Computer Vision – 6
Segmentation by Fitting

MAP-I Doctoral Programme

Miguel Tavares Coimbra

DOCTORAL PROGRAMME
IN COMPUTER SCIENCE

Outline

- The Hough Transform
- Fitting Lines
- Fitting Curves
- Fitting as a Probabilistic Inference Problem

Acknowledgements: These slides follow Forsyth and Ponce's "Computer Vision: A Modern Approach", Chapter 15.

Topic: The Hough Transform

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Fitting and Clustering

- Another definition for segmentation:
  - Pixels belong together because they conform to some model.
- Sounds like "Segmentation by Clustering"...
- Key difference:
  - The model is now **explicit**.

We have a mathematical model for the object we want to segment.
E.g. A line

Hough Transform

- Elegant method for direct object recognition
- Edges need not be connected
- Complete object need not be visible
- Key Idea: Edges **VOTE** for the possible model

Image and Parameter Spaces

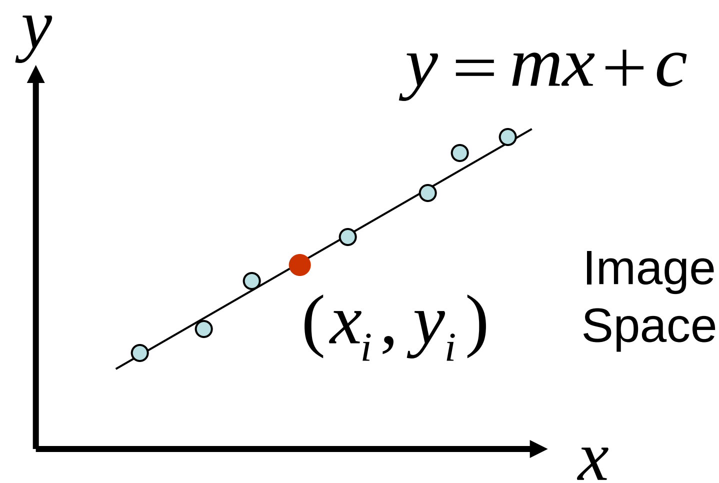
Equation of Line: $y = mx + c$

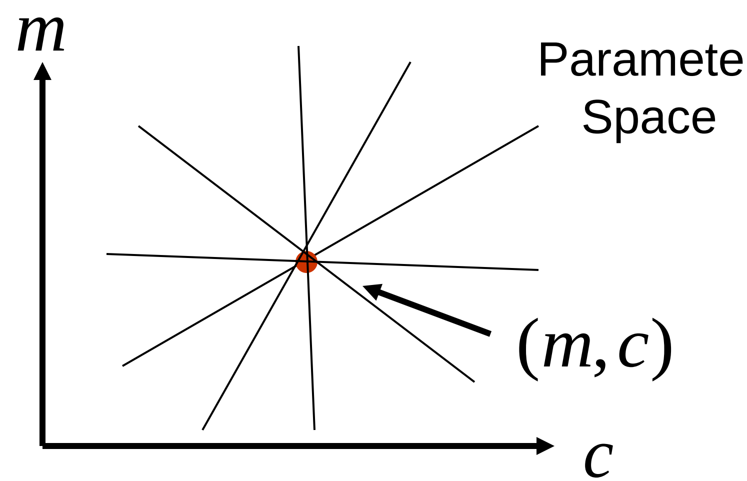
Find: (m, c)

Consider point: (x_i, y_i)

$$y_i = mx_i + c \quad or \quad c = -x_i m + y_i$$

Parameter space also called Hough Space





Line Detection by Hough Transform

Algorithm:

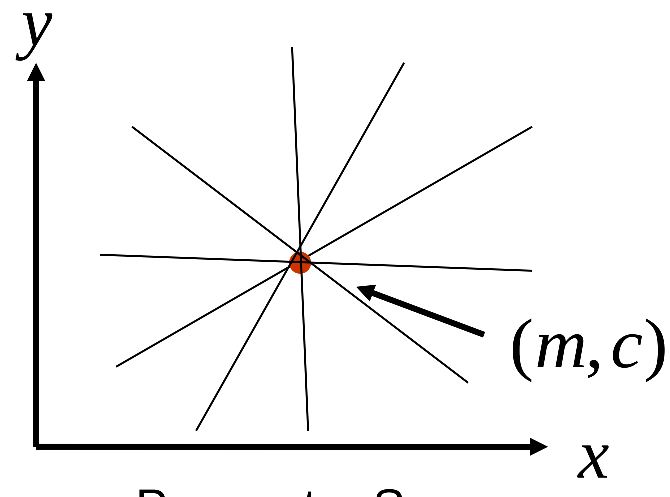
- Quantize Parameter Space (m, c)
- Create Accumulator Array $A(m, c)$
- Set $A(m, c) = 0 \quad \forall m, c$
- For each image edge (x_i, y_i) increment:

$$A(m, c) = A(m, c) + 1$$

- If (m, c) lies on the line:

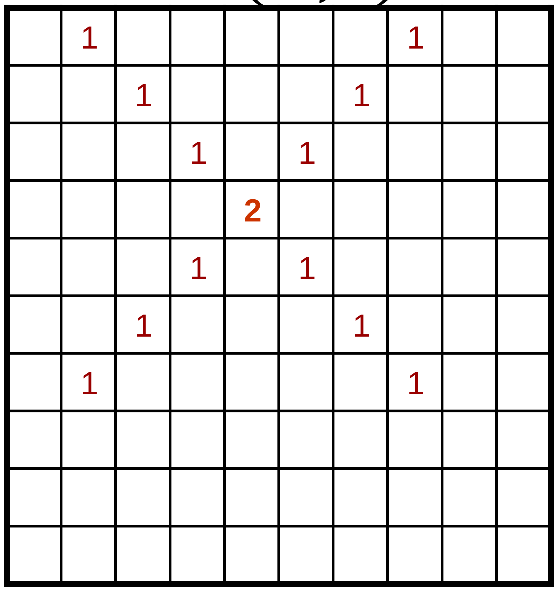
$$c = -x_i m + y_i$$

- Find local maxima in $A(m, c)$



Parameter Space

$A(m, c)$



Better Parameterization

NOTE: $-\infty \leq m \leq \infty$

Large Accumulator

More memory and computations

Improvement: (Finite Accumulator Array Size)

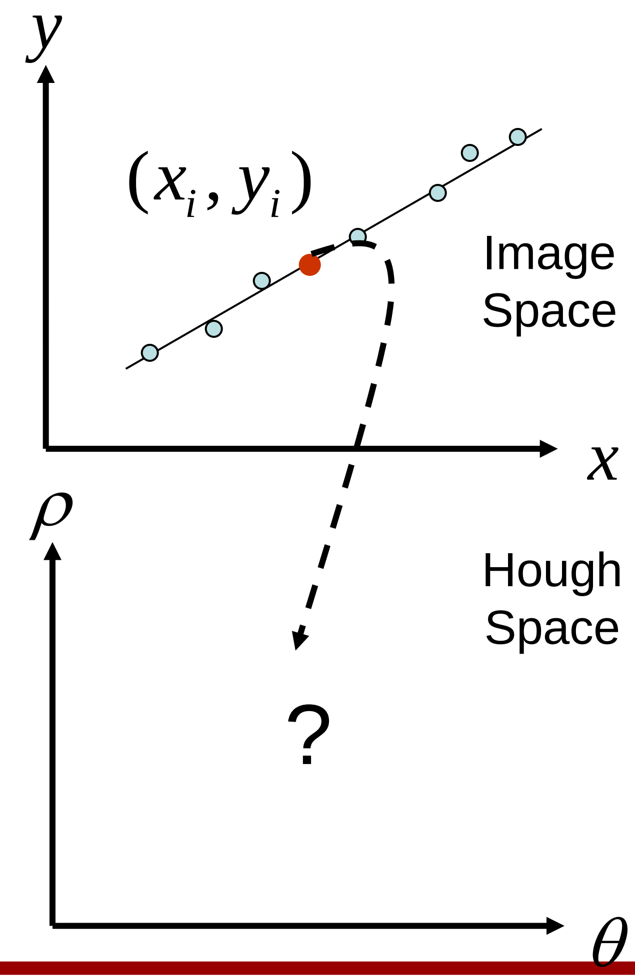
Line equation: $\rho = -x\cos\theta + y\sin\theta$

