Electricity Day-Ahead Markets: Computation of Nash Equilibria

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Abstract

In a restructured electricity sector, day-ahead markets can be modeled as a game where some players - the producers - submit their proposals. To analyze the companies' behavior we have used the concept of Nash equilibrium as a solution in these multi-agent interaction problems. In this paper, we present methods that we have developed in order to compute Nash equilibria: the adjustment process and the relaxation algorithm. The advantages of these approaches are highlighted and compared with those available in the literature.

Keywords: Electricity Market; Nash Equilibria; Adjustment Process; Relaxation Algorithm; Combinatorial Optimization.

1 Introduction

Since the beginning of the 70's, all over the world, the energy sector has undergone regulatory and operational changes; this has been analyzed e.g. in Gomes [2005] and Saraiva et al. [2002].

At the end of the 19^{th} century, electricity started to be generated, transported and distributed through low power networks, within restricted geographic areas. Then, larger companies were created which typically corresponded to vertically integrated entities and there was no competition in this sector. This kind of organization in the electricity sector implied that: the consumers could not choose an electricity company to be supplied from, the prices were defined in an administrative and sometimes unclear way, and planning activities were made with less complexity than today (also because the economic environment was less volatile). Therefore, before the oil crisis (1973), electricity companies easily made forecasts, since risk or uncertainty were not prior concerns.

This situation changed in the beginning of the 70's: high inflation and interest rates made the economic environment more volatile. Adding to this fact, the evolution of technology turned possible the deregulation of the electricity sector and its vertical unbundling. Thus, new companies were established and market mechanisms were implemented in order to create conditions for competition (Saraiva et al. [2002]). There was a transition from a monopolistic (centralised markets) to an oligopolistic situation (pool markets); each time, more companies' optimal revenues depend on the strategies of their competitors. The idea of this new approach is to move from a centralized operation to a competitive one(Conejo and Prieto [2001]).

Nowadays, many electricity markets are a pool-based auction for the purchase and sale of power. These new market rules have as aim: stimulate competition, contribute to lower electricity prices, induce economic growth and transmit an appropriate incentive for investment and new entry. However, the appropriate rules are still an ongoing research topic. In this work, we will focus in the Iberian day-ahead market mechanisms, aiming at understanding and analyzing possible market outcomes.

To analyze the companies' behavior in an electricity pool market, we apply game theory, in particular the notion of Nash equilibria (NE). We look at this pool-based auction as a non cooperative game, in which producers are the players that have to choose a strategy (a proposal) to submit. Here, their goal is to submit bids in such a way that their profits are maximized. Hence, in the electricity pool market, we are interested in finding equilibrium strategies for the producers, since an equilibrium is the best answer that each producer may have in this non-cooperative game.

Because of the complexity of the pool market structures and network constraints, the theoretical studies in this area do not reveal to be very practical. So, recently, in an attempt to predict market prices and market outcomes, more complex models have been used, but, many times they do not allow performing analytical studies. Techniques from evolutionary programming (Barforoushi et al. [2010] and Son and Baldick [2004]) and mathematical programming (Hobbs et al. [2000] and Pereira et al. [2005]) were used in these new models. For example, Pozo et al. [2010] and Pereira et al. [2005] study strategic bidding in electricity pool markets, with elastic and inelastic demand respectively, through mathematical programming. The authors Pozo and Contreras [2011] considered the case of constant, stochastic demand. They used the methodology of Pereira et al. [2005] to eliminate the bilinear terms of the generation companies' profit maximization problem, using a piecewise linear function and binary variables. They also contributed with a procedure that aims at finding all Nash equilibria in pure strategies. There, the proposals' space of strategies is discretized (unlike in our work), so they are able to focus on the use of methods to compute Nash equilibria in games with a finite strategies. However, as reported by the authors Lee and Baldick [2003], the discretization of the space of strategies can artificially eliminate some true Nash equilibria and add some equilibria that do not exist in the original game. Moreover, in order to have a good approximation of a game, the game with the discretized space of strategies would have a large number of strategies which dramatically increases the running time of the algorithms to solve the approximate game.

