1. Coach Night is trying to choose the starting lineup for the basketball team. The team consists of seven players who have been rated (on a scale of 1 = poor to 3 = excellent) according to their ball-handling, shooting, rebounding, and defensive abilities. The positions that each player is allowed to play and the player's abilities are listed in the following table:

Player	Position	Ball Handling	Shooting	Rebounding	Defense
1	G	3	3	1	3
2	C	2	1	3	2
3	G,F	2	3	2	2
4	F,C	1	3	3	1
5	G,F	3	3	3	3
6	F,C	3	1	2	3
7	G,F	3	2	2	1

   The five-player starting lineup must satisfy the following restrictions:

   (a) At least 4 members must be able to play guard, at least 2 members must be able to play forward, and at least 1 member must be able to play center.

   (b) The average ball-handling, shooting, and rebounding level of the starting lineup must be at least 2.

   (c) If player 3 starts, then player 6 cannot start.

   (d) If player 1 starts, then players 4 and 5 must both start.

   (e) Either player 2 or player 3 (or both) must start.

   Given these constraints, Coach Night wants to maximize the total defensive ability of the starting team. Formulate an IP that will help him choose his starting team.

2. Because of excessive pollution on the Momiss River, the state of Momiss is going to build pollution control stations. Three sites (1, 2, and 3) are under consideration. Momiss is interested in controlling the pollution levels of two pollutants (1 and 2). The state legislature requires that at least 80,000 tons of pollutant 1 and at least 50,000 tons of pollutant 2 be removed from the river. The relevant data for this problem are shown below. Formulate an integer optimization problem to minimize the cost of meeting the state legislature's goals.

Site	Cost of building station ($)	Cost of treating 1 ton water ($)	Amount removed (ton) per ton of water	
			Pollutant 1	Pollutant 2
1	100000	20	0.40	0.30
2	60000	30	0.25	0.20
3	40000	40	0.20	0.25

   (a) Formulate an IP to minimize the cost of meeting the state legislature's goals.

   (b) Solve it using the branch and bound method. Use GLPK or AMPL to solve the linear problems of each node, and draw the complete search tree.

   (c) Choosing an order to traverse the tree, indicate whether there are branches that could be ignored.

3. J. C. Nickles receives credit card payments from four regions of the country (West, Midwest, East, and South). The average daily value of payments mailed by customers from each region is as follows: the West, \$70,000; the Midwest, \$50,000; the East, \$60,000; the South, \$40,000. Nickles must decide where customers should mail their payments. Because Nickles can earn 20% annual interest by investing these revenues, it would like to receive payments as quickly as possible. Nickles is considering setting up operations to process payments (often referred to as lockboxes) in four different cities: Los Angeles, Chicago, New York, and Atlanta.

   The average number of days (from time payment is sent) until a check clears and Nickles can deposit the money depends on the city to which the payment is mailed, as shown in the following table.

	To city			
	1	2	3	4
From	(Los Angeles)	(Chicago)	(New York)	(Atlanta)
Region 1 West	2	6	8	8
Region 2 Midwest	6	2	5	5
Region 3 East	8	5	2	5
Region 4 South	8	5	5	2

   For example, if a check is mailed from the West to Atlanta, it would take an average of 8 days before Nickles could earn interest on the check. The annual cost of running a lockbox in any city is \$50,000. Formulate an IP that Nickles can use to minimize the sum of costs due to lost interest and lockbox operations. Assume that each region must send all its money to a single city and that there is no limit on the amount of money that each lockbox can handle.

4. Dorian Auto is considering manufacturing three types of autos: compact, midsize, and large. The resources required for, and the profits yielded by, each type of car are shown in this table:

	Compact	Midsize	Large
Steel required	1.5 ton	3.0 ton	5.0 ton
Labor required	30 hours	25 hours	40 hours
Profit yielded	2000	3000	4000

   Currently, 6,000 tons of steel and 60,000 hours of labor are available. For production of a type of car to be economically feasible, at least 1,000 cars of that type must be produced. Formulate an IP to maximize Dorian's profit.

5. **Branch-and-Bound for Machine Scheduling.** Four jobs must be processed on a single machine. The time required t_i to process each job and the date the job is due d_i are shown, for each job i, in this table:

Job	t_i	d_i
1	5	8
2	5	4
3	5	12
4	8	16

   The delay of a job is the number of days after the due date that a job is completed (if a job is completed on time or early, the job's delay is zero). In what order should the jobs be processed to minimize the total delay of the four jobs?

[source: winston]