Here $0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq \rho_{\max}$

Given points (x_i, y_i) find (ρ, θ)

Hough Space Sinusoid



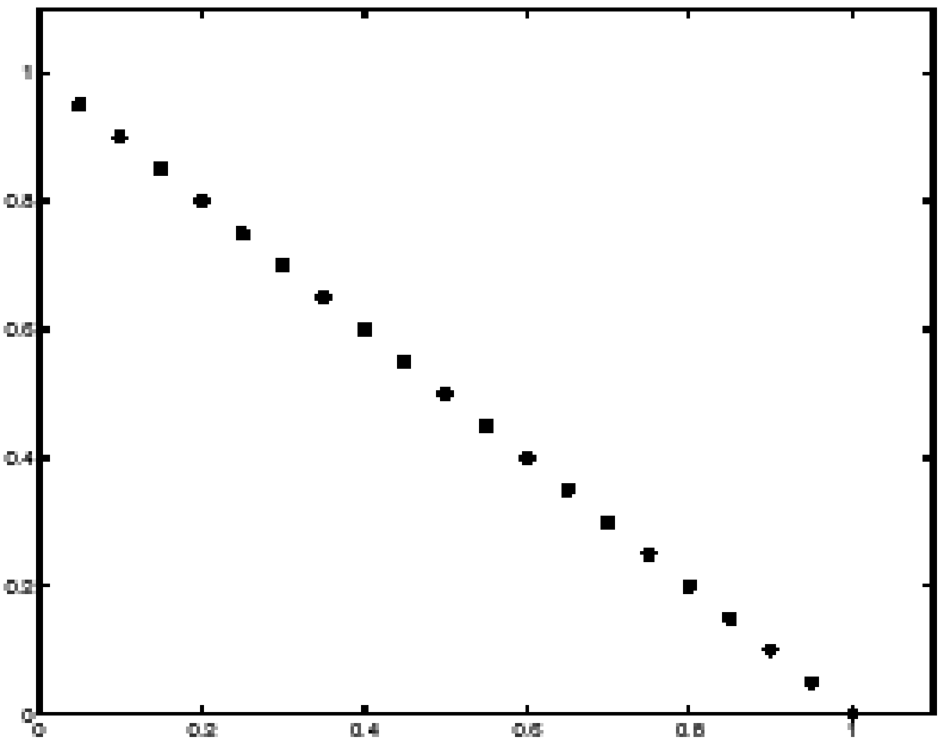
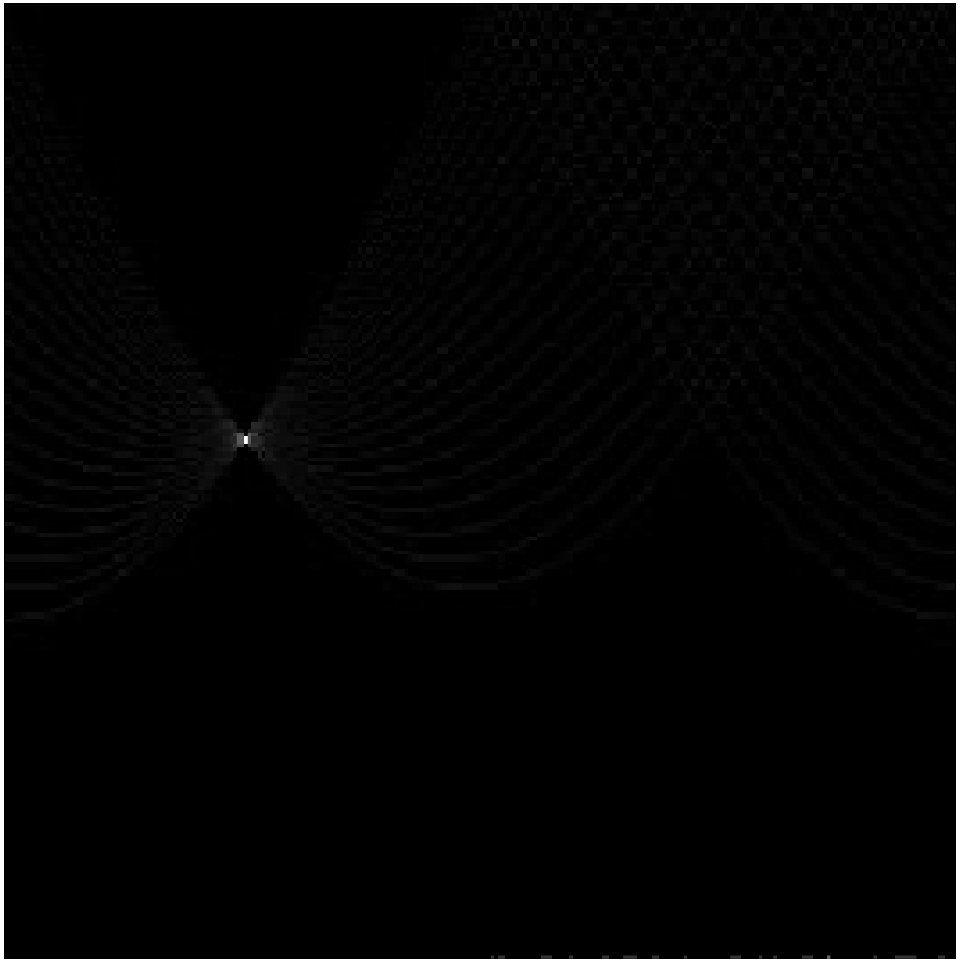
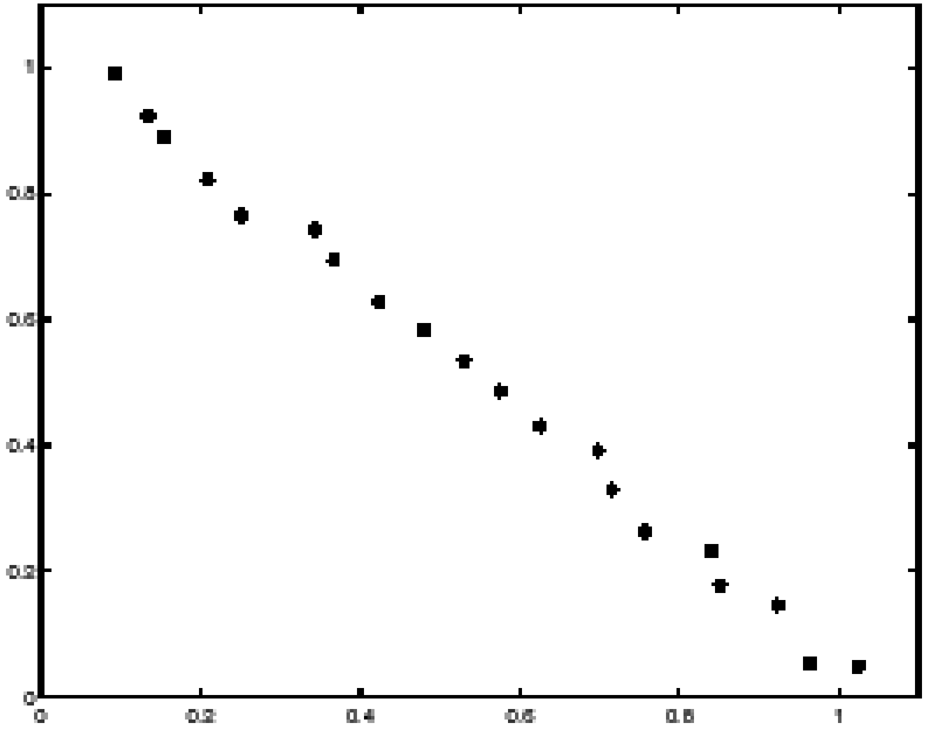