In Hasan and Galiana [2010] it is proposed a fast computation of Nash equilibria in pure strategies by observing their properties; in that work, discretization is not required. In Bajpai and Singh [2007], a particle swarm optimization method is used in order to maximize a generating company profit, while we will compute the strategies that maximize the profits simultaneously for each of the firms.

To approach this problem, we built two methods, the adjustment process (AP) and the relaxation algorithm (RA), which reveal to be very practical to find Nash equilibria. When these methods converge, a Nash equilibrium in pure strategies is found. Therefore, we contribute with procedures that could help regulators to monitor the prices and generating companies (GENCOs) to refine their bids. Moreover, the detail used in our model has not been considered yet.

This paper is organized as follows: Section 2 presents the electricity market model, Section 3 clarifies the concept of Nash equilibrium and explains the developed approaches to achieve them, Section 4 treats some examples and Section 5 presents our future work in this problem.

2 Iberian Market Model

In this section, our model will be described and notation fixed. The adopted model is based in the structure of the day-ahead electricity markets, which exist in many regions such as the Iberian Peninsula, Nord Pool, Brazil (see Pereira et al. [2005] or Barroso et al. [2006]), and Greece (see Bakirtzis et al. [2007]).

Our goal is to predict the outcomes of a given energy market; some of the details will not be considered, in order to make the model tractable. Transmission constraints will not be taken into account: a single node or zone is assumed. This occurs in some day-ahead markets such as in Portugal, Spain, UK, Sweden, China, Colombia, Peru and Brazil (see Barroso et al. [2006]).

In the pool market, consumers and producers submit their proposals to buy and sell electricity. In general, each day is divided in 24 periods of 1 hour, so there are 24 auctions. In this work, just one trading hour will be modeled. Consumers will not be considered as players since they typically have no market power. In our formulation, demand elasticity will be modeled by a parameter; this has the advantage of being a realistic approach. The demand, represented by a straight line segment $P = mQ + b$, is characterized by the real constants $m < 0$, in \$/(MWh)^2, and $b > 0$, in \$/MWh. In other words, variations in the price P, induce small variations in the quantity Q. In most of the works developed in this area, the demand is assumed as completely inelastic, so we are also contributing with a more realistic approach.

On the other hand, GENCOs will be the players of this day-ahead market, usually also called spot market (see Bakirtzis et al. [2007]). GENCOs simultaneously submit their selling proposals in the market, which correspond to pairs of quantity (MWh) and price (\$/MWh).

Let n be the number of the selling proposals. For each hour of the day we have a table T that contains all the information about the proposals of the producers and the generation costs. This table has the following form, for each row $j \in \{1, 2, \ldots, n\}$:

$$T_j = \boxed{j \mid s_j \mid q_j \mid p_j \mid c_j \mid b_j \mid a_j \mid E_j}$$

where the proposals are indexed by $j = 1, 2, \ldots, n$, s_j is the producer associated to proposal j, q_j is the proposal quantity in MWh, p_j is the price in

\$/MWh, c_j, b_j and a_j are associated cost parameters which will be described later, and E_j is the maximum production level. Then the market operator (MO), an independent agent, carries out an economic dispatch, ED, once the price and quantity bids of the producers were submitted. MO wants to find which proposals should be dispatched so that the demand is satisfied and the market clearing price P_d is minimized. The market operator organizes the generation proposals by ascending order of the prices p_j and aggregates them, forming the supply curve. Thus the intersection of this curve with the demand segment gives the market clearing price P_d and quantity Q_d, and the proposals that are dispatched. In this context, we denote as g_j the energy accepted to be produced for proposal j in the ED. See Figure 1 for an example.

