with node 1 in position 0 (\( u_1 = 0 \)).

\[
    u_j - u_i \geq x_{ij} - (n - 2)(1 - x_{ij}) \forall (i, j), j \neq 1
\]

\[
    \sum_j x_{ij} = 1 \forall i
\]

\[
    \sum_i x_{ij} = 1 \forall j
\]

\[
    y_e = x_{ij} + x_{ji} \forall e
\]

\[
    x \in \mathbb{Z}_+^A, u \in \mathbb{Z}_+^n
\]

Useful if one wants a compact valid formulation.
Formulations of CVRP

\[ x_{ij}^k = 1 \text{ if arc } (i, j) \text{ in tour of vehicle } k \]
\[ z_i^k = 1 \text{ if client } i \text{ visited by vehicle } k \]

\[
\min \sum_k \sum_{ij} c_{ij}^k x_{ij}^k
\]

\[
\sum_i x_{ij}^k = z_j^k \quad \forall \ k, j, j \neq 0,
\]

\[
\sum_j x_{ij}^k = z_i^k \quad \forall \ k, i, i \neq 0,
\]

\[
\sum_j x_{0j}^k = z_0^k \quad \forall \ k
\]

\[
\sum_i d_i z_i^k \leq C z_0^k \quad \forall \ k
\]

\[
\sum_k z_i^k = 1 \quad \forall \ i
\]

\[ x_{ij}^k, z_i^k \in \{0, 1\} \]
Formulations of CVRP with 2 Indices

\[
\begin{align*}
\min \sum_{ij} c_{ij} x_{ij} \\
\sum_{i} x_{ij} &= 1 \quad \forall \ j \neq 0, \\
\sum_{j} x_{ij} &= 1 \quad \forall \ i \neq 0, \\
\sum_{j} x_{0j} &= K \\
\sum_{ij \in \delta(S,S)} x_{ij} &\geq \left\lceil \frac{\sum_{i \in S} d_i}{C} \right\rceil \quad \forall \ S \subseteq V \\
x_{ij} &\in \{0, 1\}
\end{align*}
\]
1. Separation of $\sum_{ij \in \delta(\bar{S}, S)} x_{ij} \geq \lceil \frac{\sum_{i \in S} d_i}{C} \rceil$?

Separation of $\sum_{ij \in \delta(\bar{S}, S)} x_{ij} \geq \frac{\sum_{i \in S} d_i}{C}$ is polynomial.

2. Generalized Large Multistar

$$\sum_{e \in \delta(S)} y_e \geq \frac{2}{C} (d(S) + \sum_{i \notin S} d_j (\sum_{e \in \delta(S:\{j\})} y_e))$$
A natural idea is to have columns corresponding to feasible subtours for a vehicle.

The problem is then just to select a set of $K$ subtours such that each client is visited once.

The trouble with this is that the column generation subproblem is a prize-collecting traveling salesman problem that is almost as hard as the original problem.

So the typical approach is to look at a larger set of columns that includes all the feasible subtours, but for which the column generation problem is more tractable.
A q-route is a walk starting and ending at the depot visiting the clients 0, 1, or more times but with total demand at most $C$. Note that each time a client $i$ is visited, his demand $d_i$ is counted again.

Suppose $n = 4$, $C = 13$ and $d = (2, 4, 5, 7)$.

A feasible subtour is for example $0 \rightarrow 3 \rightarrow 2 \rightarrow 0$ with $\sum_i d_i = 9$.
A q-route, that is not a feasible subtour, is $0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0$ with $\sum_i d_i = 8$.
$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 0$ is a q-route without a 2-cycle as $\sum_i d_i = 13 \leq C$.

A minimum cost q-route without 2-cycles can be found by dynamic programming.
$f(i, j, k)$ is the min cost of a walk starting at the depot and ending with visits to $i$, then $j$ in which $k$ units are delivered.

