Computation-Limited Signals: A Channel Capacity Regime Constrained by Computational Complexity

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***Abstract*—In this letter, we introduce the computation-limited (comp-limited) signals, a communication capacity regime where the computational complexity of signal processing is the primary constraint for communication performance, overriding factors such as power or bandwidth. We present the Spectro-Computational (SC) analysis, a novel mathematical framework designed to enhance classic concepts of information theory –such as data rate, spectral efficiency, and capacity – to accommodate the computational complexity overhead of signal processing. We explore a specific Shannon regime where capacity is expected to increase indefinitely with channel resources. However, we identify conditions under which the time complexity overhead can cause capacity to decrease rather than increase, leading to the definition of the comp-limited signal regime. Furthermore, we provide examples of SC analysis and demonstrate that the OFDM waveform falls under the comp-limited regime unless the lower-bound computational complexity of the N-point DFT problem verifies as $\Omega(N)$, which remains an open challenge in the theory of computation.**

***Index Terms*—Capacity, Signal Processing, Computational Complexity, Information Theory, Fundamental Limits.**

I. INTRODUCTION

INFORMATION theory introduces power and bandwidth as the fundamental resources to describe the capacity of a noisy channel. The development of clever physical layers, coupled with the adoption of larger spectrum resources, has enabled unprecedented data rates. Consequently, the computational resources required to process more bits per signal have grown accordingly, highlighting the trade-off between signal processing complexity and data rate. Despite that, as far as we know, little knowledge have been produced to correlate these performance indicators.

Some research efforts propose unified models of computation and information theory but without concerning about the interplay between the signal processing time complexity and capacity. For instance, works such as [1] concern about whether a discrete (Turing) machine is *able* to compute a given channel capacity function. Other works bring the term "complexity" to information theory but with different meaning than that of the computation complexity, e.g., [2].

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In this letter, we present an analytical framework referred to as the "Spectro-Computational (SC) analysis". With the SC analysis, we revisit classic concepts of information theory – such as throughput, spectral efficiency and capacity – in order to account for the signal processing computational complexity overhead. Our mathematical framework enhances and generalizes concepts we previously propose to study the complexity-throughput trade-off lying in the context of specific waveforms [3], [4], [5], [6]. Based on that, we refer to a specific Shannon capacity regime to derive a novel capacity regime in which computational complexity matters more than channel resources such as bandwidth or received power.

The remainder of this letter is organized as follows. In Section II, we review the background and present the rationale for our mathematical framework. In Section III, we present the mathematical framework of the SC analysis. In Section IV, we formalize the comp-limited communication regime. In Section V, we present practical examples of the SC analysis. In Section VI we present our conclusion and future work.

II. BACKGROUND AND RATIONALE

In this section, we review some key properties of asymptotic notation that will support our analyses throughout this work (subsection II-A). In subsection II-B, we present the rationale of our proposal by discussing the interplay between computational complexity and channel resources in the Shannon communication system.

A. Asymptotic Notation

The asymptotic analysis relate functions $f(N)$ and $g(N)$ as $N \to \infty$. For the quantities of this work, we assume increasing non-negative functions. We follow the classic notation popular in the literature of analysis of algorithms.Thus, if $f(N) = \Theta(g(N))$, $f(N) = o(g(N))$ and $f(N) = \omega(g(N))$, it denotes that the order of growth of $f(N)$ is equal to, strictly lower than, or strictly higher than the order of growth of $g(N)$, respectively. Based on these notations, one can also define $O(.)$ and $\Omega(.)$ as follows,

$$f(N) = O(g(N)) \Rightarrow [f(N) = o(g(N)) \text{ or } f(N) = \Theta(g(N))]. \quad (1)$$

$$f(N) = \Omega(g(N)) \Rightarrow [f(N) = \omega(g(N)) \text{ or } f(N) = \Theta(g(N))]. \quad (2)$$

Thus, these asymptotic notations can be defined according to Eqs. (3), assuming existing limits and a real constant $c > 0$.

$$\lim_{N\to\infty} \frac{f(N)}{g(N)} = \begin{cases} c, & \text{if } f(N) = \Theta(g(N)). \quad (3a) \\ 0, & \text{if } f(N) = o(g(N)). \quad (3b) \\ \infty, & \text{if } f(N) = \omega(g(N)). \quad (3c) \end{cases}$$

B. A Case for a Time Complexity-Constrained Signal Regime

In this subsection, we firstly review the channel resources considered by Shannon to describe the capacity regimes. Then, we argue how these channel resources are related to the signal processing computational complexity and argue for a time complexity-constrained capacity regime.

