Mapping TSP to quantum annealing

PhD candidate Carla Silva[1]

Supervised by Prof. Dr. Inês Dutra[2]
Co-supervised by Prof. Dr. Ana Aguiar[3]

[1]Department of Computer Science, Faculty of Sciences, University of Porto, Porto, Portugal

[2]CINTESIS and Department of Computer Science, Faculty of Sciences, University of Porto, Porto, Portugal

[3]Instituto de Telecomunicações and Department of Electrical and Computer Engineering, Faculty of Engineering, Porto, Portugal

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Objective, contribution and motivation

Objective

The aim of this work is to perform the mapping of the travelling salesperson problem by using pseudo-Boolean constraints to a working graph of the D-Wave Systems.

Contribution

The contribution of this work concerns the suggestion of a possible mapping of combinatorial problems to quantum machines by the intersection between quantum computing and propositional logic via constraint satisfiability.

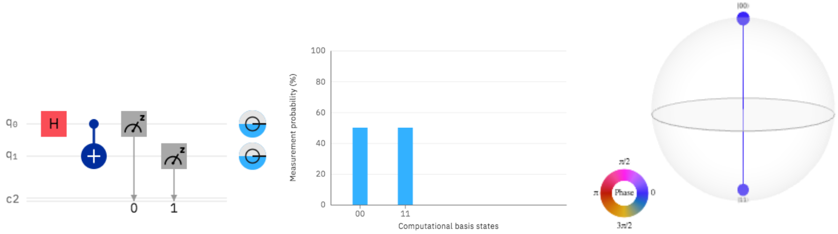
Motivation

In the context of the NP problems, the motivation of this work is to (a) map well-known optimization problems to a QC, and (b) compare classical (simulated annealing) and quantum (quantum annealing) methods.

Quantum computing

Gate Model

In the gate-base model a quantum algorithm is represented as a quantum circuit which is compiled to specific quantum gates and executed on a quantum device or simulator.



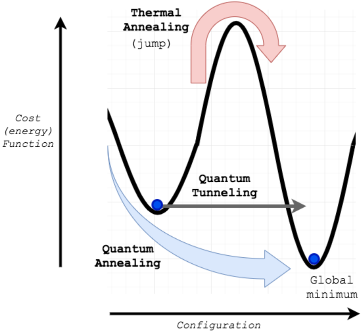
(a) *Circuit* (b) *Histogram* (c) *Q-sphere*

Figure: Bell state at IBM Quantum Experience

Quantum computing

Adiabatic Model

In the adiabatic model we encode a problem into a Hamiltonian with variables that represent *spins* which can be in one of two states embedded into the quantum hardware graph. And, we use a quantum annealer to sample low-energy states which are the optimal solutions of the problem.



Mapping combinatorial problems to constraint satisfiability

- **Combinatorial problems** consist in finding, among a finite set of objects, one/all/best solutions that satisfies a set of constraints (e.g. graph coloring problem, travelling salesperson problem (TSP), the minimum spanning tree problem (MST), or the knapsack problem).
- **Constraint satisfaction** the process of finding a solution to a set of constraints that impose conditions that the variables must satisfy.

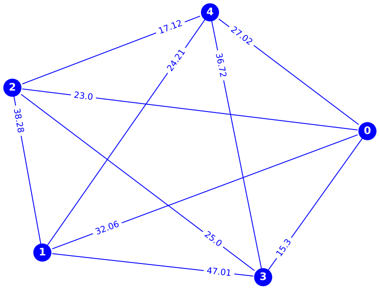
Pseudo-Boolean optimization

A combinatorial optimization method for pseudo-Boolean functions, i.e. real valued functions with 0-1 variables.

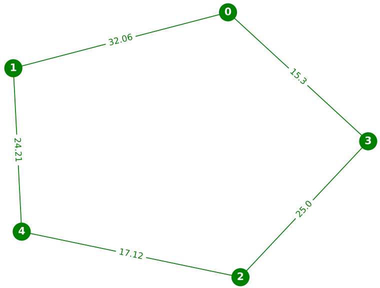
The travelling salesperson problem

Problem definition

Let $G = (V, A)$ be an undirected graph, where V is the graph's vertex set (cities) and A the set of edges. Find a tour of minimum cost (distance).



(a) $n = 5$ cities and distances



(b) Minimum distance 113.69

Scheme

Mapping problems to quantum annealing main scheme

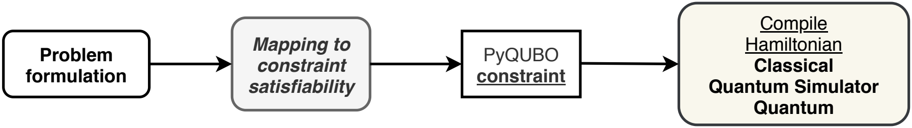


Figure: Steps

SATyrus approach

A SAT-based Neuro-Symbolic Architecture for Constraint Processing

The Integrity Constraints

1. All n cities must take part in the tour.
2. Two cities cannot occupy the same position in the tour.
3. A city cannot occupy more than one position in the tour.

Optimality Constraints

4. The cost between two consecutive cities in the tour.

SATyrus approach

- α is applied to the type (1) constraints.
- β is applied to types (2 and 3) constraint.
- Penalties for violated constraints.