$$f(i, j, k) = \min_{p: p \neq j} [f(p, i, k - d_j) + \bar{c}_{ij}]$$

Min Cost q-route is

$$\min_{i, j, k \leq C} [f(i, j, k) + \bar{c}_{0j}].$$
Let $q_j^e$ be the coefficient of edge $e$ in $q$-route $j$ with variable $\lambda_j$. One has

\[
\sum_{j=1}^{p} q_j^e \lambda_j - x_e = 0 \quad e \in E
\]

\[
\sum_{j=1}^{p} \lambda_j = K
\]

\[
\sum_{e \in \delta(i)} x_e = 2 \quad i \in V
\]

\[
x_e \geq 0 \quad e \in E
\]

\[
\lambda_j \geq 0 \quad j = 1, \ldots, p
\]
The Master Problem

\[ \sum_{e \in \delta(i)} x_e = 2 \quad i \in V \]

\[ \sum_{e \in \delta(0)} x_e = 2K \]

\[ \sum_{e \in \delta(S)} x_e \geq 2k(S) \quad S \subseteq V \]

\[ x_e \leq 1 \quad e \in E \setminus \delta(0) \]

\[ -x_e = 0 \quad e \in E \]

\[ \sum_{j=1}^{p} q_j^e \lambda_j \]

\[ \sum_{j=1}^{p} \lambda_j = K \]

\[ x_e \geq 0 \quad e \in E \]

\[ \lambda_j \geq 0 \quad j = 1, \ldots, p \]
The Master Problem

$$\begin{align*}
\min & \sum_{j=1}^{p} \sum_{e \in E} l_e q_j^e \lambda_j \\
\mu & \sum_{j=1}^{p} \sum_{e \in \delta(i)} q_j^e \lambda_j = 2 \ i \in V \\
\nu & \sum_{j=1}^{p} \sum_{e \in \delta(0)} q_j^e \lambda_j = 2K \\
\pi & \sum_{j=1}^{p} \sum_{e \in \delta(S)} q_j^e \lambda_j \geq 2k(S) \ S \subseteq V \\
\omega & \sum_{j=1}^{p} q_j^e \lambda_j \leq 1 \ e \in E \setminus \delta(0) \\
\lambda_j & \geq 0 \ j = 1, \ldots, p
\end{align*}$$

Reduced cost of $x_e$: $l_e - \mu_i - \mu_j - \sum_{e \in \delta(S)} \pi_S - \omega_e \ e \in E \setminus \delta(0)$

$l_e - \mu_j - \nu - \sum_{e \in \delta(S)} \pi_S \ e \in \delta(0)$
\( d = (2, 4, 5), C = 10 \)

\[
\begin{array}{c|cccc|c}
 cw = & w_1 & w_2 & w_3 & w_{12} & u \\
\hline
 v = 1 & 2 & 2 & 2 & 2 & = 2 & 11 \\
 v = 2 & 2 & 2 & 2 & 2 & = 2 & 8.5 \\
 v = 3 & 2 & 2 & 2 & 2 & = 2 & 10 \\
 v = 0 & 2 & 2 & 2 & 2 & = 4 \\
 1, 2 & 2 & 2 & 2 & 2 & \geq 2 \\
 1, 3 & 2 & 2 & 2 & 2 & \geq 2 \\
 2, 3 & 2 & 2 & 2 & 2 & \geq 2 \\
 1, 2, 3 & 2 & 2 & 2 & 2 & \geq 4 \\
 e = 12 & 2 & 2 & 2 & 1 & < 1 \\
\end{array}
\]

Laurence A. Wolsey

Vehicle Routing and MIP
min cost q-route: $0 - 2 - 3 - 0$ with reduced cost of -10
alternative q-route: $0 - 1 - 2 - 1 - 0$ with reduced cost of -7

<table>
<thead>
<tr>
<th>$cw$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_{12}$</th>
<th>$w_{23}$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>24</td>
<td>20</td>
<td>39</td>
<td>27</td>
<td></td>
<td>49</td>
</tr>
<tr>
<td>$v = 1$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>=</td>
<td>2</td>
</tr>
<tr>
<td>$v = 2$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>=</td>
<td>2</td>
</tr>
<tr>
<td>$v = 3$</td>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>=</td>
<td>2</td>
</tr>
<tr>
<td>$v = 0$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>=</td>
</tr>
<tr>
<td>1, 2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>≥</td>
<td>2</td>
</tr>
<tr>
<td>1, 3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>≥</td>
<td>2</td>
</tr>
<tr>
<td>2, 3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>≥</td>
<td>2</td>
</tr>
<tr>
<td>1, 2, 3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>≥</td>
<td>4</td>
</tr>
<tr>
<td>$e = 12$</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e = 23$</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Recently $q$-routes replaced by ng-routes
Note that can add constraints $\alpha_e x_e \leq \alpha_0$ and approach still works.
Robust branch-and-cut-and-price.
If cuts on the q-route variables - column generation becomes more difficult