1) Shannon Capacity Regimes: Let us consider the Additive Gaussian White Noise (AWGN) channel capacity formula of Shannon, based on which the two classic channel capacity regimes of information theory derives from, namely, the Bandwidth-Limited Regime (BLR) and Power-Limited Regime (PLR). According to Shannon, the capacity of an AWGN channel is

$$C = W \log_2(1 + \mathsf{SNR}) \quad \text{bits/second.} \quad (4)$$

$$\mathsf{SNR} = \frac{\mathcal{P}}{W N_0}. \quad (5)$$

where W is the channel bandwidth, SNR is the Signal-to-Noise Ratio (SNR), $\mathcal{P}$ is the received signal power and N_0 is the noise power spectral density. Eq. (4), establishes an upper bound for the data rate R experienced by a $B(W)$-bit message in an AWGN channel. In other words, for a symbol period of T_{sym}, data rate R and the spectral efficiency SE are given in Eq. (6) and Eq. (7), respectively.

$$R = \frac{B(W)}{T_{\text{sym}}} < C \quad \text{bits/second.} \quad (6)$$

$$\mathsf{SE} = \frac{R}{W} \quad \text{bits/second/Hertz.} \quad (7)$$

in which T_{sym} is the signal period. PLR results when SNR is very small ($\mathsf{SNR} \ll 1$). In this case, one can approximate $\log_2(1+\mathsf{SNR})$ as $\mathsf{SNR}\log_2 e$, thereby C becomes linear on $\mathcal{P}$ and is not affected by W. If SNR remains high ($\mathsf{SNR} \gg 1$) as W grows, C becomes proportional to W, leading to the BLR case. Note, however, that widening W for a fixed $\mathcal{P}$ impairs SNR due to the resulting overall noise. In this case, the regime changes from BLR to PLR as W grows.

2) Channel Resources and Time Complexity: Beyond power and spectrum, computational resources are also intrinsic to Shannon's communication system. In this system, a transmitter/receiver must be able to "*operate* on the message in some way to produce a signal" [7]. Specifically, in digital communication systems (the focus of this work), the computational resources required for this processing depend on the length of the transmitting message, which, in turn, is determined by the available channel resources. Therefore, capacity regimes such as BLR and PLR directly affect the implied computational complexity. By its turn, such complexity overhead can impair time-related performance indicators (e.g., throughput, capacity) if it is not neglected in the analysis.

3) What If Channel Resources Grow Arbitrarily?: To illustrate how time complexity can affect communication performance, consider the Shannon capacity formula (Eq. 4) under the fixed SNR regime, i.e., $\mathsf{SNR} = c$, for a real constant $c > 0$ (Eq. 5). In this regime, as $W \to \infty$, $\mathcal{P} \to \infty$ accordingly to counter the resulting noise and keep the SNR constant. Thus, in this case, $C = cW$ for some constant $c > 0$, i.e., $C = \Theta(W)$ (Eq. 3a). In practice, this means that one might expect increasing system performance if more resources are assigned to the channel. Formally, $\lim_{W\to\infty,\mathcal{P}\to\infty} C = \infty$ bits/second.

Let us now consider the effect of the time complexity in the analysis. Let $T(W)$ denote the number of computational instructions required to turn the $B(W)$-bit message into a W Hertz signal. To ensure a real-time physical layer implementation, the processor must consider the largest latency of the system, which stems from the first symbol of every transmission opportunity [6]. Based on this, the classic data rate formula of Eq. (6) rewrites to

$$R_{\text{comp}}(W) = \frac{B(W)}{tT(W) + T_{\text{sym}}} < C \quad \text{bits/second.} \quad (8)$$

Under arbitrarily large channel resources, both $B(W)$ and $T(W)$ tend to infinity. As in Shannon capacity, we assume negligible multipath effects to establish an upper-bound performance. Thus, the channel-dependent parameter T_{sym} can be considered constant. Similarly, t is a hardware-dependent constant. In this case, $T(W)$ plays a crucial role in determining whether $R_{\text{sym}}(W)$ is an increasing or decreasing function of W. Note that this condition can still hold if T_{sym} is bounded by $tT(W)$. However, such analysis would require specific models of multipath channels, which we leave for future work. If $B(W)$ grows faster than $T(W)$, i.e., $B(W) = \omega(T(W))$, then, by definition of Eq. (3c), $R_{\text{comp}}(W \to \infty) = B(W)/T(W) = \infty$. This means that complexity does not become a bottleneck for the overall system performance since $R_{\text{comp}}(W)$ grows arbitrarily on W, just as the non complexity-constrained data rate R (Eq. 6) does.

