CG – T16 – (Simple) Physics for Computer Graphics

L:CC, MI:ERSI

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Suggested reading

- D.M. Bourge, “Physics for Game Developers”, 1st Ed., O’Reilly Media, 2001

Why do we need physics?

- How do objects move?
- How much energy do they have?
- How do they stop by themselves?
- How do they float?
- How do they fly?

And that only involves movement… (Heat? Electricity? Wind? Light? Sound?)

Motion

The basic law

- Newton's second law $\vec{F} = m.\vec{a}$
- Force (N) equals mass (kg) times acceleration (ms^{-2})
- Means that accelerating an object requires an external force
- Also means that if we know this force, mass, and initial conditions we can predict object motion

Position and velocity

- If we know acceleration
- We can integrate it over time to obtain velocity
- And integrate it again to obtain position

We can predict motion!

$$\vec{v} = \int_t \vec{a}.dt$$

$$\vec{v} = \overrightarrow{v_0} + \vec{a}.t$$

$$\vec{x} = \int_t \vec{v}.dt$$

$$\vec{x} = \overrightarrow{x_0} + \overrightarrow{v_0}.t + {}^1\!/_2\,\vec{a}.t^2$$

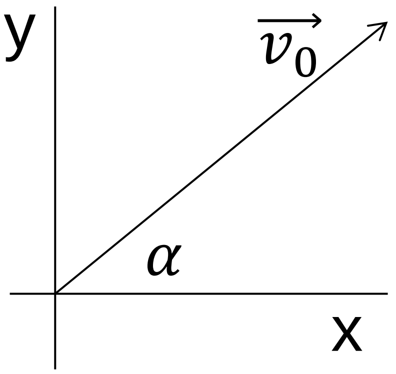
Vectors

- Note that position, acceleration and velocity are vectors
- Scalars are simpler…
- Use scalar versions of the equations for each dimension
  - x, y, z
- Separability makes things much simpler!

Example

- Break down the vector equation into its components *x* and *y*
- Use them independently
  - Great for calculating gravity effects of projectiles

$$\vec{v} = \overrightarrow{v_0} + \vec{a}.t$$



$$v_{0x} = |\overrightarrow{v_0}|.\cos\alpha$$
$$v_{0y} = |\overrightarrow{v_0}|.\sin\alpha$$

$$v_x(t) = v_{0x} + a_x(t).t$$
$$v_y(t) = v_{0y} + a_y(t).t$$

Projectile motion

- No force affects horizontal axis

  $a_x = 0$

- Gravity affects vertical axis

  $a_y = g = -9{,}8\ ms^{-2}$

- So:

$$x(t) = x_0 + v_{0x}.t$$

$$y(t) = y_0 + v_{0y}.t - {}^1/_2\, 9{,}8.t^2$$

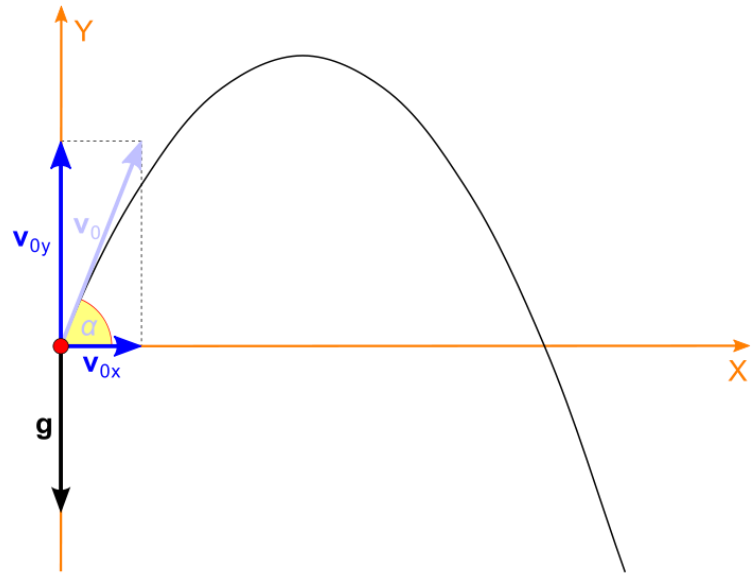


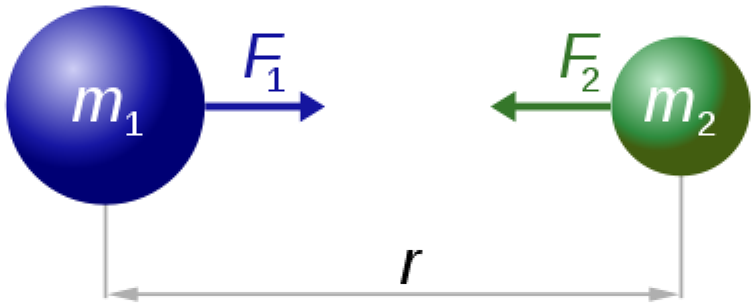
Image adapted from www.wikipedia.org

Engines

- How do I simulate an engine propelling an object?
  - I can use *force* if I know *mass*
  - I can use *acceleration* directly
- More difficult than gravity
  - Direction of acceleration is usually associated with the direction of velocity
  - Direction and magnitude of acceleration may be influenced externally: brakes, steering wheel, etc.
- Can easily combine with gravity