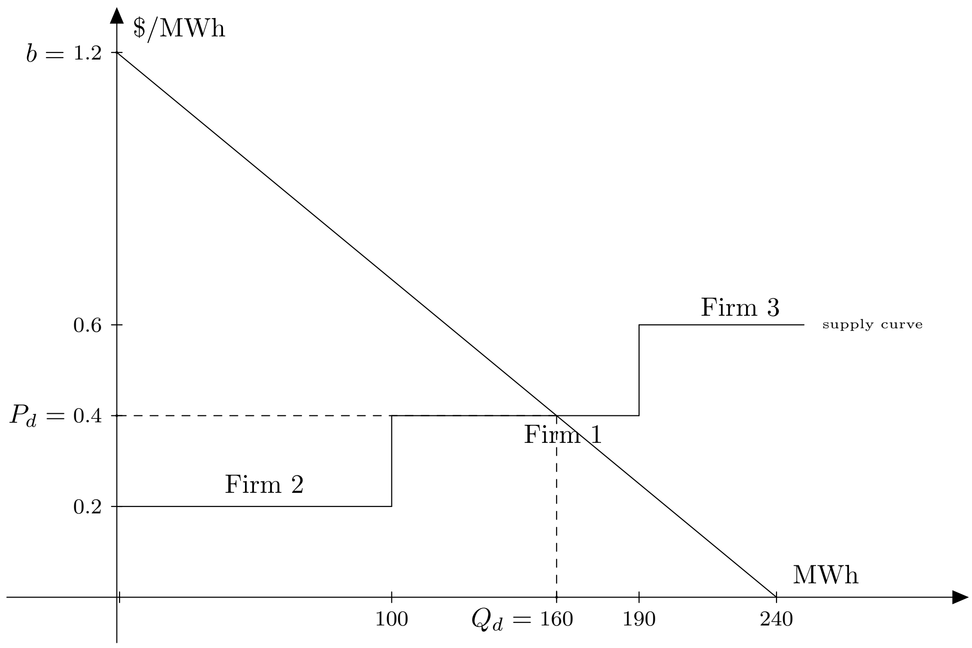

Figure 1: Economic Dispatch: the demand parameters are $m = -\frac{1}{200}$ and $b = 1.2$; Firm 1 bids $(q_1 = 90, p_1 = 0.40)$, Firm 2 bids $(q_2 = 100, p_2 = 0.20)$ and Firm 3 bids $(q_3 = 60, p_3 = 0.60)$; Firm 2 proposal is totally accepted $(q_2 = g_2)$ and Firm 1 also has the proposal accepted but with produced quantity $g_1 = 60 < q_1$.

The following function is the quadratic cost of generating unit j when producing g_j, $\forall j = 1, \ldots, n$:

$$F_j(g_j) = \begin{cases} c_j g_j^2 + b_j g_j + a_j & \text{if } g_j > 0 \\ 0 & \textit{otherwise.} \end{cases} \tag{1}$$

The parameter a_j is the fixed cost of turning on the generating unit j. Note, that in the literature mentioned before, just marginal cost is considered which simplifies the problem. The quadratic cost includes the marginal cost approach.

The revenue for each producer i is given by:

$$\Pi_i = \sum_{j \in \{A_{ED}: s_j = i\}} \left(P_d g_j - F_j \left(g_j \right) \right)$$

where g_j is the energy produced by Firm $i = s_j$ in the economic dispatch and A_{ED} is the set of accepted proposals in the economic dispatch. This profit deeply depends on the strategies of the other players, which makes the problem complex.

After clearing the market, the information about the submitted proposals is made publicly available.

Finally, as a tie breaking rule, MO divides proportional quantities proposed by the firms declaring the same price, in case they are not fully required.

3 Nash Equilibria Computation

Game theory provides important tools in economics. The concept of Nash equilibrium of a game plays a relevant role in this context. Basically, it is a probability distribution over the set of strategies of each player, such that nobody wants to change unilaterally its behavior. If some player changes his strategy with respect to a Nash equilibrium, his profit would not increase (Fudenberg and Tirole [1996]).