Penalty terms

$$\begin{cases} dist = max\{dist_{ij}\} \\ \alpha = ((n^3 - 2n^2 + n) * dist) + h \\ \beta = ((n^2 + 1) * \alpha) + h \end{cases} \tag{1}$$

- Lima PMV, Morveli-Espinoza MM, Pereira GC, Fran¸ca FMG (2005) Satyrus: a sat-based neuro-symbolic architecture for constraint processing. In: Fifth International Conference on Hybrid Intelligent Systems (HIS'05), pp 6 pp.–
- Lima PMV, Pereira GC, Morveli-Espinoza MMM, Fran¸ca FMG (2005) Mapping and combining combinatorial problems into energy landscapes via pseudo-boolean constraints. In: De Gregorio M, Di Maio V, Frucci M, Musio C (eds) Brain, Vision, and Artificial Intelligence, Springer Berlin Heidelberg, Berlin, Heidelberg, pp 308–317

SATyrus approach

- The travelling salesperson problem is represented as Boolean constraints which are mapped to an energy equation where the goal is to minimize the energy.

Energy equation

$$E = \alpha\Big[\sum_{i=1}^{n}\sum_{j=1}^{n}(1 - v_{ij})\Big] + \beta\Big[\sum_{i=1}^{n}\sum_{i'=1, i'\neq i}^{n}\sum_{j=1}^{n}(v_{ij}v_{i'j})\Big] + \beta\Big[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{j'=1, j'\neq j}^{n}(v_{ij}v_{ij'})\Big] + \Big[\sum_{i=1}^{n}\sum_{i'=1, i'\neq i}^{n}\sum_{j=1}^{n-1} dist_{ii'}\, v_{ij} v_{i'(j+1)}\Big] \quad (2)$$

- From the equation we can use any solver to find an optimal solution.

Adiabatic quantum computation

AQC

The D-Wave quantum annealing processor has the goal to minimize the energy of an Ising/QUBO configuration in a *Pegasus* graph.

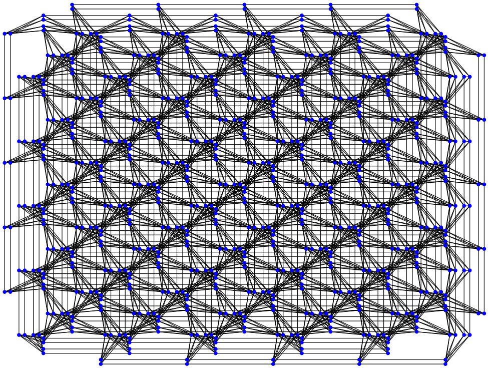


Figure: Pegasus topology.

Quadratic unconstrained binary optimization (QUBO)

- In this case, q_i is the qubit i which participates in the annealing cycle and settles into one of the final states $\{0,1\}$, with weight a_i which influences the qubit's tendency to collapse into its two possible final states, with q_i and q_j the coupler which allow one qubit to influence the other, and $b_{i,j}$ the strength which controls the influence by one qubit on another.
- In scalar notation, the objective function is:

$$\mathrm{E}_{qubo}(a_i, b_{i,j}; q_i) = \sum_i a_i q_i + \sum_{i<j} b_{i,j} q_i q_j. \tag{3}$$

where a_i are the *linear coefficients*, $b_{i,j}$ the *quadratic coefficients*, and q_i and q_j are the variables.

Experimental setup and the annealing processes

Based in metaheuristics for finding the global minimum of the objective function over a given set of candidate solutions.

- Classical

Simulated Annealing (SA)

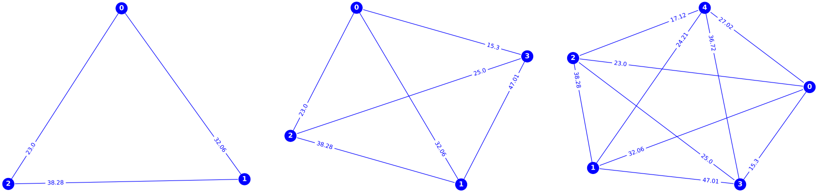
The temperature determines the probability of moving to a state of higher "energy" from a single current state.

- Quantum

Quantum Annealing (QA)

A quantum-mechanical phenomena determines the probability to change the amplitudes of all states in parallel.

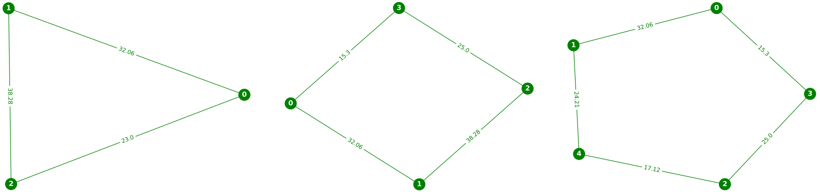
Possible tours



(a) $n = 3$ (b) $n = 4$ (c) $n = 5$

Figure: All distances.

Optimal tours



(a) $n = 3$,
$distance = 93.34$

(b) $n = 4$,
$distance = 110.64$

(c) $n = 5$,
$distance = 113.69$

Figure: Minimum distances.

Optimal solutions

- **Conditions to ensure optimal solutions**:
  - No broken constraints: Let $c \in \mathbb{R} : c = 0$
  - Maximum occurrences: O_{max}
  - Minimum energy: E_{min}
  - $\exists n$ edges: Let $v_{ij} \in \{0, 1\} = 1$
  - Different cities for each vertex of an edge: $i \neq j$
  - Same number of edges and vertices: $n_{edges} = n_{vertices}$
  - Connected graph: *True*

We ensure the minimum distance tour.

Conclusions

- Both QA and SA attained optimal solutions.
- Increasing the number of sweeps in the simulated annealing algorithm had impact to achieve the optimal solutions.

Future work

- We intend to expand the experiments to larger graphs.