Gravitational force

- Any two objects with mass attract each other
  - Newton's law of universal gravitation
- Direction of force
  - Line containing the *centers of mass* of the two objects
- How come earth's gravitational pull is constant then?
  - It is not…



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

$G = 6.674\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$

Image adapted from www.wikipedia.org

Warp speed

- Near the speed of light
  - Mass increases with velocity
  - Mass deforms space
  - Things get messy
- What to do?
  - Go read Stephen Hawking
  - Cheat in your space combat simulation

Energy of moving objects

Kinetic energy

- Things in motion have *energy*
  - Defined as the *work* needed to accelerate a body of a given *mass* from rest to its stated *velocity*
  - Measured in *joules*
- Classic mechanics
  - Kinetic energy of a non-rotating rigid body:

$$E_k = {}^{1}/_{2}\, m v^2$$

Potential energy

- Things not moving also have ‘potential’ energy
  - Energy due to the position of the various objects of a system
- Most common potential energy
  - Gravity $E_p = m.g.h$
  - Higher objects have higher energy than lower objects with the same mass
  - Others: elastic, electric, magnetic

Conservation of energy

- Law of conservation of energy
  - The total amount of *energy* in an *isolated system* remains constant over time
- Isolated system
  - Physical system without any external exchange of matter or energy
- Great for approximating many real-world situations!

Back to our projectiles

- Projectile going up
  - Velocity decreasing – lower kinetic energy
  - Height increasing – higher potential energy
- Projectile going down
  - Vice-versa
- What about engines?
  - External energy source
  - Not an isolated system!

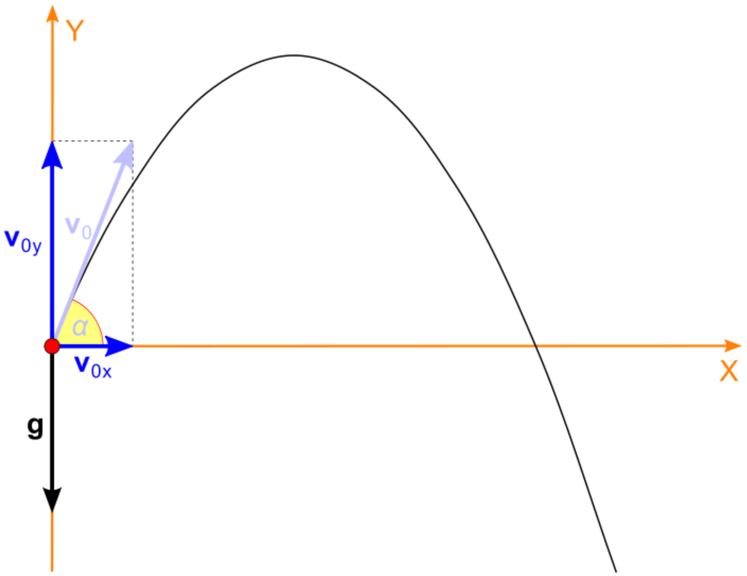


Image adapted from www.wikipedia.org

Object collision

- What happens when my projectile falls to the ground?
  - Law of conservation of energy
  - No external forces were applied
  - What happened to the kinetic energy?
- Ground must generate an opposing force that stops the projectile
  - Which could break or deform the ground…
- Energy is typically converted into heat
  - Explaining why even a small asteroid falling on earth can create a huge explosion…

Momentum

- What happens when two objects collide?
  - All collisions conserve *momentum*
  - Not all collisions conserve *kinetic energy*
- What is *momentum*?

$$p = m.v$$

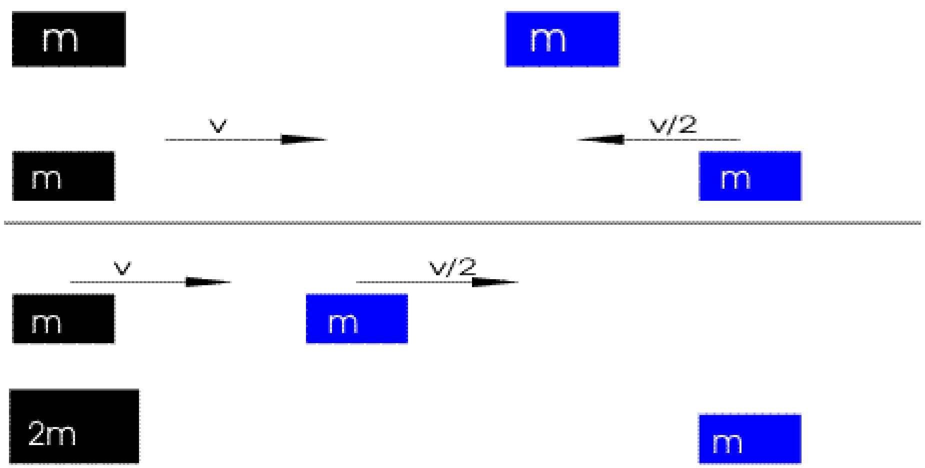
  - Product of *mass* and *velocity* of an object
  - *Conserved* in a *closed system*
- The momentum of a system of particles is the sum of their momenta

