Note on the Cournot and Stackelberg Competitions: is it worth to be the last playing?

Margarida Carvalho
INESC TEC and Faculdade de Ciências da Universidade do Porto, Portugal
margarida.carvalho@dcc.fc.up.pt

João Pedro Pedroso
INESC TEC and Faculdade de Ciências da Universidade do Porto, Portugal
jpp@fc.up.pt

Technical Report Series: DCC-2015-01

U. PORTO

FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

Departamento de Ciência de Computadores

Faculdade de Ciências da Universidade do Porto
Rua do Campo Alegre, 1021/1055,
4169-007 PORTO,
PORTUGAL

Tel: 220 402 900   Fax: 220 402 950
http://www.dcc.fc.up.pt/Pubs/

Note on the Cournot and Stackelberg Competitions: is it worth to be the last playing?

Margarida Carvalho
INESC TEC and Faculdade de Ciências da Universidade do Porto, Portugal
margarida.carvalho@dcc.fc.up.pt

João Pedro Pedroso
INESC TEC and Faculdade de Ciências da Universidade do Porto, Portugal
jpp@fc.up.pt

September 2015

Abstract

In this brief note, a competition on a new product produced by two firms sharing a market is analyzed both from a Cournot and Stackelberg point of view. In contrast with classical models, setup costs are considered in the firms' production costs, leading to ambiguities in their best response strategies.

Our aim is to establish the different equilibria strategies that may arise from these two competition versions. Moreover, our goal is to open the discussion of whether it might be worth for a player to wait for the opponents to move, forcing a Stackelberg equilibrium to be played.

Keywords: Cournot Competition, Stackelberg Competition, Setup costs, Nash Equilibria.

1 Introduction

The Cournot competition (CG) [1] was one of the earliest games analyzed. It models an economic market in which the participating firms produce a homogeneous product and each firm's output level influences the market price and thus, their profits. In this model, the firms play simultaneously.

Stackelberg competition (SG) [2] reflects markets in which there is a set of firms called the leaders and another set of firms called the followers. First, the leaders simultaneously take their decisions and then, the followers observe the selected leaders' strategies and, simultaneously, choose their strategies. In this paper, we focus in a SG where in each of the two stages a CG is played.

We restrict our analyzes to rational strategies computed using the concept of Nash equilibrium [3] and Stackelberg equilibrium [2].

In order to approximate reality, a combinatorial component in the firms' costs is introduced. Setup costs are included representing the cost of turning on the production machine or an investment in a new production technology. This is a novel model, since research on Cournot and Stackelberg competitions focused on problems without combinatorial components, and the optimization models have focused on single agent problems.

While in our model a Cournot equilibrium always exists (which can be proven, *e.g.*, using the fact that the Cournot competition is a potential game [4]), a Stackelberg equilibrium might not. Thus, in order to overcome the possible absence of a Stackelberg equilibrium, the notion of ϵ-Stackelberg equilibrium is introduced which might lead to interesting cases with the followers out of the market.

We discuss the relevance of comparing the solutions (equilibria) for these two economic markets. A related discussion is presented in [5] where the authors search for games in which Nash and Stackelberg equilibria coincide.

Section 2 describes the CG and SG, the solution concept for games, and provides illustrative examples to make the exposition clear. In Section 3, differences in the solutions for these games dynamics are discussed. We end this note highlighting the conclusions and further research to be done in this context in Section 4.

2 The Models

Define $\mathcal{M} = \{1, 2, \ldots, m\}$ as the set of players. In both games, player p must plan her production by deciding if production is going to take place or not, $y_p = 1$, $y_p = 0$, respectively, and the quantity to be produced $q_p \geq 0$, for $p \in \mathcal{M}$. The tuple $z_p = (y_p, q_p)$ denotes player p decision. The game parameters are f_p, c_p and P which represent the setup production cost, the unit production cost and the unit market price function, respectively.

The interactions among the players occur through the unit market price P, this is, P depends on the decisions (strategies) of all players. The typical function modeling the unit market price is a linear function depending on the total product quantity in the market:

$$P(Q) = (a - bQ)^+ \tag{2.1}$$

where $y^+ = \max(y, 0)$ and $Q = \sum_{p=1}^{m} q_p$. The parameter $a > 0$ represents the market size and $b > 0$ the price elasticity.