Conversely, if $B(W) = o(T(W))$ then $R_{\text{comp}}(W \to \infty) = B(W)/T(W) = 0$ (Eq. 3b). In other words, in the fixed SNR regime of information theory, the upper bound for $R_{\text{comp}}(W)$ grows linearly on W. By contrast, when the computational complexity is considered in the $B(W) = o(T(W))$ case, that Shannon capacity bound becomes meaningless, since $T(W)$ causes $R_{\text{comp}}(W)$ to behave as a decreasing function of W. Moreover, if $T(W)$ is asymptotically optimal, this effect cannot be reversed except by increasing the computational resources as $T(W)$ grows. Therefore, there might exist regimes in which the maximum achievable data rate is limited by computational – rather than channel – resources.

III. Fundamentals of Spectro-Computational Complexity

Throughout this section, we evolve the classic definitions of information theory to account for the signal processing time complexity overhead. The resulting analytical framework we refer to as the "SC" analysis. The term "SC" dates back to

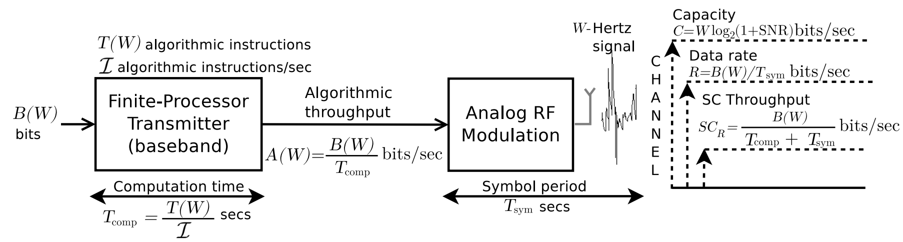

Fig. 1. Communication system model of the SC analysis (receiver-side omitted).

our earliest work [6], in which we concerned about the trade-off between spectral efficiency and computational complexity lying in a specific waveform. To avoid ambiguity with the classic definitions, we will adopt the same nomenclature of our original work to designate each novel enhanced definition. Therefore, in what follows, we revisit the classic information theory definitions of data rate R (or throughput, Eq. 6) and spectral efficiency SE (Eq. 7) to introduce the enhanced homologous concepts of "SC throughput" SC_R (Eq. 11) and "SC efficiency" $\mathsf{SC}_{\mathsf{SE}}$ (Eq. 13), respectively. We leave the definition of the "SC capacity" to Section IV.

1) System Model and Assumptions: Fig. 1 illustrates the transmitter of our communication system for a W-Hertz channel. The analysis is reciprocal for the receiver side. We concern about the throughput experienced by a *particular* $B(W)$-bit message. The message is turned into a baseband signal by a "finite-baseband transmitter" that can perform $\mathcal{I} > 0$ instructions per second. This quantity represents all computational resources allocated to executing the algorithmic instructions. It is assumed to be finite due to the fundamental limits of chipset manufacturing [8]. To turn the message into a signal, $T(W)$ computational instructions need to be performed (i.e., 'time complexity'). The number of bits $B(W)$ sets the input length for the algorithms in the baseband processor. Since it is a function of W in the fixed SNR-regime, the complexity T can be written as a function of W as well.

2) Computational Time and Algorithmic Throughput: The baseband signal processing runtime T_{comp} and the *algorithmic throughput* $A(W)$ of the baseband processor in Fig. 1 are defined in Eq. (9) and Eq. (10), respectively.

$$T_{\text{comp}} = \frac{T(W)}{\mathcal{I}} \quad \text{seconds.} \tag{9}$$

$$A(W) = \frac{B(W)}{T_{\text{comp}}} = \frac{\mathcal{I}B(W)}{T(W)} \quad \text{bits/second.} \tag{10}$$

The "Analog RF Modulation" block converts the signal to analog and performs the carrier modulation. The entire process corresponds to the symbol duration T_{sym} seconds.