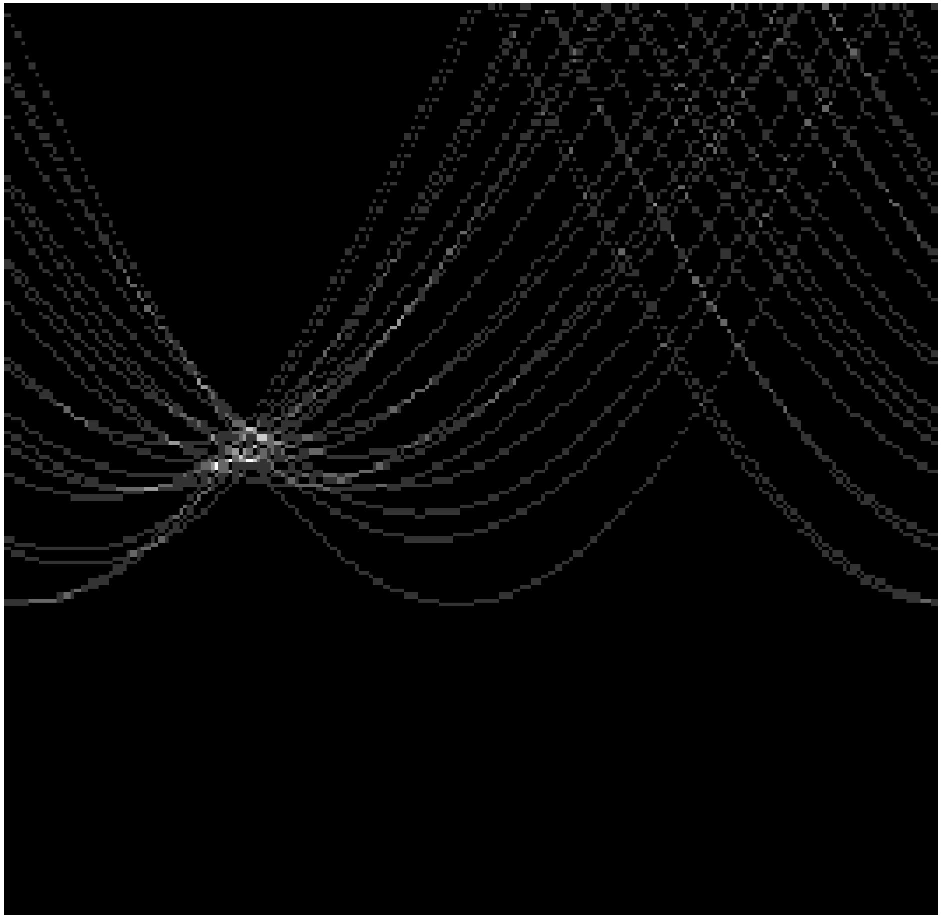
Image space

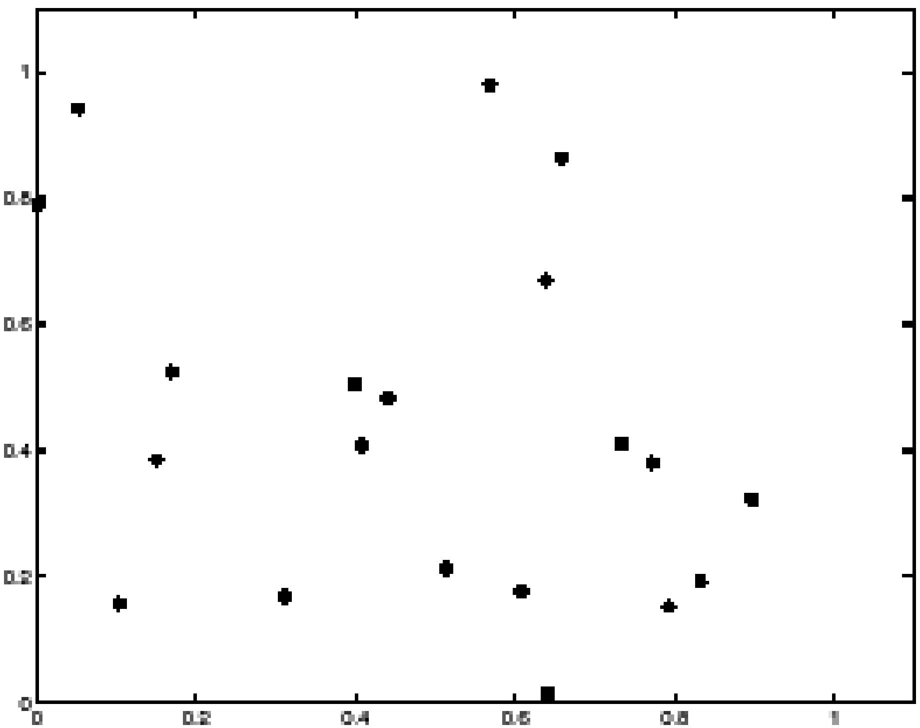


Votes

Horizontal axis is θ, vertical is rho.