What about time-windows?
Similar BCP approach.
Column generation may be easier - less choice.
Cut generation more difficult - cuts based on infeasibility.
The Inventory Routing Problem: One Period

Laurence A. Wolsey
Vehicle Routing and MIP
The Inventory Routing Problem

Laurence A. Wolsey

Vehicle Routing and MIP
Model 1: Single Period Inventory Routing Problem

\[
\begin{align*}
\sum_{i \in I} x^i & \leq Cz^0, \quad x^i \leq W^i z^i, \quad z^0 \geq z^i, \quad i \in I \\
\sum_{j \in I_0} y^{ij} & = \sum_{j \in I_0} y^{j} = z^i, \quad j \in I_0, \\
\sum_{i \in S \cup \{0\}} \sum_{j \in I \setminus S} y^{ij} & \geq z^i, \quad S \subseteq I,
\end{align*}
\]

**Capacitated Subtours**

\(x, \in R^+_T, z^i \in \{0, 1\} \quad i \in I, \ z^0 \in \{0, 1, \ldots, K\}, \ y^{ij} \in \{0, 1\}^A\)
Where do the capacities come from?

\[ W_i^t = \min\{ C, d_i^t + \bar{S}_i \} \]

What about the capacitated subtours?

\[ C \sum_{i \in I \setminus S} \sum_{j \in S} y_{ij} \geq \sum_{i \in S} x^i \]

where \( S \subseteq I \) and \( I_0 = I \cup \{0\} \).
Direct a flow of $x^k$ from the origin to each client $k \in I$: $f_{ijk}$ is the flow $i (i, j)$ with destination $k \in I$

$$\sum_{i \in I_0} f_{ijk} - \sum_{i \in I} f_{jik} = 0 \quad \forall j \neq k, \forall k$$

$$\sum_{i \in I_0} f_{ijk} = x^k \quad k = j, \forall k$$

$$\sum_{k \in I} f_{ijk} \leq C_{y^i} \quad \forall (i, j)$$

$$f_{ijk} \leq W^k y^i \quad \forall i, j, k$$
Suppose $\mu_{ij} \geq 0$ for all $(i, j) \in A$ and $S \subseteq I$, then

$$\sum_{(i,j) \in A} \mu_{ij} y_{ij} \geq \sum_{i \in S} x^i$$

is a valid inequality if

$$\sum_{(i,j) \in P} \mu_{ij} \geq \min(C, \sum_{i \in v(P) \cap S} W^i)$$

for every subtour $P$ beginning and ending at the depot, where $v(P) \subseteq I$ are the clients on subtour $P$. 

Laurence A. Wolsey
S_1, S_2 is a partition of S

$$
\sum_{(I_0 \setminus S, S_1)} Cy^{ij} + \sum_{(I_0 \setminus S, S_2)} W^j y^{ij} + \sum_{(S_2, S_1)} (C - W^i)y^{ij} \\
+ \sum_{(S_2, S_2)} \min(C - W^i, W^j)y^{ij} \geq \sum_{i \in S} x^i
$$
Example of a Particular Metric Inequality

\[
\sum_{(I^0 \setminus S, S_1)} Cy^{ij} + \sum_{(I^0 \setminus S, S_2)} W^{j}y^{ij} + \sum_{(S_2, S_1)} (C - W^{i})y^{ij}
\]
\[
+ \sum_{(S_2, S_2)} \min(C - W^{i}, W^{j})y^{ij} \geq \sum_{i \in S} x^{i}
\]

\(|I^0| = 3, C = 300, W = (289, 123, 76), S = \{1, 2, 3\}, S_1 = \{1\}, S_2 = \{2, 3\}\)

\[
300y^{01} + 123y^{02} + 76y^{03} + (300 - 123)y^{21} + (300 - 76)y^{31}
\]
\[
+ \min(76, 300 - 123)y^{23} + \min(123, 300 - 76)y^{32} \geq x^{1} + x^{2} + x^{3}
\]
Add $s_{t-1}^i + x_t^i = D_t^i + s_t^i$ and IRP model for periods 1, \ldots, T.