Definition 3.1. In a game with a set N of n players, a vector $p^{NE} = \left(p_1^{NE}, p_2^{NE}, \ldots, p_n^{NE} \right)$, where p_i^{NE} specifies a probability distribution over the set of strategies of player i, is a Nash equilibrium if $\forall i \in \{1, 2, \ldots, n\}$:

$$\Pi_i \left(p^{NE} \right) \geq \Pi_i \left(p_i, p_{-i}^{NE} \right) \forall p_i \in S_i$$

where Π_i is the utility of player i, S_i is his space of strategies and $p_{-i}^{NE} = \{ p_j : j \in N, j \neq p_i \}$.

In our case, the strategies of each player are the proposals, so in a Nash equilibrium, we have the probability of choosing (q_j, p_j) over the set $S_j = [0, E_j] \times [0, b]$ (space of strategies), where b is the maximum price at which the consumers buy electricity (see Section 2 where the demand is defined). A Nash equilibrium in which each player plays with probability one a certain strategy is called an equilibrium in pure strategies.

In current electricity markets, the producers have to communicate the market operator their proposals for each hour of the following day. We admit that each producer predicts exactly the demand for each hour and knows the technology of his competitors. Our goal is to find the best strategy for each company.

The methods that we use in this paper only provide pure Nash equilibria. As seen in the related literature, algorithms to find Nash equilibria in day-ahead markets use a finite strategy approach both in prices and in quantities. We wanted to avoid this treatment since: for a finite game to be tractable the number of firms and the space of strategies can not be very large; there is no guarantee that the equilibria from the finite game are equilibria of the original game.

3.1 Adjustment Process

We will apply an adjustment process (AP) to find out the Nash equilibria of this non-cooperative game. An adjustment process, sometimes refereed as Gauss-Seidel-type method (Facchinei and Kanzow [2007]), is an iterative process in which each player adjusts his strategy according to the past iterations. This is a learning process. It is easy to find examples in which this method diverges or has chaotic behavior, indicating that this process does not always work. However, if a solution is found, then it is a Nash equilibrium. In Contreras et al. [2004] a very similar process is presented, but there the decision variables are only the quantities.

Our method can be described with the pseudo code of Algorithm 1. In

Algorithm 1 The adjustment process pseudo code.

1: initialise with an information table T, $\epsilon > 0$, and demand parameters m and b;
2: let S_i be the set of proposals of the producer i and k the number of producers;
3: **repeat**
4: $\quad X \leftarrow$ thrid and fourth column of T;
5: $\quad$ **for** $i = 1$ to k **do**
6: $\quad\quad q_{S_i}, p_{S_i} \leftarrow \arg\max_{q_j, p_j, j \in S_i} \sum_{s_j \in A_{ED} \cap S_i} (P_d g_j - F_j(g_j))$;
7: $\quad\quad$ update the third and fourth column of M with q_j and p_j for $j \in S_i$;
8: $\quad$ **end for**
9: $\quad Y \leftarrow$ third and fourth column of T;
10: $\quad \Delta \leftarrow ||Y - X||$;
11: **until** $\Delta < \epsilon$

short, in each step every producer finds the strategy that maximizes his profit assuming that the other players are going to follow the strategy of the previous iteration. The process stops when two iterations are sufficiently close to each other meaning that the current table T is a Nash equilibrium, because nobody made a significant change in his behavior. In fact, when $\Delta = 0$, T is exactly a Nash Equilibrium. It is important to notice that the maximization process, in step six, needs a method able to tackle non-smooth functions, as the profit of the companies is a function with discontinuities.

The most important step in our adjustment process is the maximization of the producers's profits. To solve this problem, we have used the MATLAB implementation of a global optimization method developed by Ismael Vaz and Luís Vicente, see Vaz and Vicente [2007]. In this method, called PSwarm, we only need to evaluate the objective function values resulting from pattern search and particle swarm, so this is exactly what we needed in our adjustment process.

