A Point-free Approach to Bidirectional Transformation

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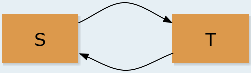
Bidirectional Transformations

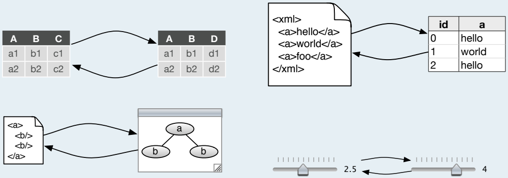
Bidirectional languages

- derive two unidirectional transformations from a specification



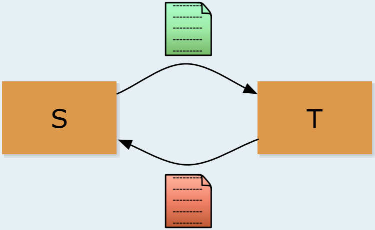
- clean semantics: consistency properties
- compositional

Bidirectional languages exist for...



Motivation - Specification

- bidirectional behaviour is defined by sophisticated procedures



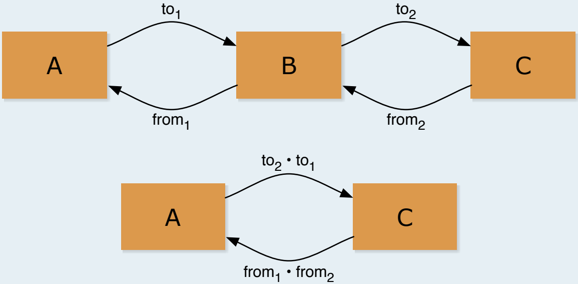
- hard to prove properties about the bidirectional transformations

Goal

- a good framework to prove properties?

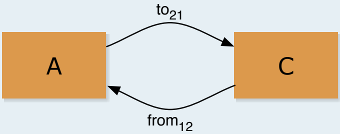
Motivation - Calculation

- compositionality = cluttering



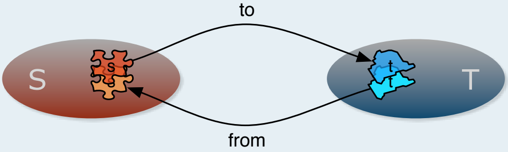
Goal

- how to calculate an optimised transformation?



Motivation - Totality

- non-total transformations



- partial laws

$$to \circ from = id$$

$$to \circ from \sqsubseteq id$$

- allow more expressive languages
- but may disallow desired updates

Goal

- need to control the partiality: which updates are valid?

A point-free design

- An application domain (Trees)

$$\textbf{data}\ Maybe\ a = Nothing \mid Just\ a$$
$$\textbf{data}\ [a] = [\,] \mid a : [a]$$

- A syntax for combinators

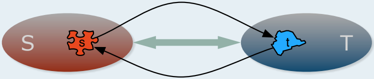
$$id : A \to A$$
$$\circ : (B \to C) \to (A \to B) \to (A \to C)$$
$$\pi_1 : A \times B \to A$$
$$\times : (A \to C) \to (B \to D) \to (A \times B \to C \times D)$$

- A set of calculation/simplification laws

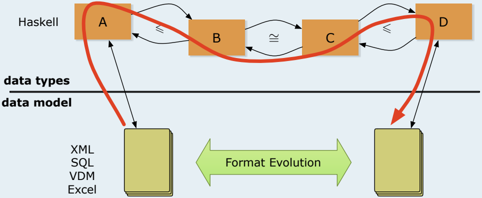
$$f \circ (g \circ h) = (f \circ g) \circ h \qquad \circ\text{-Assoc}$$
$$\pi_1 \circ (f \triangle g) = f \wedge \pi_2 \circ (f \triangle g) = g \qquad \times\text{-Cancel}$$
$$(f \times g) \circ (h \triangle i) = f \circ h \triangle g \circ i \qquad \times\text{-Absor}$$

2LT Framework

- a bidirectional two-level transformation:

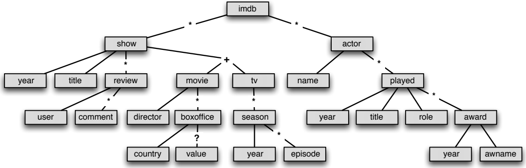


- Two-level Type-level transformation and corresponding value-level transformations.
- an experimental framework for model transformations:

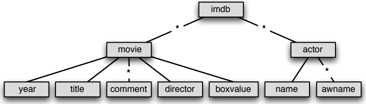


- Type-safe Type-checking guarantees that the data migration functions are well-formed in relation to the transformation