1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1




0
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0.4
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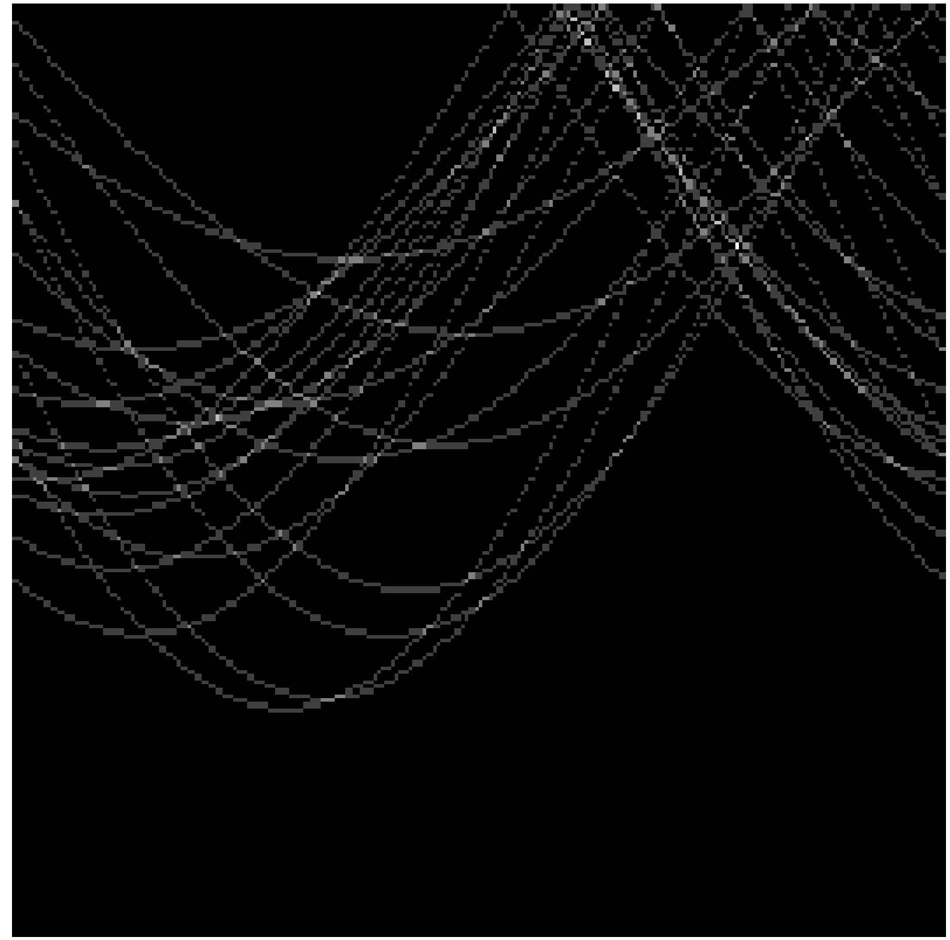
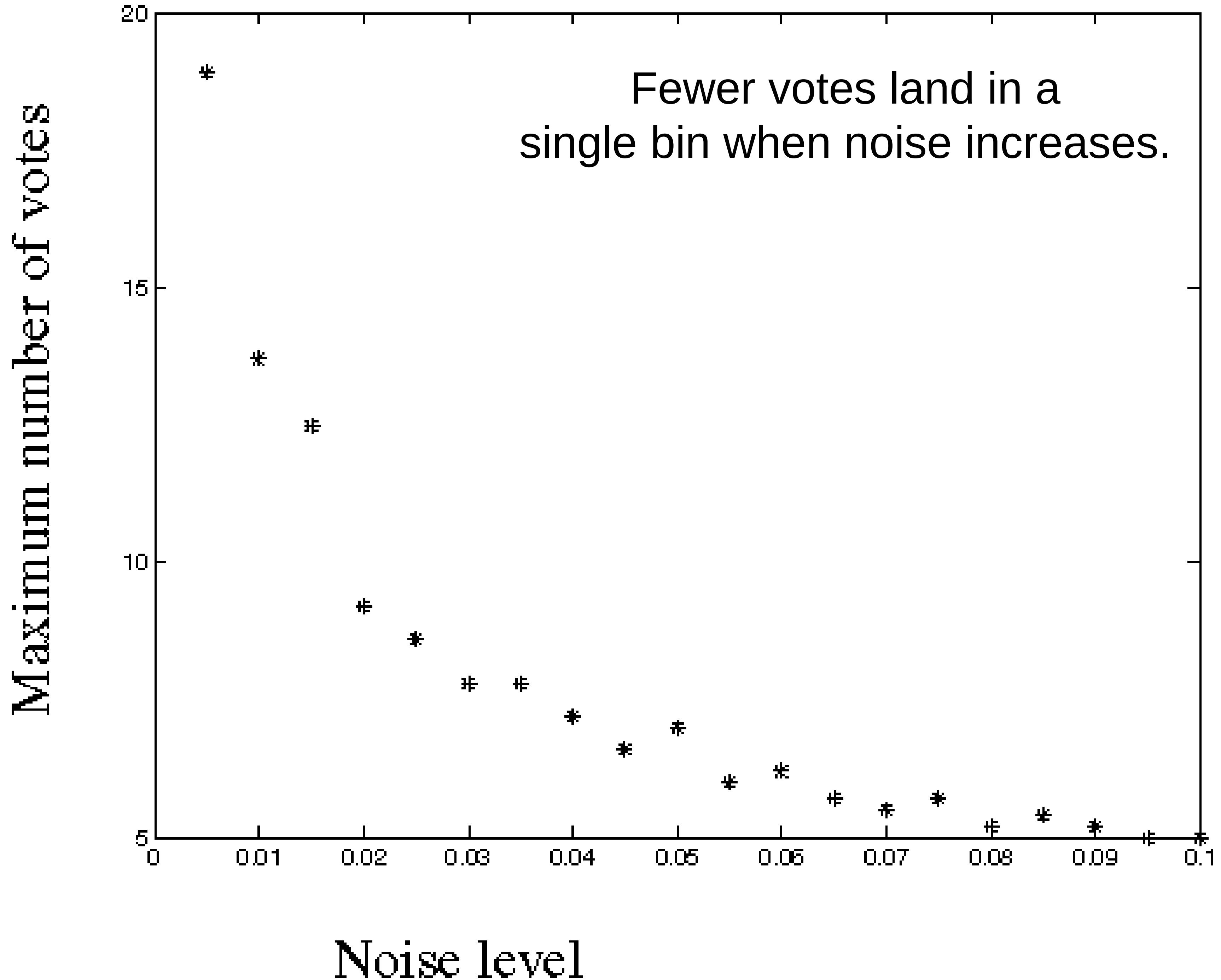
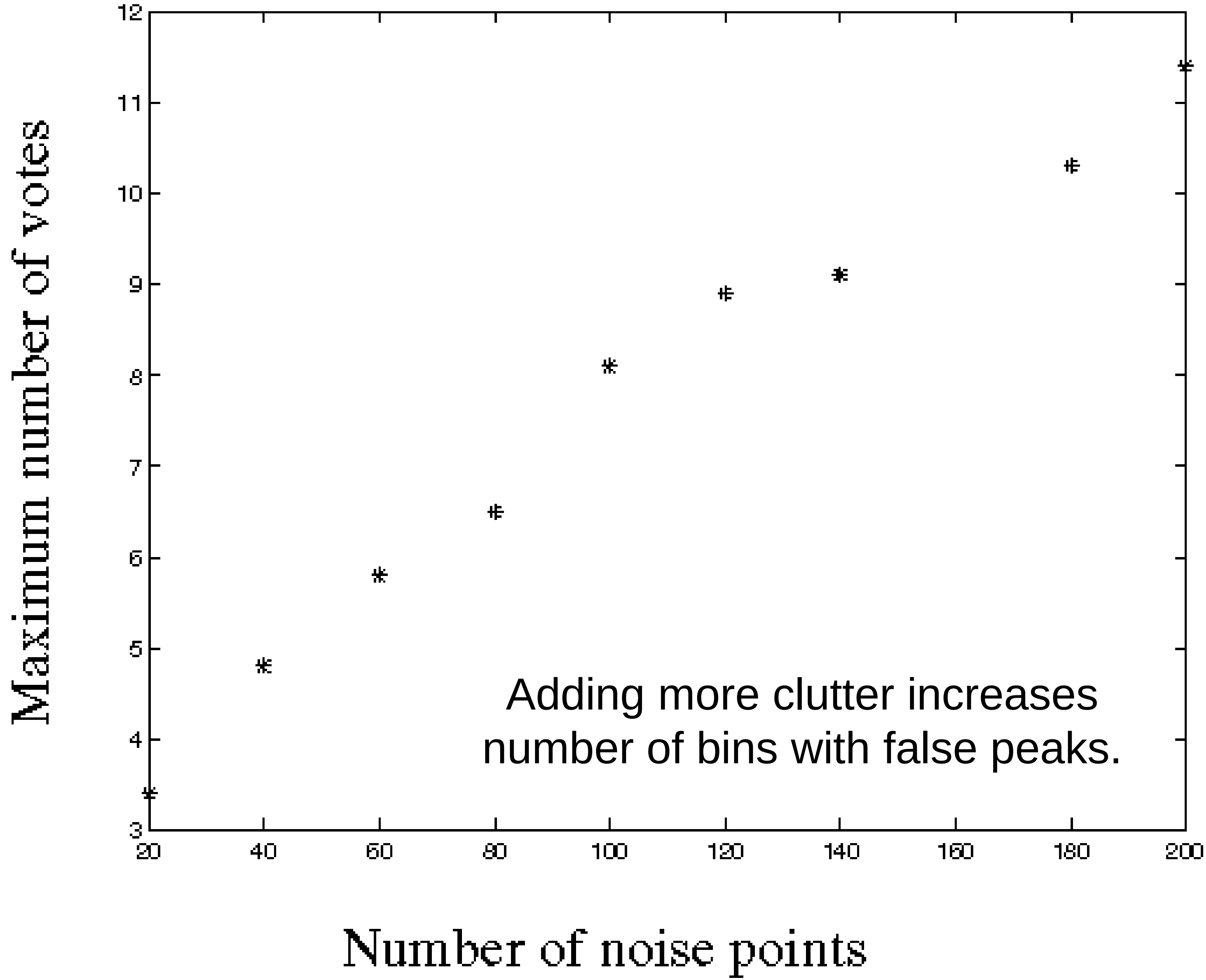
0.8
1