At this point, we are ready to describe for each player p the set of feasible strategies X_p:

$$0 \leq q_p \leq M_p y_p \tag{2.2a}$$

$$y_p \in \{0, 1\} \tag{2.2b}$$

where M_p is the production capacity of player p. Constraint (2.2a) ensures that if there is production, $q_p > 0$, then $y_p = 1$ and the capacity limitation is satisfied. Constraint (2.2b) forces y_p to be binary.

The main difficulty of this model is in the players decision of production, this is, whether their revenue will be higher than the production costs. See Figure 2.1.

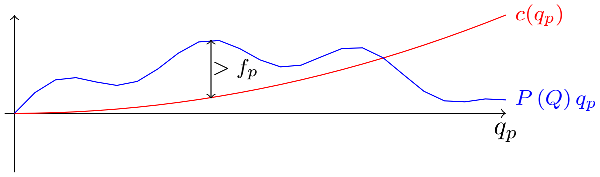

Figure 2.1: Profit Vs Production Cost.

The Cournot and Stackelberg competitions will be described in sections 2.1 and 2.2, respectively.

2.1 Cournot Game (CG)

In the Cournot game, the players move simultaneously. In this way, each player p problem can be described as

$$\underset{z_p \in X_p}{\text{maximize}} \quad \Pi_p = P(Q)q_p - f_p y_p - c_p q_p \tag{2.3}$$

The objective function (2.3) is player p profit.

A solution to CG means to find players' strategies such that Problem (2.3) is solved simultaneously for $p \in \mathcal{M}$. In other words, we are interested in computing *Cournot equilibria* of this game. A Cournot equilibrium is a profile of feasible strategies $(\hat{z}_1, \ldots, \hat{z}_m) \in \prod_{p=1}^{m} X_p$ such that for $p \in \mathcal{M}$, $\forall z_p \in X_p$, it holds

$$\Pi_p\left(\hat{z}_1, \ldots, \hat{z}_p, \ldots, \hat{z}_m\right) \geq \Pi_p\left(\hat{z}_1, \ldots, \hat{z}_{p-1}, z_p, \hat{z}_{p+1}, \ldots, \hat{z}_m\right) \tag{2.4}$$

meaning that, none of the players has advantage to unilaterally deviate from the equilibrium.

The CG is a potential game which implies the existence of a Cournot equilibrium (shown in [6]). However, uniqueness cannot be guaranteed, as shown in the following example.

Example 2.1 *Consider an instance with two players, $m = 2$, such that the market price parameters are $a = 20$ and $b = 1$. The production parameters are $M_1 = +\infty$, $M_2 = +\infty$, $f_1 = 0$, $c_1 = 10$, $f_2 = 25$ and $c_2 = 5$.*

If player p produces at equilibrium, her optimal strategy is determined by deriving Π_p in terms of q_p and equalizing to zero (note that Π_p is strictly concave when $P(Q) > 0$ and $y_p = 1$). Therefore, player p optimal strategy when production takes place is

$$\frac{(a - bq_k - c_p)^+}{2b} \tag{2.5}$$

with $p \neq k$. Player p only produces if the associated profit is greater than the setup cost. Hence, the best response is

$$q_p^*(q_k) = \begin{cases} \frac{a-bq_k-c_p}{2b}, & \text{if } a - bq_k - c_p \geq 2\sqrt{bf_p} \\ 0, & \text{if } a - bq_k - c_p \leq 2\sqrt{bf_p}. \end{cases} \tag{2.6}$$

Figure 2.2 illustrates these best responses and puts in evidence the existence of two equilibria: $(y_1, q_1, y_2, q_2) = (1, \frac{5}{3}, 1, \frac{20}{3})$ and $(y_1, q_1, y_2, q_2) = (1, 5, 0, 0)$.