3) Spectro-Computational Throughput: Thus, in addition to T_{comp}, the signal carrying the $B(W)$-bit message will take T_{sym} seconds. Based on this, we define the SC data rate (or throughput) $\mathsf{SC}_R(W)$ as

$$\mathsf{SC}_R(W) = \frac{B(W)}{T_{\text{comp}} + T_{\text{sym}}} = \frac{B(W)}{\frac{T(W)}{\mathcal{I}} + T_{\text{sym}}} \quad \text{bits/second.} \tag{11}$$

Note that the algorithmic throughput $A(W)$ (Eq. 10) corresponds to a special case of the SC throughput $\mathsf{SC}_R(W)$ (Eq. 11) when the channel time T_{sym} is set to 0. Besides, as customary in the theory of computation, one may also consider the algorithmic performance independent of a specific hardware. In our case, this stems by setting $\mathcal{I} = 1$. Thus,

$$A(W) = \mathsf{SC}_R(W) = \frac{B(W)}{T(W)} \quad \text{if } \mathcal{I} = 1 \text{ and } T_{\text{sym}} = 0. \tag{12}$$

The particular condition of Eq. (12) holds, for example, for an asymptotic analysis of $\mathsf{SC}_R(W)$ on $W \to \infty$. In this case, the constants T_{sym} and $\mathcal{I}$ can be neglected. Thus, the terms *algorithmic throughput* and *SC throughput* can be interchangeable for the asymptotic analyses presented throughout this work. Similarly, note also that the rate units bits/instruction and bits/second are interchangeable in these cases.

4) Spectro-Computational Efficiency: Based on the SC throughput (Eq. 11), we introduce the SC Efficiency (SCE) to enhance the classic definition of SE (Eq. 7) with time complexity. This is given in Eq. (13).

$$\mathsf{SC}_{\mathsf{SE}}(W) = \frac{\mathsf{SC}_R(W)}{W} \quad \text{bits/second/Hertz.} \tag{13}$$

Homologously to Eq. (12), $\mathsf{SC}_{\mathsf{SE}}(W)$ (Eq. 13) can also be expressed in bits/instruction/Hertz if $\mathcal{I} = 1$ and $T_{\text{sym}} = 0$. Next, we build on the definitions of this section to formalize a novel complexity-constrained capacity regime.

IV. Computation-Limited Signals

In this section, we build upon the definitions of Section III to define a capacity regime constrained by computational resources.

A. SC Algorithmic Capacity

We define the SC (algorithmic) capacity $\mathsf{SC}_C(W)$ of a waveform as the asymptotic upper bound for the SC throughput $\mathsf{SC}_R(W)$ of Eq. (11), i.e., $\mathsf{SC}_R(W) = O(\mathsf{SC}_C(W))$. Assuming the fixed SNR regime discussed in Section II-B3, the constants $\mathcal{I}$ and T_{sym} in Eq. (11) can be neglected as $W \to \infty$. Thus, our analysis follows based on $\mathsf{SC}_R(W)$ of Eq. (12). In that case, the upper-bound $\mathsf{SC}_C(W)$ is defined as the ratio

$$\mathsf{SC}_C(W) = \frac{B_{\max}(W)}{\mathcal{L}(W)} \quad \text{bits/second.} \tag{14}$$

and

$$\mathsf{SC}_R(W) = O(\mathsf{SC}_C(W)). \tag{15}$$

In Eq. (14), $B_{\text{max}}(W)$ and $\mathcal{L}(W)$ stand for the highest and lowest orders of growth that can be assumed for the numerator $B(W)$ and the denominator $T(W)$, respectively. We assume that the maximum number of bits passing through the baseband processor grows linearly with the spectrum. As we discuss in section V-B, this assumption is verified in practical network standards. Therefore, under the fixed SNR regime, it results

$$B_{\text{max}}(W) = \Theta(C) = cW \quad \text{bits.} \tag{16}$$

In turn, $\mathcal{L}(W)$ stands for the asymptotic lower bound of a computational problem. This lower bound complexity may be hard to derive in some cases, as is the case of the N-point DFT problem required by an Orthogonal Frequency Division Multiplexing (OFDM) signal (as discussed in Section V).