Mechanics of the Hough Transform

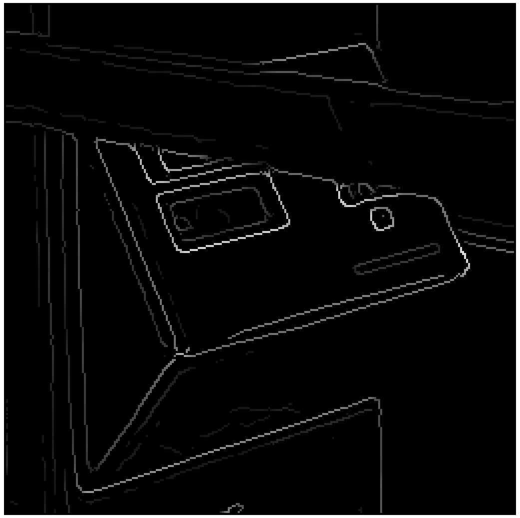
- Difficulties
  - how big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - Count the peaks in the Hough array
  - Treat adjacent peaks as a single peak
- Which points belong to each line?
  - Search for points close to the line
  - Solve again for line and iterate


Fewer votes land in a
single bin when noise increases.
Maximum number of votes
Noise level
20
15
10
5
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

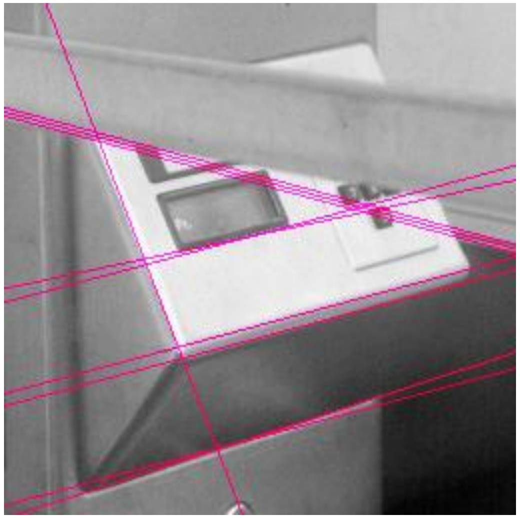

Maximum number of votes
12
11
10
9
8
7
6
5
4
3
20
40
60
80
100
120
140
160
180
200
Adding more clutter increases
number of bins with false peaks.
Number of noise points