A bidirectional transformation: XML querying



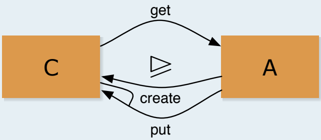
$$
\begin{aligned}
imdb &= shows \times actors \\
shows &= map\ ((id \times reviews) \times id) \circ filter\_l \\
&\quad \circ\ map\ distr \circ map\ (id \times (movie + tv)) \\
reviews &= length \circ concat \circ map\ \pi_{comments} \\
movie &= id \times boxoffices \\
boxoffices &= sum \circ filter\_r \circ map\ \pi_{value} \\
tv &= concat \circ map\ \pi_{episodes} \\
actors &= map\ (id \times awards) \\
awards &= map\ \pi_{awname} \circ concat \circ map\ \pi_{awards}
\end{aligned}
$$



Data abstraction

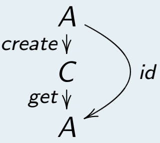
Lenses

$$get \quad : C \rightarrow A$$
$$create : A \rightarrow C$$
$$put \quad : A \times C \rightarrow C$$



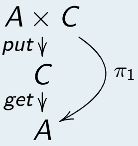
Properties for well-behaved lenses

- CreateGet



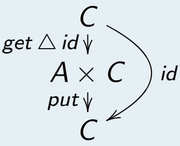
$$get \circ create = id$$

- PutGet



$$get \circ put = \pi_1$$

- GetPut



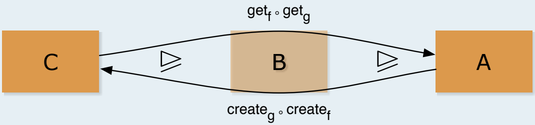
$$put \circ (get \bigtriangleup id) = id$$

Composition as a lens

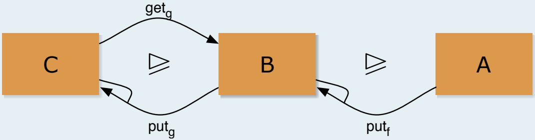
Lens composition

$$\forall f : B \trianglerighteq A,\ g : C \trianglerighteq B.\ \ f \circ g : C \trianglerighteq A$$

$get = get_f \circ get_g$ $\qquad$ $create = create_g \circ create_f$



$$put = put_g \circ (put_f \circ (id \times get_g) \triangle \pi_2) : A \times C \to C$$



$$id \circ f = f = f \circ id \qquad \text{ID-NAT}$$

$$f \circ (g \circ h) = (f \circ g) \circ h \qquad \circ\text{-ASSOC}$$

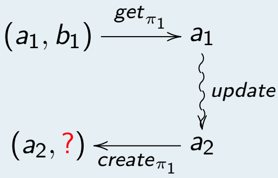
Products - Projection lenses

Drop element

$$\forall f : A \to B.\ {\pi_1}^{f} : A \times B \trianglerighteq A$$

$$\begin{aligned}
get &: A \times B \to A \\
get &= \pi_1 \\
create &: A \to A \times B \\
create &= id \vartriangle f \\
put &: A \times (A \times B) \to A \times B \\
put &= id \times \pi_2
\end{aligned}$$

- Choice of create



Properties

$$\begin{aligned}
get \circ create &= \pi_1 \circ (id \vartriangle f) &&= id \\
get \circ put &= \pi_1 \circ (id \times \pi_2) &&= \pi_1 \\
put \circ (get \vartriangle id) &= (id \times \pi_2) \circ (\pi_1 \vartriangle id) &&= \pi_1 \vartriangle \pi_2 = id
\end{aligned}$$

Products - More lenses

Split

- $f \triangle g : A \to B \times C$ is not a abstraction $\Leftarrow$ duplication

Product

$$\forall f : A \trianglerighteq C, g : B \trianglerighteq D.\ f \times g : A \times B \trianglerighteq C \times D$$

$$swap : A \times B \trianglerighteq B \times A$$

$$assoc : A \times (B \times C) \trianglerighteq (A \times B) \times C$$

Some laws

$$id \times id = id \qquad \times\text{-}\textsc{Functor-Id}$$

$$(f \times g) \circ (h \times i) = f \circ h \times g \circ i \qquad \times\text{-}\textsc{Functor-Comp}$$

$$\pi_1{}^h \circ (f \times g) = f \circ \pi_1{}^{create_g \circ h \circ get_f} \qquad \pi_1\text{-}\textsc{Nat}$$

$$swap \circ (f \times g) = (g \times f) \circ swap \qquad swap\text{-}\textsc{Nat}$$

$$\pi_1{}^f \circ swap = \pi_2{}^f \qquad swap\text{-}\textsc{Cancel}$$

Injections

- $i_1 : A \to A + B$ and $i_2 : B \to A + B$ are not abstractions $\Leftarrow$ insert information