3.2 Relaxation Algorithm

The relaxation algorithm is similar to the AP, but it uses the Nikaido-Isoda function (N-I function). This function transforms the difficult problem of computing a Nash equilibrium in to a far simpler optimization problem (Krawczyk and Zuccollo [2006]).

Definition 3.2. Let Π_{s_j} be the payoff function for player s_j and S a collective strategy space. The Nikaido-Isoda function $\psi : S \times S \longrightarrow \mathbb{R}$ is defined as:

$$\psi(x, y) = \sum_{s_j \in F} \left[\Pi_{s_j} \left(y_{s_j} | x \right) - \Pi_{s_j}(x) \right]$$

The N-I function can be interpreted as the improvement that a player will received by changing his action from x_{s_j} to y_{s_j} while all other players continue to play according to x. So $\psi(x, y)$ represents the sum of these improvements.

Note that $\psi(x, x) = 0$, then the maximum value that this function can take for a given x is nonnegative. When either x or y are a Nash equilibrium, the N-I function is everywhere non-positive since no player can improve his payoff. Therefore, when the N-I function can not be made significantly positive for a given y we have approximately reached a Nash equilibrium. This fact is used to define a stopping criteria for the relaxation algorithm that is an $\varepsilon > 0$ such that when

$$\underset{y \in S}{\text{maximize}} \, \psi(x^s, y) < \varepsilon$$

the Nash equilibrium would be achieved with a sufficient degree of precision.

The relaxation algorithm is an iterative process for finding NE. As in the adjustment process, the relaxation algorithm is initialized with a feasible information table and in each step the N-I function is maximized. When the stopping criteria ε is achieved, the algorithm stops.

An important remark about the RA should be made in this context. In the way that the RA was described, there is a high probability that it would not converge for our electricity market problem. Therefore, we made a crucial adaptation. Note that when a strategy of a generating company has profit zero, there are others strategies that also lead to zero profit and these equivalent strategies (in terms of profit) are a problem for the convergence of the RA. In order to overcome this convergence problem, we add in each iteration of the RA a cycle that calculates the previous and current profit of each generating company. For each generating company with profit zero in these consecutive steps, we do not update its strategies.

4 Results

Finally, we will see the effectiveness of our algorithms in the game that electricity markets represent. We added another stopping criteria for AP and for the RA, in order to force these algorithms to stop. This new stopping criteria corresponds to a maximum number of iterations.

The computations were made in a computer with an Intel Core Duo CPU processor at 2.20GHz, running Microsoft Windows XP.

In this section, we are going to study two cases: a simple oligopoly market and a duopoly market for which we will progressively increase the number of generating units in order to see the limitations of our approach.

4.1 Case study: Oligopoly Market

We will consider a pool market with five generating companies. All GENCOs, except Producer C and E, have two generating units. Let the initial information table be Table 1 and let the demand be modeled by

$$P = 150 - \frac{150}{2000}Q.$$

The adjustment process and the relaxation algorithm where applied in 10 in-

j	s_j	q_j	p_j	$c_j \times 10^6$	$b_j \times 10^6$	a_j	E_j
1	Producer A	300	26	20	400	24	300
2	Producer A	300	27	30	300	24.5	300
3	Producer B	550	57	30	400	54	550
4	Producer B	150	61	10	450	60	150
5	Producer C	300	40	100	600	30	300
6	Producer D	300	48	66	65	40	300
7	Producer D	400	50	54	35	45	400
8	Producer E	200	49	24	25	47	200

Table 1: Initial information matrix.

dependent runs, using this initial table. Recall that our methods are stochastic, since the optimization procedure PSwarm is stochastic.