Real World Example

Original

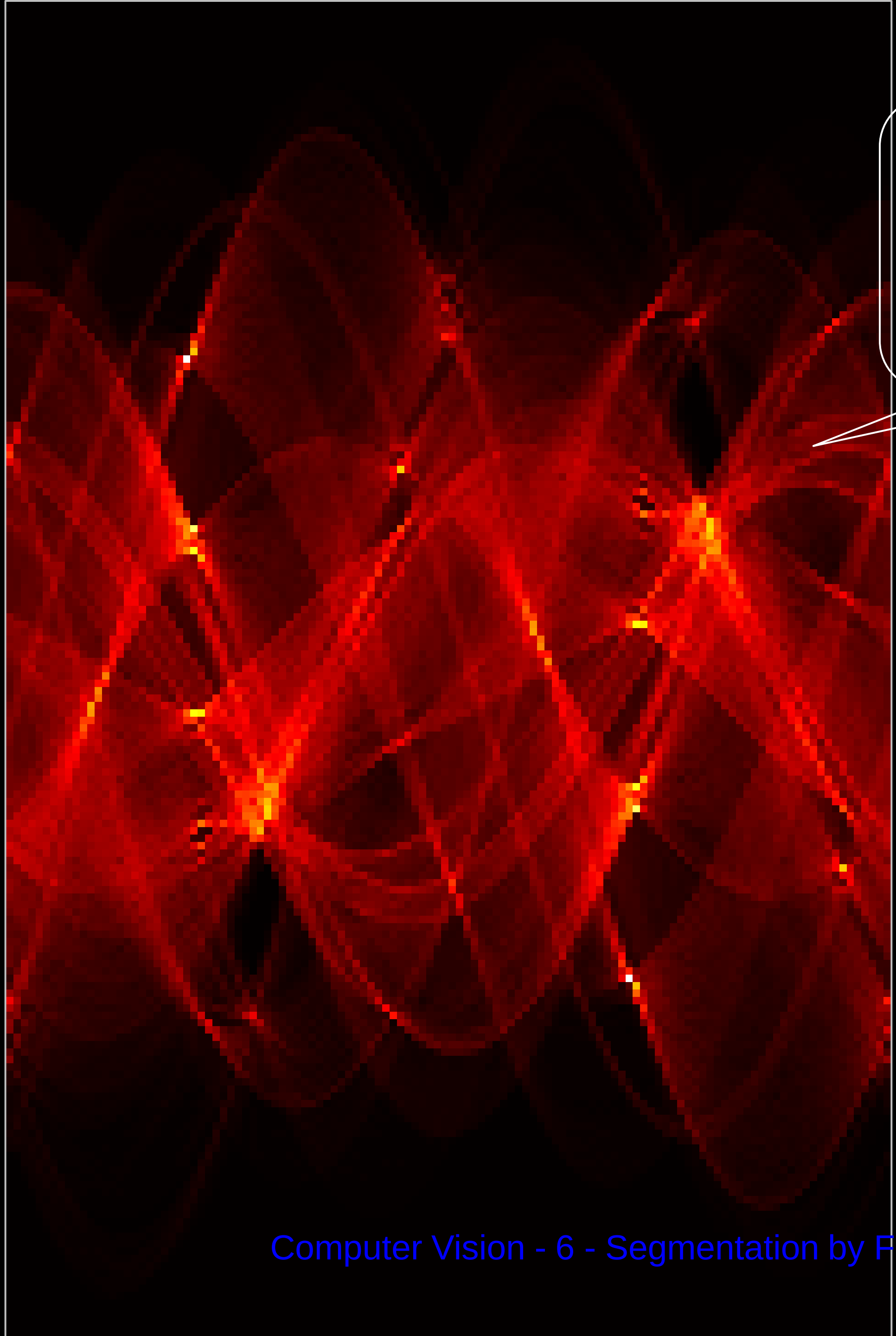


Edge Detection



Found Lines

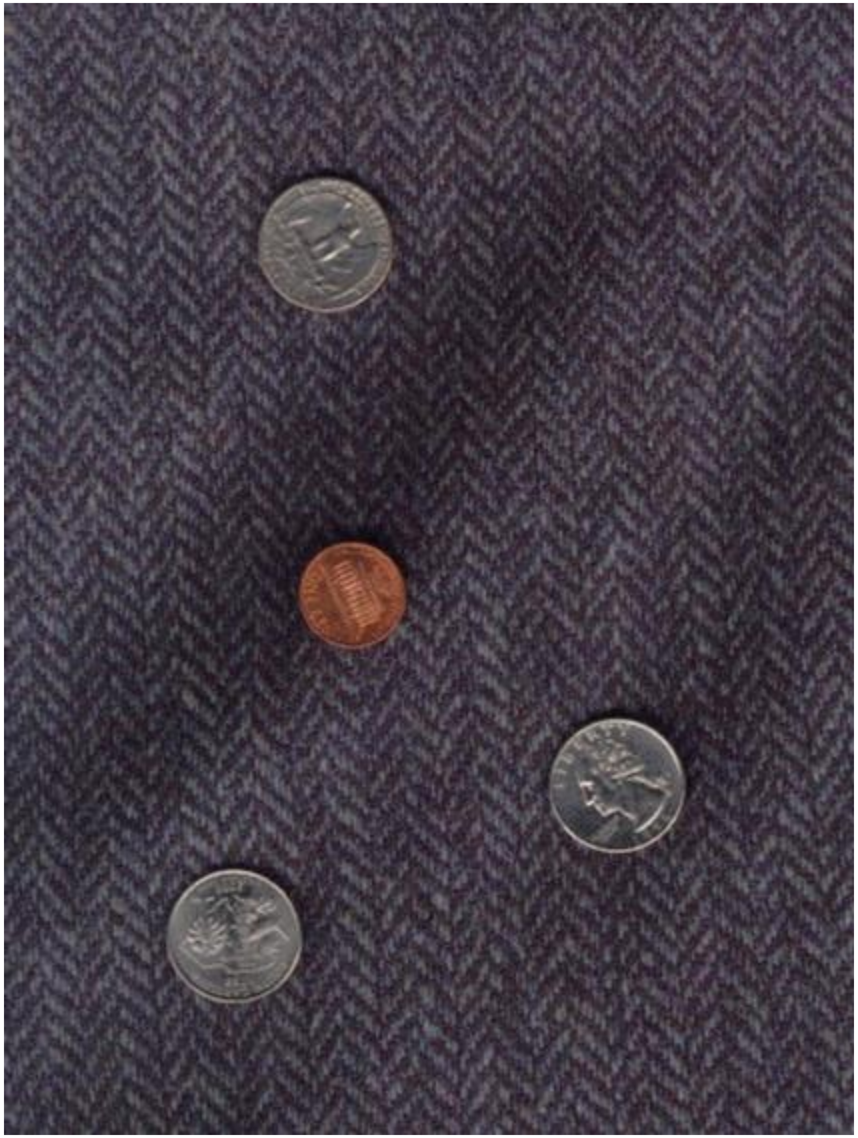
Parameter Space



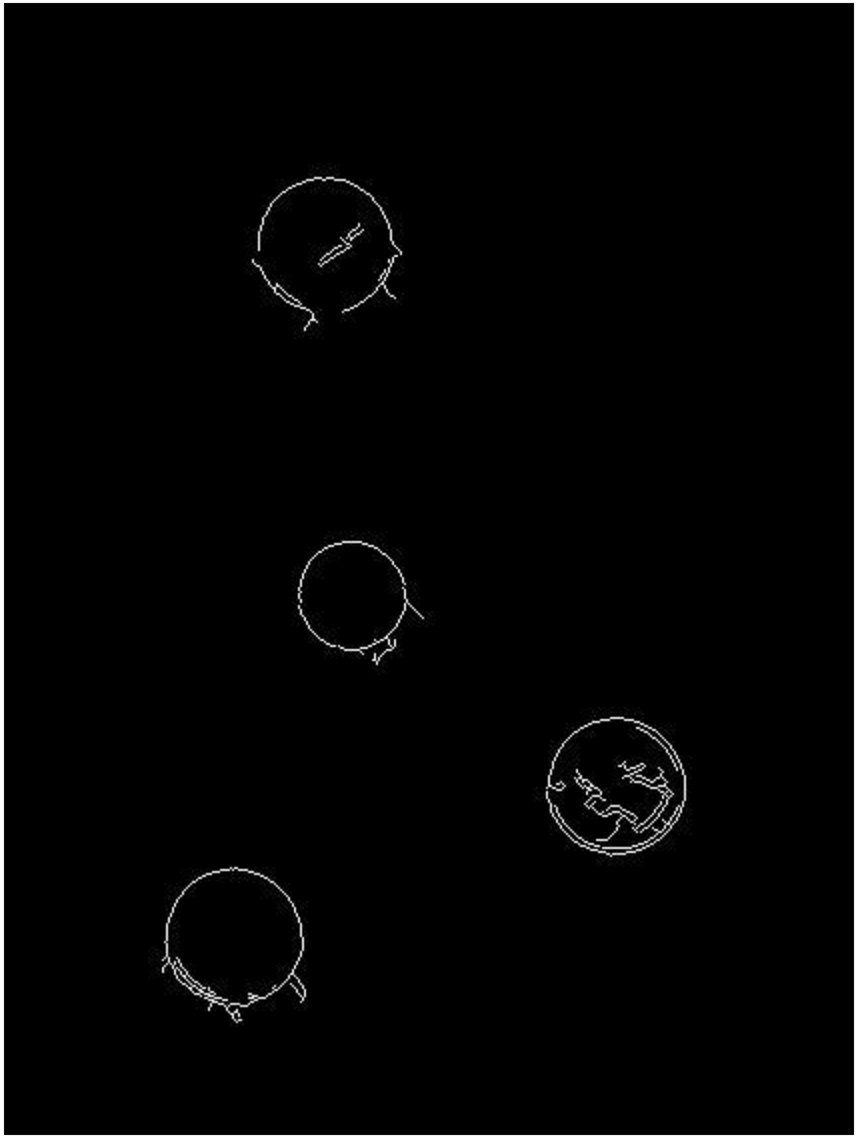
Where are the lines?

Other shapes

Original



Edges when using circle model



Topic: Fitting Lines

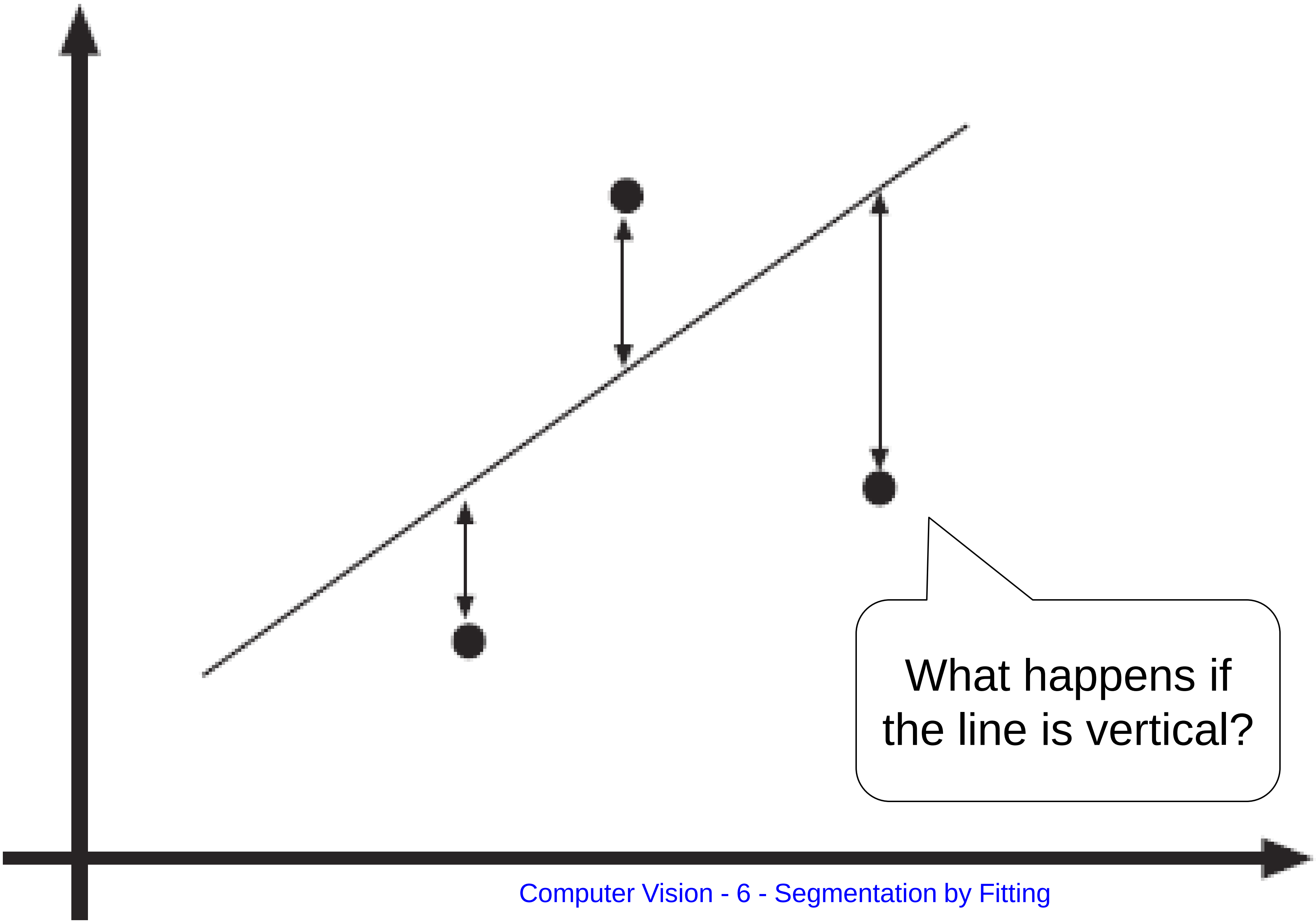
- The Hough Transform
- Fitting Lines
- Fitting Curves
- Fitting as a Probabilistic Inference Problem