$$p = p_1 + p_2 = m_1 v_1 + m_2 v_2$$

Elastic collisions

- *Momentum* is conserved
- *Total kinetic energy* is conserved
- Solvable system of equations

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \,.$$



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Inelastic collision

- *Momentum* is conserved
- *Kinetic energy* is <u>not</u> conserved
- Coefficient of restitution
  – Fractional value representing the ratio of speeds after and before an impact

Images adapted from www.wikipedia.org

Why do moving objects stop?

(without collisions or brakes…)

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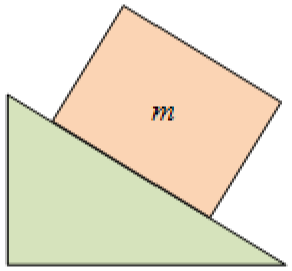
Reason #1 - Friction

- Force resisting the relative motion of solid surfaces in contact
  - Actually this is *dry kinetic friction…*
- Coulomb friction

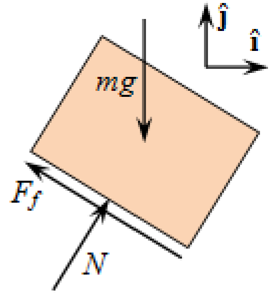
$$F_f \leq \mu F_n$$

- Does not depend on velocity!
- Depends on the *normal force* between two surfaces

A block on a ramp



Free body diagram of just the block



Images adapted from www.wikipedia.org

Reason #2 - Drag

- Forces which act on a solid object in the direction of the relative fluid flow
  - Air resistance
  - Fluid resistance
- Depends on velocity and the object's cross-sectional area

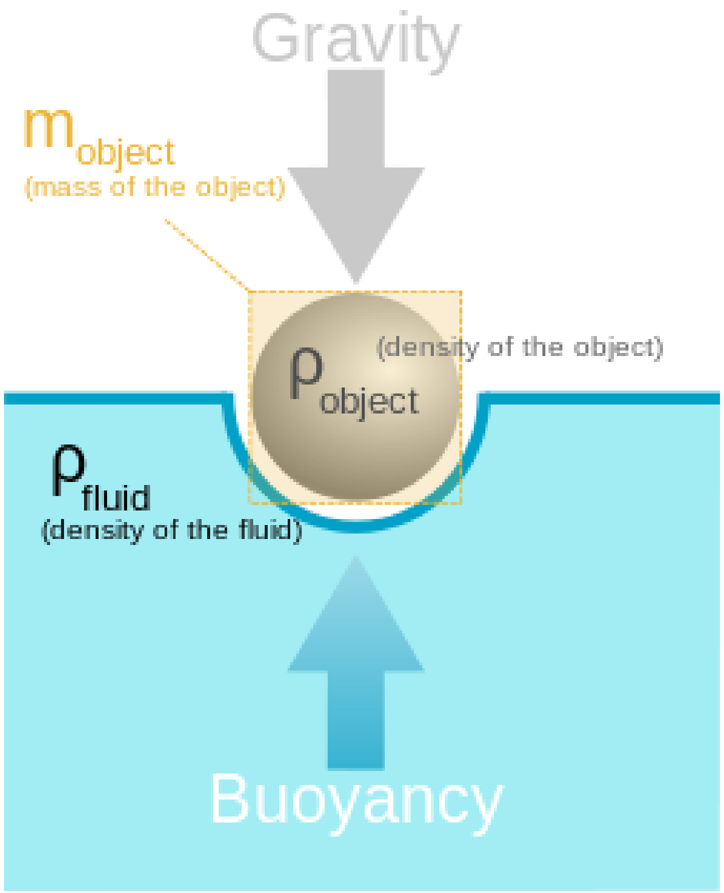
$$F_D = \frac{1}{2} \rho v^2 C_D A$$

- More complex than friction
- Use simple models (Stokes', Newton…)

Why do things float?

Buoyancy

- Archimedes' principle
  - A body immersed in a fluid suffers an upward force equal to the weight of the fluid the body displaces
- Objects float if they are less dense than the fluid they are in
  - Can you model such an object falling on a fluid?



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How do explosions work?
How can I model turbulence?
How do things fly?
Why do cars get lighter as they go faster?

...

Go read about physics!

Physics in space? ☺

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http://youtu.be/o8TssbmY-GM