For the AP a solution was found in an average of 27 iterations and 336 seconds. In the second algorithm a solution was found in an average of 103 iterations and 424 seconds. Here, the solutions found were equivalent, in the sense that all of them lead to the same economic dispatch. For instance, a final information table of the adjustment process was Table 2. The final value of Δ

j	s_j	q_j	p_j	$c_j \times 10^6$	$b_j \times 10^6$	a_j	E_j
1	Producer A	300.0000	0.4000×10^{-3}	20	400	24	300
2	Producer A	300.0000	0.3000×10^{-3}	30	300	24.5	300
3	Producer B	412.9585	7.5032	30	400	54	550
4	Producer B	97.8811	71.6682	10	450	60	150
5	Producer C	300.0000	0.6000×10^{-3}	100	600	30	300
6	Producer D	300.0000	0.6500×10^{-4}	66	65	40	300
7	Producer D	400.0000	0.3500×10^{-4}	54	35	45	400
8	Producer E	200.0000	0.2500×10^{-4}	24	25	47	200

Table 2: Final information matrix reached by the AP.

(distance between consecutive iterations) was equal to zero in all the experiments made, which means that PSwarm is not able to improve this solution. Therefore, it is expected that this represents an equilibrium. Observing the economic dispatch of this solution, we can conclude that this is indeed a NE. The economic dispatch of Table 2 is Table 3. Note that the only proposal that is not accepted is proposal 4, which is the one with the highest fixed cost, and that $P_d = 7.50320$ is the optimal price for proposal 3 of Producer B. The profit due to proposal 3 is:

$$\left(\frac{p_3 - b}{m} - Q\right) p_3 - c_3 \left(\frac{p_3 - b}{m} - Q\right)^2 - b_3 \left(\frac{p_3 - b}{m} - Q\right) - a_3,$$

where Q is the sum of the quantities that were accepted, except the dispatch

j	s_j	g_j	p_j
8	Producer E	200.0000	0.2500×10^{-4}
7	Producer D	400.0000	0.3500×10^{-4}
6	Producer D	300.0000	40.6500×10^{-4}
2	Producer A	300.0000	0.3000×10^{-3}
1	Producer A	300.0000	0.4000×10^{-3}
5	Producer C	300.0000	0.6000×10^{-3}
3	Producer B	412.9585	7.5032

Table 3: Economic dispatch of the information Table 2.

quantity of proposal 3. A stationary point of this function is:

$$p_3 = \frac{2c_3 b + 2c_3 Qm - bm - b_3 m - Qm^2}{2\,(c_3 - m)} \approx 7.503199$$

to which corresponds a quantity $Q = 1800$. Therefore, the information Table 2 is an equilibrium of this instance.

4.2 Case study: Duopoly Market

In what follows only the duopoly situation will be considered. Our aim is to increase the number of generating units of each firm, until our methods stop converging in a limited time. We should note that we consider for all computations the same PSwarm parameters, however for larger instances more time should be given to PSwarm.

In these instances, the optimization steps are difficult, and as the instances grow it becomes very hard to prove that a final information table is an equilibrium. A practical method for checking if we have found an equilibrium is the following. If PSwarm is not able to improve the solutions in the final optimization step, then with high probability it has found an optimum and the solution is likely to be a NE.

As the running time for these experiments became rather high, from now on we will do only one observation of the algorithms for each instance.

Consider a pool market with two generating companies: Producer A and Producer B. Let the demand be modeled as

$$P = 150 - \frac{150}{2000} Q.$$

Let the initial information Table be 4.

We will start by solving the case where Producer A only has the generating units 1, 2, 3 and 4, and Producer B has the generating units 11, 12, 13 and 14 - Case 1. The computed NE are in Table 5. The solution given by the RA is not an equilibrium, since applying the adjustment process to its solution, it does not stop in the first iteration, so producers have benefit in changing their strategies. On the other hand the solution of the AP seems to be a Nash equilibrium. Furthermore the market clearing price ($P_d = p_3$) coincides with the stationary point:

$$p_3 = \frac{2c_3 b + 2c_3 Qm - bm - b_3 m - Qm^2}{2\,(c_3 - m)} \approx 30.01219$$

