1. Widgetco is about to introduce a new product (product 3 ). One unit of product 3 is produced by assembling 1 unit of product 1 and 1 unit of product 2. Before production begins on either product 1 or 2 , raw materials must be purchased and workers must be trained. Before products 1 and 2 can be assembled into product 3 , the finished product 2 must be inspected. A list of activities and their predecessors and of the duration of each activity is given in Table 12. Draw a project diagram for this project.

| Activity | Description | Predecessors | Duration (days) |
| :---: | :--- | :---: | :---: |
| A | train workers | - | 6 |
| B | purchase raw materials | - | 9 |
| C | produce product 1 | A,B | 8 |
| D | produce product 2 | A,B | 7 |
| E | test product 2 | D | 10 |
| F | assemble products 1 and 2 | C,E | 12 |

(a) Draw a project diagram for this project.
(b) Determine the total float (folga total) for each activity and the critical path.
(c) Use linear programming to find a critical path, providing an interpretation of the optimal values of the dual variables.
(d) Suppose that by allocating additional resources to an activity, Widgetco can reduce the duration of any activity by as many as 5 days. The cost per day of reducing the duration of an activity is:

| A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 20 | 3 | 30 | 40 | 50 |

Determine the allocation of resources that minimizes the cost of concluding the project in 25 days.
(e) Consider now that the duration of each activity is a random variable following a beta distribution, with the following parameters:

| Activity | $a$ | $b$ | $m$ |
| :---: | :---: | :---: | :---: |
| A | 2 | 10 | 6 |
| B | 5 | 13 | 9 |
| C | 3 | 13 | 8 |
| D | 1 | 13 | 7 |
| E | 8 | 12 | 10 |
| F | 9 | 15 | 12 |

Using PERT, determine the probability that the project will be completed within 35 days. [source: Winston]
2. Consider the following network of activities, where it is assumed that the duration of each of the activities is not precisely known.


Estimates for the minimum, maximum, and most likely duration for each of the activities are given in the following table (in days).

| Activity | $a$ | $b$ | $m$ |
| :---: | ---: | ---: | ---: |
| $(1,2)$ | 4 | 8 | 6 |
| $(1,3)$ | 2 | 8 | 4 |
| $(2,4)$ | 1 | 7 | 3 |
| $(3,4)$ | 6 | 12 | 9 |
| $(3,5)$ | 5 | 15 | 10 |
| $(3,6)$ | 7 | 18 | 12 |
| $(4,7)$ | 5 | 12 | 9 |
| $(5,7)$ | 1 | 3 | 2 |
| $(6,8)$ | 2 | 6 | 3 |
| $(7,9)$ | 10 | 20 | 15 |
| $(8,9)$ | 6 | 11 | 9 |

Using PERT, determine the probability that the project will be completed in less than 40 days.
3. When an accounting firm audits a corporation, the first phase of the audit involves obtaining "knowledge of the business." This phase of the audit requires the following activities:

| Activity | Description | Immediate <br> predecessors | Duration <br> (days) |
| :---: | :--- | :---: | :---: |
| A | Determining terms of engagement | - | 3 |
| B | Appraisal of auditability risk and materiality | A | 6 |
| C | Identification of types of transactions and possible | A | 14 |
|  | errors |  |  |
| D | Systems description | C | 8 |
| E | Verification of systems description | D | 4 |
| F | Evaluation of internal controls | E | 8 |
| G | Design of audit approach | F | 9 |

(a) Draw the project network and determine the total float for each activity and the critical path.
(b) Set up the LP that can be used to find the project's critical path.
(c) Assume that the project must be completed in 30 days. The duration of each activity can be reduced by incurring the costs, and up to a maximum reduction, as follows:

| Activity | Cost/day | Maximum possible reduction (days) |
| :---: | :---: | :---: |
| A | 100 | 3 |
| B | 80 | 4 |
| C | 60 | 5 |
| D | 70 | 2 |
| E | 30 | 4 |
| F | 20 | 4 |
| G | 50 | 4 |

Formulate and solve a linear program that can be used to determine the minimum cost of meeting the project deadline.
[source: Winston]
4. Consider the following network of activities, where it is assumed that the duration of each of the activities is not precisely known and can be modeled with a a beta distribution.


Estimates for the minimum, maximum, and most likely duration for each of the activities are given in the following table (in days).

| Actividade | $a$ | $b$ | $m$ |  | Actividade | $a$ | $b$ | $m$ |
| :---: | ---: | ---: | ---: | :---: | :---: | ---: | ---: | ---: |
| $(1,2)$ | 4 | 8 | 6 |  | $(7,10)$ | 5 | 28 | 18 |
| $(1,3)$ | 2 | 8 | 4 |  | $(8,9)$ | 6 | 11 | 9 |
| $(2,4)$ | 1 | 7 | 3 |  | $(8,11)$ | 7 | 18 | 10 |
| $(3,4)$ | 6 | 12 | 9 |  | $(9,10)$ | 2 | 2 | 2 |
| $(3,5)$ | 5 | 15 | 10 |  | $(10,14)$ | 10 | 40 | 25 |
| $(3,6)$ | 7 | 18 | 12 |  | $(11,12)$ | 5 | 20 | 10 |
| $(4,7)$ | 5 | 12 | 9 |  | $(11,13)$ | 4 | 18 | 12 |
| $(5,7)$ | 1 | 3 | 2 |  | $(12,13)$ | 1 | 3 | 2 |
| $(6,8)$ | 2 | 6 | 3 |  | $(12,14)$ | 8 | 12 | 10 |
| $(7,9)$ | 10 | 20 | 15 |  | $(13,14)$ | 7 | 22 | 10 |

Using simulation, solve the following questions.
(a) Determine the average duration of the project, and compare it with the solution of the analytical approach using CPM.
(b) Determine the probability that the project will be completed in less than 50 days.
(c) Determine the probability that task $(1,3)$ is critical.
(d) Determine average project durations under the following circumstances:
i. Activities $(5,7)$ and $(6,8)$ must start at the same time;
ii. Activity $(5,7)$ only needs to be performed if $(3,5)$ ends after $(3,4)$.

Compare them with the previous value.