"Conditional" choice

$$\forall p : C \to 2, f : A \trianglerighteq C, g : B \trianglerighteq C.\ (f \nabla g)^p : A + B \trianglerighteq C$$

$$\begin{array}{lcl}
get & : & A + B \to C \\
get & = & get_f \nabla get_g \\
create & : & C \to A + B \\
create & = & (create_f + create_g) \circ p\,? \\
put & : & C \times (A + B) \to A + B \\
put & = & (put_f + put_g) \circ distr
\end{array}$$

- Choice of create

$$C \xrightarrow{p?} C + C \xrightarrow{create_f + create_g} A + B$$

Sum combinator

$$\forall f : A \trianglerighteq C, g : B \trianglerighteq D.\ \circ (f + g)^{h,i} : A + B \trianglerighteq C + D$$

- Choice of put

$$(C + D) \times (A + B)$$

$$\downarrow dists$$

$$C \times A \; + \; C \times B \; + \; D \times A \; + \; D \times B$$

$$\downarrow put_f \quad \downarrow \ldots \quad \downarrow \ldots \quad \downarrow put_g$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

Some laws

$f \circ (g \nabla h)^p = (f \circ g \nabla f \circ h)^{p \circ create_f}$	+-Fusion
$(f \nabla g)^p \circ (h + i)^{j,k} = (f \circ h \nabla g \circ i)^p$	+-Absor
$(id + id)^{f,g} = id$	+-Functor-Id
$(f + g) \circ (h + i) = f \circ h + g \circ i$	+-Functor-Comp
$(f + g)^{j,k} \circ (h + i)^{l,m} = (f \circ h + g \circ i)^{o,p} \Leftrightarrow \ldots$	+-Comp

Recursive lenses

Some recursive lenses

$$\begin{array}{ll} \mathit{length} : [A] \trianglerighteq \mathbb{N} & \mathit{map} : (A \trianglerighteq B) \to [A] \trianglerighteq [B] \\ \mathit{get}\ [\,] = 0 & \mathit{get}\ f\ [\,] = [\,] \\ \mathit{get}\ (x : xs) = (\mathit{get}\ xs) + 1 & \mathit{get}\ f\ (x : xs) = \mathit{get}_f\ x : \mathit{get}\ xs \end{array}$$

- How can we bidirectionalise recursive lenses?

Recursive point-free combinators

$$\begin{array}{ll} \mathit{in}_F : F\ \mu F \trianglerighteq \mu F & \forall f : F\ A \trianglerighteq A.\ (\![f]\!)_F : \mu F \trianglerighteq A \\ \mathit{out}_F : \mu F \trianglerighteq F\ \mu F & \forall f : A \trianglerighteq F\ A.\ [\![f]\!]_F : A \trianglerighteq \mu F \end{array}$$

- Recursive lenses as bidirectional folds and unfolds

"Lensified" examples

$$\begin{array}{ll} \mathit{length}^A : [A] \trianglerighteq \mathbb{N} & \mathit{map} : (A \trianglerighteq B) \to [A] \trianglerighteq [B] \\ \mathit{length}^a = (\![\mathit{in}_N \circ (\mathit{id} + \pi_2{}^{\underline{a}})]\!)_L & \mathit{map}\ f = (\![\mathit{in}_L \circ (\mathit{id} + f \times \mathit{id})]\!)_L \end{array}$$

- Good: reuse existing point-free combinators!

Recursive lenses - Laws

Some recursive laws

$$in_F \circ out_F = id \;\wedge\; out_F \circ in_F = id \qquad \textit{in-out}\text{-}\textsc{Iso}$$

$$(\!|in_F|\!)_F = id \qquad (\!|\cdot|\!)\text{-}\textsc{Reflex}$$

$$(\!|f|\!)_F \circ in_F = f \circ F\ (\!|f|\!)_F \qquad (\!|\cdot|\!)\text{-}\textsc{Cancel}$$

$$f \circ (\!|g|\!)_F = (\!|h|\!)_F \Leftarrow f \circ g = h \circ F\ f \qquad (\!|\cdot|\!)\text{-}\textsc{Fusion}$$