j	s_j	q_j	p_j	$c_j \times 10^6$	$b_j \times 10^6$	a_j	E_j
1	Producer A	300	26	20	400	24	300
2	Producer A	300	27	30	300	24.5	300
3	Producer A	550	57	30	400	54	550
4	Producer A	150	61	10	450	60	150
5	Producer A	200	22	2	3000	20	200
6	Producer A	180	26	10	120	34	180
7	Producer A	50	27	30	300	23.5	50
8	Producer A	350	57	35	600	38	350
9	Producer A	250	61	12	400	36	250
10	Producer A	150	22	2	800	29.1	150
11	Producer B	300	40	100	600	30	300
12	Producer B	300	48	66	65	40	300
13	Producer B	400	50	54	35	45	400
14	Producer B	200	49	24	25	47	200
15	Producer B	100	21	20	100	21	100
16	Producer B	125	10	500	800	10	125
17	Producer B	220	25	30	310	35	220
18	Producer B	200	28	28	400	35.2	200
19	Producer B	110	20	10	450	30.1	110
20	Producer B	60	19	1	300	20	60

Table 4: Initial information matrix.

where $Q = 1200$, as before. Therefore, we are strongly convinced that this is an equilibrium; the optimization process is not able to improve the solution, and the stationary conditions are met.

Now, consider that Producer A has the generating units 1, 2, 3, 4, and 5, and Producer B has the generating units 11, 12, 13, 14 and 15 - Case 2. Here, for the computed NE of the AP, Producer A has profit of about 9137 \$ and Producer B has profit of about 33923 \$. On the other hand, for the computed NE of the RA, Producer A has profit of about 18698 \$ and Producer B of about 37367 \$. Thus, we found two possible outcomes. The most predictable outcome is the one computed by the RA, since the profit of each producer is higher in the solution of the RA than in the solution of the AP (among the equilibria found). This instance may have more than two Nash equilibria, but with the initial information table used, only two potential Nash equilibria were found.

Notice that, as usually happens with fixed point algorithms and with non linear optimization, the starting point is important for obtaining convergence; as the situations get more complex, this becomes a crucial factor. Here, we were able to find a solution but the running time of our algorithms would considerably decrease if our initial solution was close to an equilibrium.

Case 3 consists of Producer A having the generating units from 1 to 10 and Producer B has the generating units 11, 12, 13, 14 and 15. Once again, two different NE were computed. The computed NE of the AP gives a profit of about 9126 \$ to Producer A and 33921 \$ to Producer B, and the RA solution gives a profit of approximately 18664 \$ to Producer A and approximately 37334 \$ to Producer B. This last equilibrium is more interesting than the one computed by AP, since both producers have a higher profit.

Finally, in Case 4, Producer A has the generating units from 1 to 10 and Producer B has the generating units from 11 to 20. Here, we considered a maximum of 800 iterations for each of the developed methods. The adjustment process and the relaxation algorithm were not able to converge in 800 steps, so their final output was not an equilibrium. Obviously, one of the reasons for not obtaining convergence is the starting point. Therefore, we tried another initial information table. In order to try to obtain convergence we simplified our problem setting: $c_j = b_j = 0$ for $j = 1, \ldots, 20$, this means, just fixed costs are considered. Again, neither of our two methods was able to converge in this case. However, if $c_j = a_j = 0$ for $j = 1, \ldots, 20$ our methods easily converge, showing the complexity that fixed costs introduce in the problem. This solution is not helpful as starting solution to the previous cases; the fixed costs significantly change the situation. We also tried to adapt the parameters of PSwarm, to no avail. Without further improvements this instance seems to be above the limits of our algorithms.