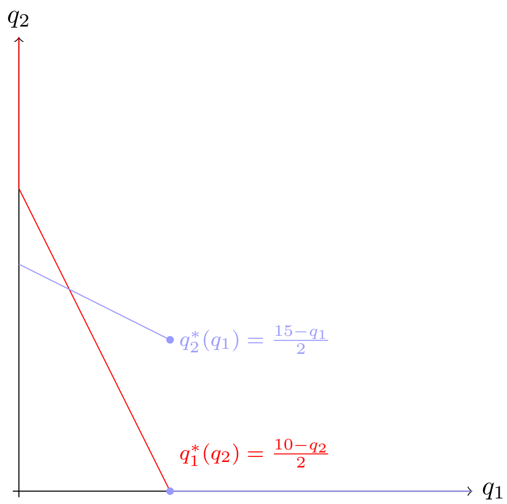

Figure 2.2: Players best responses.

2.2 Stackelberg Game (SG)

In the Stackelberg game (SG), there are players that move first, called the leaders, and the remaining called followers that observe their strategies and move next. Let us assume that players in $\mathcal{L} = \{1, \dots, l\}$ take the rule of leaders and players in $\mathcal{F} = \{l+1, \dots, m\}$ the rule of followers. The game can be described by the following bilevel optimization problems for each leader $p \in \mathcal{L}$:

$$\underset{z_p \in X_p}{\text{maximize}} \;\; \Pi_p = P\left(\sum_{k=1}^{l} q_k + \sum_{k=l+1}^{m} q_k\right) q_p - f_p y_p - c_p q_p \tag{2.7a}$$

$$\begin{aligned} &\text{where, for each } k \in \mathcal{F}, \\ &q_k \text{ solves the follower's problem:} \\ &\underset{z_k \in X_k}{\text{maximize}}\, \Pi_k = P(\sum_{k=1}^{l} q_k + \sum_{k=l+1}^{m} q_k) q_k - f_k y_k \\ &\quad + \sum_{p=1}^{l} h_k^p y_p y_k - c_k q_k \end{aligned} \tag{2.7b}$$

where h_k^p is the reduction in player k setup cost due to observation of the investment (technology) of the leaders. An optimal solution of (2.7) is a *Stackelberg equilibrium*. A profile of strategies $(z_1, \ldots, z_m) \in \prod_{p=1}^{m} X_p$ is bilevel feasible to player p problem (2.7) if $(z_{l+1}, \ldots, z_m)$ is a Cournot equilibrium given $(z_1, \ldots, z_l)$. In contrast to CG, SG might be undefined and thus, fail to have an equilibrium. There are two reasons for this to occur: due to no assumption about how cooperative with the leaders are the followers and due to the discontinuity of the followers' optimal solutions. When player $k \in \mathcal{F}$ has multiple optimal strategies for a fixed leaders' strategy which results in distinct profits for them, problem (2.7) is not well defined. In this context, it is typical for two-players games to decide about an *optimistic or pessimistic formulation.* Under the optimistic formulation, the follower chooses among her optimal solutions the one that benefits the most the leader. On the other hand, the pessimistic formulation assumes that among follower's optimal solutions the one that penalizes the most the leader is the chosen one. However, even after adopting one of these formulations, given the discontinuity in the follower optimal solution due to the introduction of setup costs, only an ϵ-*Stackelberg equilibrium* might be guaranteed to exist as defined in [7]. An ϵ-*Stackelberg equilibrium* is a profile of strategies $(z_1, \ldots, z_m) \in \prod_{p=1}^{m} X_p$ such that it is bilevel feasible for all $p \in \mathcal{L}$ and no leader $p \in \mathcal{L}$ can increase more than ϵ her profit by deviating to another bilevel feasible strategy. In the next example, this situation is illustrated by an instance with a well defined Stackelberg equilibrium under the optimistic formulation and with only a ϵ-Stackelberg equilibrium if the pessimistic formulation is assumed.

Example 2.2 *Consider the game parameters of Example 2.1 and let player 1 be the leader and player 2 the follower. Assume that there is no reduction in the follower's cost,* $h_2^1 = 0$.