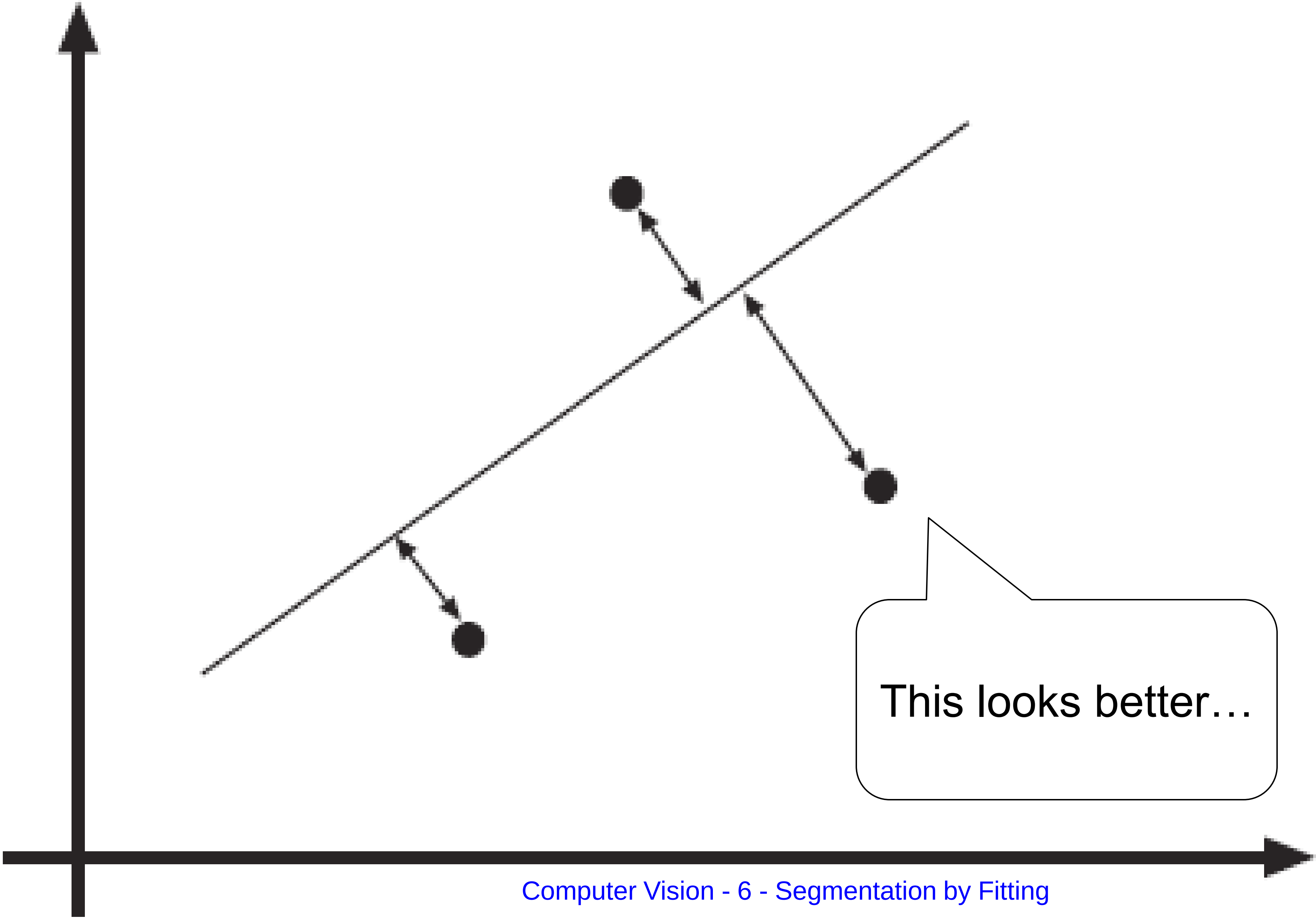
Least Squares

- Popular fitting procedure.
- Simple but biased (why?).
- Consider a line:

$$y = ax + b$$

- What is the line that best predicts all observations (x_i, y_i)?
  - Minimize: $$\sum_i (y_i - a x_i - b)^2$$


What happens if
the line is vertical?


This looks better…

Total Least Squares

- Works with the actual distance between the point and the line (rather than the vertical distance).
- Lines are represented as a collection of points where:

$$ax + by + c = 0$$

- And:

$$a^2 + b^2 = 1$$

Again... Minimize the error, obtain the line with the 'best fit'.

Point correspondence

- We can estimate a line but, **which points are on which line**?
- Usually:
  - – We are fitting lines to edge points, so…
  - – Edge directions can give us hints!
- What if I only have isolated points?
- Let's look at two options:
  - – Incremental fitting.
  - – Allocating points to lines with K-means

Incremental Fitting

- Start with connected *curves* of edge points
- Fit *lines* to those points in that curve.
- Incremental fitting:
  - Start at one end of the *curve*.
  - Keep fitting all points in that curve to a line.
  - Begin another line when the fitting deteriorates too much.
- Great for closed curves!

```
Put all points on curve list, in order along the curve
empty the line point list
empty the line list

Until there are two few points on the curve
  Transfer first few points on the curve to the line point list
  fit line to line point list

  while fitted line is good enough
    transfer the next point on the curve
    to the line point list and refit the line
  end

  transfer last point back to curve
  attach line to line list
end
```

K-means allocation

- What if points carry no hints about which line they lie on?
- Assume there are *k* lines for the *x* points.
- Minimize: $$\sum_{\text{lines}} \sum_{\text{points}} dist(\text{line}, \text{point})^2$$
- Iteration:
  - Allocate each point to the closest line.
  - Fir the best line to the points allocated to each line.

```
Hypothesize k lines (perhaps uniformly at random)
or
hypothesize an assignment of lines to points
and then fit lines using this assignment

Until convergence
  allocate each point to the closest line
  refit lines
```

Topic: Fitting Curves

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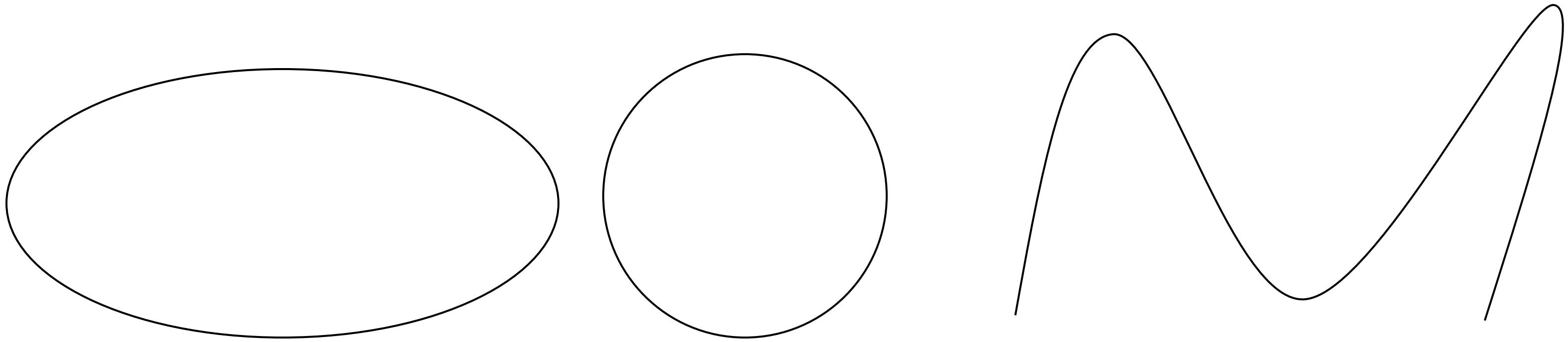
Fitting Curves

- In principle, **same as fitting lines**.
  - Minimize distances of points to the curve.
- However, how can I tell the **distance between a point and a curve**?
  - Get your geometry book and **solve it**.
  - Get your engineering book and **approximate it**.