Generating Units	Case 1				Case 2				Case 3			
	AP		RA		AP		RA		AP		RA	
	q	p	q	p	q	p	q	p	q	p	q	p
1	91.9105	32.8732	300.0000	0.7564×10^{-2}	208.9650	20.8283	298.9506	31.0921	131.2359	0.1772×10^{-1}	149.4873	0.6257
2	110.3156	74.3004	0.0076	150.0000	283.5197	26.2544	299.8458	37.5062	75.1397	142.9125	298.3602	37.5032
3	525.4269	30.0122	550.0000	0.7563×10^{-2}	429.6769	28.1552	535.1755	50.9074	25.1954	39.2026	362.3090	53.6314
4	99.1894	121.2487	0.0076	73.8584	61.1236	103.7434	129.4398	84.2723	44.6941	107.6113	150.0000	150.000
5	-	-	-	-	137.7379	137.2047	200.0000	150.0000	74.9234	113.0211	200.0000	121.1563
6	-	-	-	-	-	-	-	-	78.2181	47.7785	180.0000	120.4974
7	-	-	-	-	-	-	-	-	15.6677	113.3583	0.0006	121.7018
8	-	-	-	-	-	-	-	-	325.6291	26.4845	208.0022	103.9335
9	-	-	-	-	-	-	-	-	249.9982	26.2528	249.1455	5.9538
10	-	-	-	-	-	-	-	-	49.6786	97.8927	150.0000	133.0288
11	300.0000	0.6000×10^{-3}	113.9848	57.6625	300.0000	0.6000×10^{-3}	300.0000	0.4394×10^{-1}	300.0000	0.6000×10^{-3}	296.3713	150.0000
12	300.0000	0.6500×10^{-4}	300.0000	43.1421	300.0000	0.6500×10^{-4}	300.0000	0.4394×10^{-1}	300.0000	0.6500×10^{-4}	300.0000	0.1717×10^{-2}
13	400.0000	0.3500×10^{-4}	316.2496	0.7573×10^{-2}	400.0000	0.3500×10^{-4}	400.0000	0.4394×10^{-1}	400.0000	0.3400×10^{-4}	400.0000	0.4214×10^{-3}
14	200.0000	0.2500×10^{-4}	186.6182	99.1516	200.0000	0.2500×10^{-4}	0.9133	150.0000	200.0000	0.2500×10^{-4}	200.0000	14.2777
15	-	-	-	-	100.0000	0.1000×10^{-3}	0.0439	150.0000	100.0000	0.1000×10^{-3}	100.0000	0.6279×10^{-3}
16	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-	-	-	-	-	-
20	-	-	-	-	-	-	-	-	-	-	-	-
CPU time	281		283		778		617		778		946	
# Iter	56		96		149		179		210		275	

Table 5: Results of the AP and of the RA when applied to Cases 1, 2 and 3. The CPU time is in seconds.

5 Conclusions

In order to solve real instances of the Iberian day-ahead market, a practical method to compute NE is imperative, since the problem is extremely complex to be treated analytically. In an attempt to deal with this problem we have built two procedures: the adjustment process (AP) and the relaxation algorithm (RA). We have been able to tackle small to medium instances which, despite not being real problems size, allowed us to find equilibria and the numerous traps that may appear during the search phases. Both methods are iterative and in each step they follow the same idea of adjusting the players' strategies.

The simple case with constant marginal costs was already treated in the literature, though using a discretization of the space of strategies; the methods proposed here work without changing the space of strategies. The size of the test instances solved in our work is similar to those solved in the literature. Moreover, we were able to tackle small instances considering a more realistic structure of the productions costs (quadratic costs), providing a contribution in this area. In the literature this problem is often formulated in mathematical programming in order to be solved by general purpose mixed-integer optimization solvers. However, mathematical optimization solvers are not able to find optimal solutions x of the form $x = y - \epsilon$, where ϵ is infinitesimal, and during this work we have seen the importance of considering such strategies for some instances.

Concerning future development, a possible direction is the improvement of the optimization procedures, and another one is the refinement of the AP and the RA. We always used the strategies of the last iteration in the AP and RA, but the use of an estimation based on all the past iterations may have some good properties in the convergence of the methods. In addition, the exploration of the structure of the problem, as we made in the RA adaptation (we keep some strategies unchanged into the following iteration in order to improve convergence), is likely to lead to further advances.

In conclusion, we gave a small contribution in this area, but many problems remain open.

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