Optimistic formulation *Player* 2 *optimal strategy is as computed in* (2.6) *with the difference that when* $q_1 = 5$*, player* 2 *chooses* $q_2 = 0$ *in order to cooperate with the leader. This makes it easy to conclude that* $(y_1, q_1, y_2, q_2) = (1, 5, 0, 0)$ *is a Stackelberg equilibrium.*

Pessimistic formulation *Player* 2 *optimal strategy is as computed in* (2.6) *with the difference that when* $q_1 = 5$*, player* 2 *chooses* $q_2 = 5$ *in order to penalize the leader's profit. Note that player* 1 *bilevel feasible region is a noncompact set which might lead to the non-existence of a solution to the optimization problem* (2.7) *(see [8] for discussion in the conditions for solution existence in the context of bilevel optimization).*

Player 1 *maximum profit is attained when* $q_2 = 0$ *and* $q_1 = 5$*, however such profile of strategies does not represent an equilibrium since it is not bilevel feasible. Thus, it is clear that player* 1 *rational strategy is to produce* $q_1 = 5 + \sqrt{\epsilon}$ *with* $\epsilon > 0$*, in order to force player* 2 *to be out of the market. In fact,* $(y_1, q_1, y_2, q_2) = (1, 5 + \sqrt{\epsilon}, 0, 0)$ *is an* ϵ*-Stackelberg equilibrium.*

3 Models' discussion

In practice, there are cases where identifying if the economic market is modeled according with a Cournot or Stackelberg competition is obvious due to the significant difference in the market power associated with the firms. However, this is not always the case. Furthermore, one can even ask if in a Cournot Competition in which the rules do not impose that decisions are taken simultaneously, is it worth for a player to wait for the opponent(s) to play first?

In order to clarify the question in hand, we compare the players profits under the Cournot and Stackelberg equilibria for the parameters of example 2.2.

Example 3.1 *Consider the game parameters of Example 2.1 and* $h_2^1 = h_1^2 = 0$.
Observe the players profits for the following profiles of strategies:

Cournot equilibrium:

$$(y_1, q_1, y_2, q_2) = (1, \frac{5}{3}, 1, \frac{20}{3}), \quad \Pi_1 = \frac{25}{9} \simeq 2.78, \qquad \Pi_2 = \frac{175}{9} \simeq 19.44$$

Cournot equilibrium:

$$(y_1, q_1, y_2, q_2) = (1, 5, 0, 0), \quad \Pi_1 = 25, \qquad \Pi_2 = 0$$

Stackelberg equilibrium: optimistic formulation:

$$(y_1, q_1, y_2, q_2) = (1, 5, 0, 0), \quad \Pi_1 = 25, \qquad \Pi_2 = 0$$

ϵ*-Stackelberg equilibrium: pessimistic formulation:*

$$(y_1, q_1, y_2, q_2) = (1, 5 + \sqrt{\epsilon}, 0, 0), \quad \Pi_1 = 25 - \epsilon, \qquad \Pi_2 = 0.$$

For this instance, player 2 *does not have advantage in waiting for player* 1 *to impose her strategy, since that would mean to be out of the market.*

What if player 2 *was the leader? Since player* 1 *will react according to* (2.6)*, player* 2 *optimal strategy is* $q_2 = 10$ *forcing player* 1 *to be out of the market* ($q_1^*(10) = 0$)*. In this Stackelberg equilibrium the players profits are* $\Pi_1 = 0$ *and* $\Pi_2 = 25$*. Therefore, as player* 2*, player* 1 *has no advantage in waiting to observe player* 2 *production plan.*

In conclusion, for this instance, both players have incentive to play first, which seems to naturally imply that a Cournot model is the more suitable to represent the game.

After the analyzes of the previous example, it seems evident that none of the players would benefit by playing last if the reduction on their production costs is zero. Note that the market price model $P(Q)$ adopted, makes the profit functions concave, which implies that a player optimal quantity to be introduced in the market will decrease as the rival quantity increases (and the profits behave analogously according with these changes in the quantities). Thus, the player moving first has a clear advantage since she can impose a certain quantity of the product in the market. This implies a reduction of the market to be shared by the followers. However, if a player can get some advantage by observing the opponents' technology, it happen that she will have incentive to move last, as shown in the next example.