$$C \sum_{u=k}^{t} \sum_{(i,j) \in (I_0 \setminus S, S)} y_{u}^{ij} \geq \sum_{u=k}^{t} \sum_{i \in S} x_{u}^{i} = \sum_{i \in S} \sum_{u=k}^{t} x_{u}^{i} \geq \sum_{i \in S} (D_{kt}^i - s_{k-1}^i)$$

With $s_{k-1}^i \leq U_i^i$, above is of the form:

$$\sigma + Cv \geq b, \quad 0 \leq \sigma \leq h, \quad v \in \mathbb{Z}.$$ 

This gives the inequalities:

$$v \geq \left\lceil \frac{b - h}{C} \right\rceil \quad \text{and} \quad \sigma + \rho v \geq \rho \left\lfloor \frac{b}{C} \right\rfloor.$$
Table: VMIRP-SUB: computational results for the instances with $n = 50$ and $T_{\text{max}} = 6$

<table>
<thead>
<tr>
<th>No</th>
<th>$LB_{\text{Ini}}$</th>
<th>$LB_{\text{LS}}$</th>
<th>$LB_{\text{Cut}}$</th>
<th>BUB</th>
<th>Nodes</th>
<th>Time</th>
<th>Blb</th>
<th>Bub</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>l1</td>
<td>8375</td>
<td>9754</td>
<td>9754</td>
<td>9966</td>
<td>15123</td>
<td>1485</td>
<td>9901</td>
<td>9976</td>
<td>86400</td>
</tr>
<tr>
<td>l2</td>
<td>8952</td>
<td>10516</td>
<td>10523</td>
<td>10632</td>
<td>65</td>
<td>334</td>
<td>10632</td>
<td>10632</td>
<td>2536</td>
</tr>
<tr>
<td>l3</td>
<td>8725</td>
<td>10376</td>
<td>10391</td>
<td>10511</td>
<td>3972</td>
<td>1876</td>
<td>10511</td>
<td>10511</td>
<td>1355</td>
</tr>
<tr>
<td>l4</td>
<td>8628</td>
<td>10243</td>
<td>10243</td>
<td>10513</td>
<td>166667</td>
<td>18016</td>
<td>10513</td>
<td>10513</td>
<td>60289</td>
</tr>
<tr>
<td>l5</td>
<td>8386</td>
<td>9860</td>
<td>9899</td>
<td>10113</td>
<td>2500</td>
<td>2327</td>
<td>10113</td>
<td>10113</td>
<td>2416</td>
</tr>
<tr>
<td>l6</td>
<td>8417</td>
<td>9945</td>
<td>9948</td>
<td>10148</td>
<td>1900</td>
<td>2318</td>
<td>10114</td>
<td>1014x</td>
<td>86400</td>
</tr>
<tr>
<td>l7</td>
<td>8355</td>
<td>9776</td>
<td>9776</td>
<td>9982</td>
<td>284288</td>
<td>28195</td>
<td>9982</td>
<td>9982</td>
<td>14698</td>
</tr>
<tr>
<td>l8</td>
<td>8385</td>
<td>10015</td>
<td>10066</td>
<td>10299</td>
<td>878</td>
<td>1360</td>
<td>10253</td>
<td>10229</td>
<td>86400</td>
</tr>
<tr>
<td>l9</td>
<td>8484</td>
<td>9897</td>
<td>9904</td>
<td>10010</td>
<td>819</td>
<td>801</td>
<td>10010</td>
<td>10010</td>
<td>6326</td>
</tr>
<tr>
<td>l10</td>
<td>8014</td>
<td>9546</td>
<td>9546</td>
<td>9659</td>
<td>2425</td>
<td>2081</td>
<td>9659</td>
<td>9659</td>
<td>3523</td>
</tr>
<tr>
<td>h1</td>
<td>29508</td>
<td>29862</td>
<td>29906</td>
<td>30189</td>
<td>1235</td>
<td>645</td>
<td>30189</td>
<td>30189</td>
<td>3036</td>
</tr>
<tr>
<td>h2</td>
<td>27983</td>
<td>29601</td>
<td>29615</td>
<td>29790</td>
<td>133</td>
<td>357</td>
<td>29790</td>
<td>29790</td>
<td>3334</td>
</tr>
<tr>
<td>h3</td>
<td>27830</td>
<td>29634</td>
<td>29657</td>
<td>29791</td>
<td>219</td>
<td>809</td>
<td>29791</td>
<td>29791</td>
<td>4020</td>
</tr>
<tr>
<td>h4</td>
<td>29517</td>
<td>31241</td>
<td>31241</td>
<td>31518</td>
<td>1424</td>
<td>1618</td>
<td>31518</td>
<td>31518</td>
<td>5737</td>
</tr>
<tr>
<td>h5</td>
<td>27413</td>
<td>28993</td>
<td>29021</td>
<td>29240</td>
<td>199</td>
<td>565</td>
<td>29240</td>
<td>29240</td>
<td>684</td>
</tr>
<tr>
<td>h6</td>
<td>30008</td>
<td>31621</td>
<td>31630</td>
<td>31903</td>
<td>367</td>
<td>1048</td>
<td>31903</td>
<td>31903</td>
<td>28320</td>
</tr>
<tr>
<td>h7</td>
<td>27933</td>
<td>29397</td>
<td>29397</td>
<td>29734</td>
<td>7988</td>
<td>1703</td>
<td>29734</td>
<td>29734</td>
<td>13561</td>
</tr>
<tr>
<td>h8</td>
<td>23923</td>
<td>25692</td>
<td>25692</td>
<td>25954</td>
<td>328</td>
<td>1202</td>
<td>25954</td>
<td>25954</td>
<td>21552</td>
</tr>
<tr>
<td>h9</td>
<td>28467</td>
<td>29863</td>
<td>29884</td>
<td>30193</td>
<td>390</td>
<td>822</td>
<td>30193</td>
<td>30193</td>
<td>20581</td>
</tr>
<tr>
<td>h10</td>
<td>29508</td>
<td>31101</td>
<td>31101</td>
<td>31338</td>
<td>83</td>
<td>488</td>
<td>31338</td>
<td>31338</td>
<td>1879</td>
</tr>
</tbody>
</table>
Table: Computational results for the instances 1n15T6 with \( n = 15 \) and \( T_{\text{max}} = 6 \)