Some derived laws for lists

$$map\ id = id \qquad \textit{map}\text{-}\textsc{Id}$$

$$map\ f \circ map\ g = map\ (f \circ g) \qquad \textit{map}\text{-}\textsc{Fusion}$$

$$filter\_l \circ map\ (f + g)^{h,i} = map\ f \circ filter\_l \qquad \textit{filter\_l}\text{-}\textsc{Map}$$

$$filter\_r \circ map\ (f + g)^{h,i} = map\ g \circ filter\_r \qquad \textit{filter\_r}\text{-}\textsc{Map}$$

$$length^{v} \circ map\ f = length^{create_f\ v} \qquad \textit{length}\text{-}\textsc{Map}$$

Summary (Lens language)

Grammar for lens combinators

$$
\begin{aligned}
Lens &::= id \mid Lens \circ Lens \mid !^f \mid Prod \mid Sum \mid Iso \mid Rec \\
Prod &::= {\pi_1}^f \mid {\pi_2}^f \mid Lens \times Lens \\
Sum &::= (Lens \nabla Lens)^p \mid (Lens + Lens)^{f,g} \\
Iso &::= assoc \mid assoc^{-1} \mid coassoc \mid coassoc^{-1} \\
&\quad \mid \ swap \mid coswap \mid distl \mid distr \\
Rec &::= in_F \mid out_F \mid F\ \cdot \mid (\![\cdot]\!)_F \mid [\![\cdot]\!]_F
\end{aligned}
$$

Notable exceptions

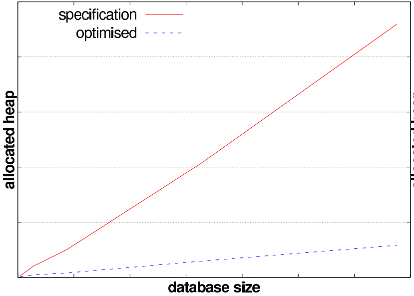
$$
\begin{aligned}
NonLens ::= {} & i_1 : A \to A + B \mid i_2 : B \to A + B \\
\mid \ & \underline{\cdot} : 1 \to B \\
\mid \ & \cdot \triangle \cdot : (A \to B) \to (A \to C) \to (A \to B \times C)
\end{aligned}
$$

Optimisation

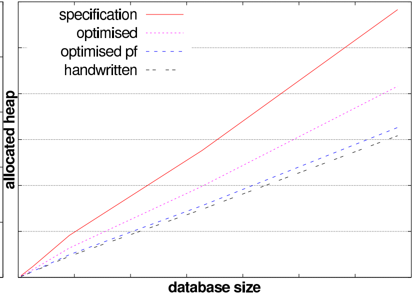
- calculational laws $\Rightarrow$ automated optimisation tool
- how about an handwritten definition?
- simpler example

$$\begin{array}{l}\textbf{type}\ Person = (Name, Gender)\ \textbf{data}\ Gender = M \mid F \\ women\ :\ [Person] \trianglerighteq \mathbb{N} \\ women = length \circ filter\_r \circ map\ (out_G \circ \pi_{Gender})\end{array}$$

running example



women example



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Practical Aspects of Declarative Languages, 2007.

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Mathematics of Program Construction, 2010.

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Algebraic Specialization of Generic Functions for Recursive Types.
Mathematically Structured Functional Programming, 2008.

Demos: Haskell++

- `http://hackage.haskell.org` ⇒ `pointless-lenses`
- `http://hackage.haskell.org` ⇒ `pointless-rewrite`

Conclusions

Pros

+ Bidirectional language from standard point-free combinators
+ Clear bidirectionalisation: straightforward proofs
+ Support for recursive lenses (w/ termination conditions)
+ Lens bidirectional calculus: reasoning and optimisation of complex transformations
+ (Some) choice of backward transformation

Cons

− Expressiveness (perfect abstractions)
− Opaque isomorphism combinators

Further discussion

Currently

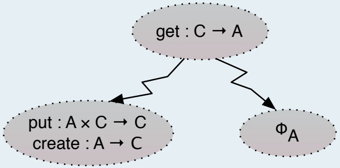
- no full products (π_1,π_2, $\triangle$) or sums (i_1,i_2, ∇)
- counterexample (duplication):

$$id \triangle id : A \trianglerighteq A \times A id \triangle id : A \trianglerighteq A \times A_\phi$$

- why is it not a valid lens (in our setting)?

$$\phi_{A\times A} = \{ (a_1, a_2) \in A\times A \,|\, a_1 = a_2 \}$$

Going relational... (current work)



- total transformations for the restricted domains