Example 3.2 *Consider an instance with three players,* $m = 3$*, such that the market price parameters are* $a = 53$ *and* $b = \frac{1}{2}$*. The production parameters are* $M_1 = M_2 = M_3 = +\infty$*,* $f_1 = f_3 = 0$*,* $f_2 = 60$*,* $c_1 = 26$*,* $c_2 = 21$ *and* $c_3 = 10$*.*

There is a Cournot equilibrium such that player 1 and 3 share the market, and player 2 has no incentive to participate in the market. This equilibrium is $(q_1, q_2, q_3) = (\frac{22}{3}, 0, \frac{118}{3})$ *with profits* $\Pi_1 \simeq 26.89$*,* $\Pi_2 = 0$ *and* $\Pi_3 \simeq 773.56$*. Consider now a Stackelberg competition where the leader is player 1, the followers are player 2 and 3 and the reduction on setup costs are* $h_2^1 = 30$ *and* $h_3^1 = 200$*. A Stackelberg equilibrium is* $(q_1, q_2, q_3) = (6, 12, 34)$ *with profits* $\Pi_1 \simeq 6$*,* $\Pi_2 = 42$ *and* $\Pi_3 \simeq 778$*. Thus, player 2 and 3 have incentive to let player 1 move first.*

4 Conclusion

In this brief note, a combinatorial component was added to two classical economic models by considering setup production costs. If these fixed costs are meaningful in comparison with production costs, the study of equilibria becomes complex. The optimization problem of each player has non continuous decision variables, which complicates the computation of equilibria and it can even lead to their non existence in the Stackelberg Competition. Therefore, for the latter case, we introduced the notion of ϵ-Stackelberg equilibrium.

Along the paper, we exposed the second issue mentioned above (the existence of equilibria), but our purpose was also to highlight the different outcomes for a same instance under a Cournot and Stackelberg competition. As the examples have shown, if followers' reduction on costs by observation of the leaders' technology is zero, the outcomes of a follower in a Stackelberg equilibrium lead to smaller profits than in a Cournot equilibrium. However, in the presence of multiple Cournot-equilibria (as in the example presented), since it may not be clear which one will be selected by the players, a Stackelberg model can be more suitable if one of the players is risk averse. Additionally, a Stackelberg model can be more suitable if the reduction in production costs for some set of players is sufficient to give them advantage over Cournot equilibria.

Fundamentally, our note compares the players' solutions when they move simultaneously, versus when they move sequentially. Thus, it concerns any game in which there are no rules about the order in which players must take a strategic decision.

In conclusion, the ideas discussed in this note show the relevance of developing tools for the computation of equilibria in order to understand which kind of game dynamics is more suitable for each instance. It should be recalled that the combinatorial component of this models is crucial to obtain realistic situations.

Acknowledgments

The first author acknowledges the support of the Portuguese Foundation for Science and Technology (FCT) through a PhD grant number SFRH/BD/79201/2011, POPH/FSE program.

References

[1] A. A. Cournot, Researches into the Mathematical Principles of the Theory of Wealth, Translated in 1897 by N.T. Bacon, with a bibliography of Mathematical Economics by Irving Fisher. The Macmillan Company, New York, 1838.

[2] H. V. Stackelberg, The Theory of the Market Economy, Oxford University Press, Oxford, 1952.

[3] D. Fudenberg, J. Tirole, Game Theory, 5th Edition, MIT Press, Cambridge, MA, 1996.

[4] D. Monderer, L. S. Shapley, Potential games, Games and Economic Behavior 14 (1) (1996) 124 – 143. doi:http://dx.doi.org/10.1006/game.1996.0044.
URL http://www.sciencedirect.com/science/article/pii/S0899825696900445

[5] C. d'Aspremont, L.-A. Gérard-Varet, Stackelberg-solvable games and pre-play communication, Journal of Economic Theory 23 (2) (1980) 201 – 217. doi:10.1016/0022-0531(80)90006-X.
URL http://www.sciencedirect.com/science/article/pii/002205318090006X

[6] M. Carvalho, C. Telha, M. Van Vyve, Competitive uncapacitated lot-sizing game, Working paper.

[7] M. Breton, A. Alj, A. Haurie, Sequential Stackelberg equilibria in two-person games, Journal of Optimization Theory and Applications 59 (1) (1988) 71–97. doi:10.1007/BF00939867.
URL http://dx.doi.org/10.1007/BF00939867

[8] L. Vicente, G. Savard, J. Judice, Discrete linear bilevel programming problem, Journal of Optimization Theory and Applications 89 (3) (1996) 597–614. doi:10.1007/BF02275351.
URL http://dx.doi.org/10.1007/BF02275351