<table>
<thead>
<tr>
<th></th>
<th>Subtour</th>
<th>LS</th>
<th>Prop1</th>
<th>Gen</th>
<th>Cover</th>
<th>BLB</th>
<th>BUB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l1 )</td>
<td>2</td>
<td>4880.7</td>
<td>5581.1</td>
<td>5681.8</td>
<td>5732.8</td>
<td>5803.6</td>
<td>5987.4</td>
</tr>
<tr>
<td>( l1 )</td>
<td>3</td>
<td>5640.7</td>
<td>6222.0</td>
<td>6541.4</td>
<td>6580.9</td>
<td>6684.5</td>
<td>6861.07</td>
</tr>
<tr>
<td>( l1 )</td>
<td>4</td>
<td>6527.8</td>
<td>7049.5</td>
<td>7484.6</td>
<td>7525.6</td>
<td>7622.3</td>
<td>7320.32</td>
</tr>
<tr>
<td>( l1 )</td>
<td>5</td>
<td>7466.0</td>
<td>7697.3</td>
<td>8377.4</td>
<td>8480.9</td>
<td>8512.7</td>
<td>7574.4</td>
</tr>
<tr>
<td>( h1 )</td>
<td>2</td>
<td>11388.5</td>
<td>12196.1</td>
<td>12305.6</td>
<td>12367.0</td>
<td>12369.2</td>
<td>12624.7</td>
</tr>
<tr>
<td>( h1 )</td>
<td>3</td>
<td>12150.2</td>
<td>12820.6</td>
<td>13165.7</td>
<td>13248.6</td>
<td>13310.4</td>
<td>13517.6</td>
</tr>
<tr>
<td>( h1 )</td>
<td>4</td>
<td>13039.4</td>
<td>13642.5</td>
<td>14111.8</td>
<td>14217.6</td>
<td>14240.1</td>
<td>13999.8</td>
</tr>
<tr>
<td>( h1 )</td>
<td>5</td>
<td>13975.3</td>
<td>14540.1</td>
<td>15006.9</td>
<td>15109.4</td>
<td>15156.1</td>
<td>14262.2</td>
</tr>
</tbody>
</table>
The MIP systems are amazing, but sometimes one can still help.

Thank you for your attention
CVRP


IRP