Implicit Curves

- Coordinates satisfy some parametric equation.
- If the equation is **polynomial** then the curve is said to be **algebraic**.



Curve	equation
Line	$ax + by + c = 0$
Circle, center (a, b) and radius r	$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$
Ellipses (including circles)	$ax^2 + bxy + cy^2 + dx + ey + f = 0$ where $b^2 - 4ac < 0$
Hyperbolae	$ax^2 + bxy + cy^2 + dx + ey + f = 0$ where $b^2 - 4ac > 0$
Parabolae	$ax^2 + bxy + cy^2 + dx + ey + f = 0$ where $b^2 - 4ac = 0$
General conic sections	$ax^2 + bxy + cy^2 + dx + ey + f = 0$

Distance to an Implicit Curve

- Distance between point *(x,y)* and **closest point** of the curve *(u,v)*.
- So, this distance vector:
  - Contains the point.
  - Is normal to the curve: $\phi(u,v)$
- Therefore:

$$\phi(u,v) = 0 \text{ since } (u,v) \text{ is on the curve}$$

$$s = (x,y) - (u,v) \text{ is normal to the curve}$$

Calculating the distance

- We can derive an equation to this distance but solving it is not always trivial.
- Even simple polynomial curves can make this problem rather daunting.
- Several ways to approximate distances.
- Want to dig deeper?
  - Forsyth and Ponce, Chapter 15.3.

Parametric Curves

- Coordinates are given as **parametric functions** of a **parameter**.

Curves	Parametric form	parameters
Circles centered at the origin	$(rsin(t), rcos(t))$	$\theta = r$ $t \in [0, 2\pi)$
Circles	$(rsin(t) + a, rcos(t) + b)$	$\theta = (r, a, b)$ $t \in [0, 2\pi)$
Axis aligned ellipses	$(r_1 sin(t) + a, r_2 cos(t) + b)$	$\theta = (r_1, r_2, a, b)$ $t \in [0, 2\pi)$
Ellipses	$(\cos\phi\,(r_1 sin(t) + a) - \sin\phi\,(r_2 cos(t) + b)\,,$ $\sin\phi\,(r_1 sin(t) + a) + \cos\phi\,(r_2 cos(t) + b))$	$\theta = (r_1, r_2, a, b, \phi)$ $t \in [0, 2\pi)$
cubic segments	$(at^3 + bt^2 + ct + d, et^3 + ft^2 + gt + h)$	$\theta = (a, b, c, d, e, f, g, h)$ $t \in [0, 1]$

Topic: Fitting as a PIP

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Error models

- Total least squares fitting seems ‘reasonable’ but it’s in fact… arbitrary.
  - It does not take into account the error model of our data.
  - We should thus be asking the following question:
- How did the points come to not lie on a line in the first place?

Maximizing the likelihood

- Let's assume our errors are Gaussian noise perturbations to our line.

$$N(0,\sigma)$$

$$\mathrm{au} + \mathrm{bv} + \mathrm{c} = 0, \mathrm{a}^2 + b^2 = 1$$

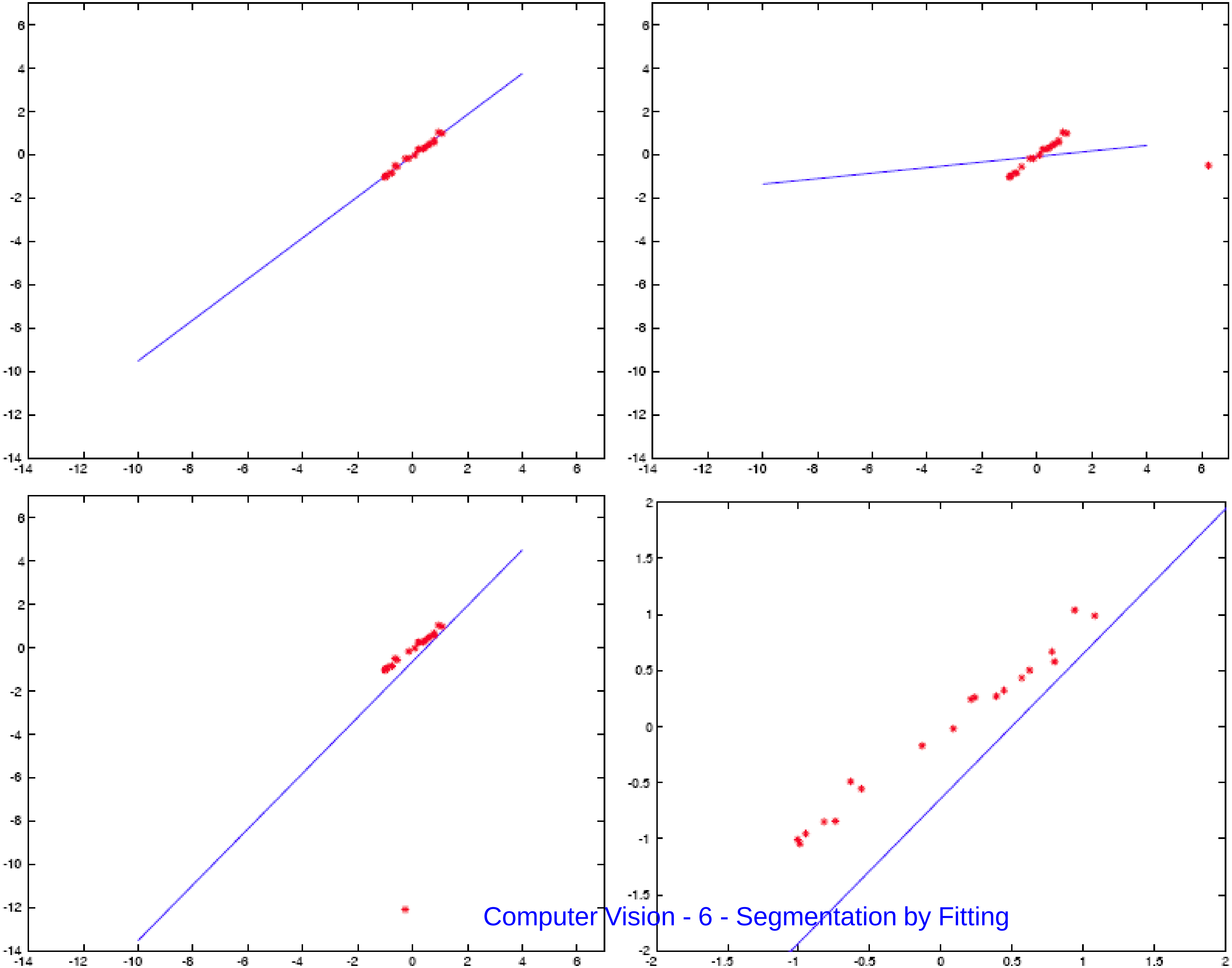
$$P(\mathrm{meas.} \mid a,b,c) = \prod_i P(x_i, y_i \mid a,b,c)$$

- We can thus maximize our log-likelihood:

$$-\frac{1}{2\sigma^2}\sum_i (ax_i + by_i + c)^2$$

Difficulties

- Robustness
  - TLS places a huge weight on large errors.
  - Leads to severe fitting errors.
  - Our solution must be **robust** to errors.
- Missing data
  - Some points are noise.
  - Some points come from real lines.
  - Distinguishing them is vital for a good fitting!


6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-14 -12 -10 -8 -6 -4 -2 0 2 4 6
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2 -1.5 -1 -0.5 0 0.5 1 1.5 2

Robust fitting

- How do we minimize the effect of these **outliers**?
  - **M-estimators** - Give noise ‘heavier tails’.
  - Study data points and identify outliers.
  - **RANSAC** – Search for points that ‘appear to be good’ and use those.
- Want to know more?
  - Forsyth and Ponce, Chapter 15 and 16.

Resources

- Forsyth and Ponce, Chapter 15