Problem B - Influência nas Eleições

The election season is approaching, and a certain candidate needs to present his popularity report to the media and the National Organization of Influence (ONI in portuguese). Over N days, the public influence of this candidate was measured. For a given day i, the influence I_i is an integer that can fluctuate between positive, negative, or 0 values.

In previous elections, candidates were required to group their influence values in the popularity report into fixed-duration periods (for example, grouping them into consecutive week-long periods). However, this time, candidates are free to group them as they see fit.

Part I

To make a good media impression, our candidate has decided to display only a contiguous period of his sequence of daily influence values, whose sum of values is maximum. Help him find that period!

Given the sequence $I_1, I_2, \ldots, I_N$ of candidate's daily influence values over N days, determine the largest sum of values $I_i + I_{i+1} + \cdots + I_j$ $(1 \leq i \leq j \leq N)$ in a non-empty contiguous subsequence.

Example

Suppose $N = 9$ and the candidate's public influence on each day is:

1	2	3	4	5	6	7	8	9
-2	1	-3	4	-1	2	1	-5	4

If we sum, for example, the candidate's influence between days 1 and 3, we get $(-2)+1+(-3) = -4$. But, if we take the sum of the values between days 4 and 7, we get $4 + (-1) + 2 + 1 = 6$.

1	2	3	4	5	6	7	8	9
-2	1	-3	4	-1	2	1	-5	4

1	2	3	4	5	6	7	8	9
-2	1	-3	4	-1	2	1	-5	4

It can be seen that 6 is the highest value we can obtain by summing consecutive elements of the sequence, so the answer in this case is 6.

Constraints

The following limits are guaranteed in all test cases of this part that will be submitted to the program:

$1 \leq \boldsymbol{N} \leq 10^5$	Number of days
$-10^9 \leq \boldsymbol{I_i} \leq 10^9$	Candidate's influence on day i

The test cases of this part of the problem are organized into two groups with different additional constraints:

Subtask	Points	Additional Constraints
1	15	$\boldsymbol{N} \leq 100$
2	25	No additional constraints

Part II

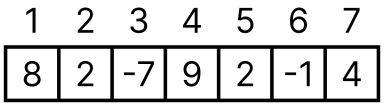
To please the ONI, our candidate must now divide his sequence of daily influence values into contiguous periods of variable duration, so that his **popularity** in each period is strictly positive.

The **popularity** in a period is equal to the sum of the daily influence values in that period minus the duration (in days) of that period. For example, the **popularity** of the period $[1, -4, 2]$ is equal to $(1 + (-4) + 2) - 3 = -4$, and the popularity of the period $[2]$ is equal to $2 - 1 = 1$.

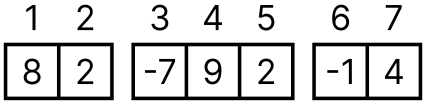
Given the sequence $\boldsymbol{I_1}, \boldsymbol{I_2}, \ldots, \boldsymbol{I_N}$ of candidate's daily influence values over $\boldsymbol{N}$ days, help the candidate determine in how many ways it is possible to partition the sequence into periods of **strictly positive popularity**. The total number of partitions in the partition is irrelevant. As the number of partitions can be very large, **you should output the result modulo $10^9 + 7$** (see note below on modulo).

Example

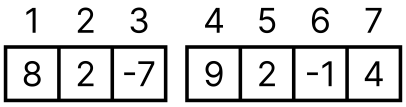
Suppose $\boldsymbol{N} = 7$ and the candidate's public influence on each day is:



One possible partition is



because the popularity in all periods is positive: $(8 + 2) - 2 = 8$, $(-7 + 9 + 2) - 3 = 1$, and $(-1 + 4) - 2 = 1$. On the other hand, the partition



is not valid because the popularity of the first period is $(8 + 2 - 7) - 3 = 0$.

Constraints

The following limits are guaranteed in all test cases of this part that will be submitted to the program:

$1 \leq \boldsymbol{N} \leq 10^5$	Number of days
$-10^9 \leq \boldsymbol{I_i} \leq 10^9$	Candidate's influence on day i

The test cases of this part of the problem are organized into two groups with different additional constraints:

Subtask	Points	Additional Constraints
3	35	$\boldsymbol{N} \leq 1000$
4	25	No additional constraints

Note about modulo

The modulo operator in C/C++/Java/Python is the symbol %. In Pascal, the operator is the expression `mod`. Some important properties about this operator that can be useful for your solution are:

- `(a + b) % n` is equal to `((a % n) + (b % n)) % n`
- `(a - b) % n` is equal to `((a % n) - (b % n) + n) % n`
- `(a * b) % n` is equal to `((a % n) * (b % n)) % n`

Summary of Subtasks

The test cases of the problem are organized into four groups with different additional constraints:

Subtask	Points	Part	Additional Constraints
1	15	Part I	$\boldsymbol{N} \leq 100$
2	25	Part I	No additional constraints
3	35	Part II	$\boldsymbol{N} \leq 1000$
4	25	Part II	No additional constraints

Input Format

The first line contains an integer $\boldsymbol{P}$, corresponding to the part that the test case represents. If it is 1, then the test case refers to Part I, if it is 2, then it refers to Part II. Regardless of the part, the input has the same format.

The second line contains an integer $\boldsymbol{N}$, indicating the number of days. Finally, there is one last line containing $\boldsymbol{N}$ integers, which correspond to the candidate's daily influence values $\boldsymbol{I_1}, \boldsymbol{I_2}, \ldots, \boldsymbol{I_N}$.

Output Format

Part I

The output should contain an integer: the largest sum of influence values of a non-empty contiguous subsequence of the $\boldsymbol{N}$ days.

Part II

The output should contain an integer: the number of ways to partition the $\boldsymbol{N}$ days into contiguous subsequences of **strictly positive popularity**, **modulo** $\mathbf{10^9 + 7}$.

Example 1 Input

```
1
9
-2 1 -3 4 -1 2 1 -5 4
```

Example 1 Output

```
6
```

Example 1 Description

This example corresponds to the example of Part I mentioned in the statement.

Example 2 Input

```
2
7
8 2 -7 9 2 -1 4
```

Example 2 Output

```
13
```

Example 2 Description

This example corresponds to the example of Part II mentioned in the statement.

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