B. The Comp-Limited Signal Regime

To ensure the SC throughput does not nullify as the channel resources grow, the waveform design might satisfy Lemma 1.

Lemma 1 (Condition of Scalability)**.** *Under the fixed SNR regime of the Shannon capacity (discussed in Section II-B3), the SC throughput (Eq. 12) nullifies as $W \to \infty$ unless $B(W) = \Omega(T(W))$.*

Proof. The condition of scalability is such that

$$\mathsf{SC}_R(W \to \infty) = \lim_{W \to \infty} \frac{B(W)}{T(W)} \quad > \quad 0. \tag{17}$$

To ensure Ineq. 17 holds, the time complexity must grow as fast as $B(W)$ at most, i.e. $B(W) = \Omega(T(W))$. It means either $B(W) = \Theta(T(W))$ (Eq. 3a) or $B(W) = \omega(T(W))$ (Eq. 3c). If none of these cases holds, then $B(W) = o(T(W))$ does, since the conditions of Eq. 3 are mutually exclusive. Under this latter case, it follows from Eq. (3b) that $\mathsf{SC}_R(W \to \infty) = 0$. Therefore, the SC throughput $\mathsf{SC}_R(W)$ nullifies as $W \to \infty$ unless $B(W) = \Omega(T(W))$. $\square$

A particular signal implementation may not satisfy Lemma 1. In some cases, overcoming that is just a matter of devising and implementing asymptotically faster signal baseband algorithms e.g., [4]. We are particularly interested in checking whether the SC capacity $\mathsf{SC}_C(W)$ remains greater than 0 as W increases. If it does not, then it indicates that the signal data rate is constrained by computational resources. This limitation arises because the number of algorithmic instructions cannot be improved beyond the asymptotic lower bound $\mathcal{L}(W)$ present in $\mathsf{SC}_C(W)$. We refer to this as the comp-limited regime (Def. 1).

Definition 1 (Comp-Limited Signal Regime)**.** *A signal waveform is limited by computation (comp-limited) if its SC (algorithmic) capacity $\mathsf{SC}_C(W)$ (Eq. 14) nullifies as $W \to \infty$.*

Def. 1 translates the fact that there exist conditions under which the computational resources of the baseband processor must grow arbitrarily (i.e., $\mathcal{I} \to \infty$) to prevent the time complexity to nullify capacity as $W \to \infty$. *Therefore, in this regime, capacity is limited by the computational – rather than spectrum or power – resources.*

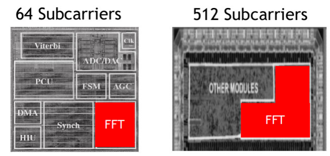

Fig. 2. FFT baseband processor comparison: 512-point (right-hand side) vs. 64-point (left-hand side). The 512-point processor produces a $8\times$ faster signal at the penalty of consuming larger computational resources. We show that this gain nearly halves if the signal data rate also accounts for runtime and both processors are provided the same computational resources.

V. Examples

In this section, we demonstrate different use cases of the SC analysis.

A. Common Parameters

For the analyses of this section, we assume a N-subcarrier OFDM signal spaced by Δf Hz each and a symbol period of $T_{\text{sym}} = 1/\Delta f$ seconds. Given an M-point constellation diagram, the number of bits per subcarrier is $\log_2 M$. Since M grows on the SNR, this number is constant in the fixed SNR regime. Thus, the total number of bits in the OFDM frame solely depends on N and we will assume it as equal to N without loss of generality.

B. A Fairer Wi-Fi Data Rate Comparison

The IEEE 802.11ac standard claims a data rate improvement of $8\times$ in comparison to its legacy IEEE 802.11a counterpart. This results from widening the bandwidth by a factor of $8\times$ keeping the legacy OFDM symbol period of $T_{\text{sym}} = 3.2$ μs unchanged (without considering cyclic prefix, CP). However, such an improvement comes at a cost that is not captured by the classic data rate formula (Eq. 6). Consider, for example, a DFT computation of roughly $T_{\text{DFT}}(N) = N \log_2 N$ algorithmic instructions [3]. Thus, increasing N from 64 (IEEE 802.11a) to 512 (IEEE 802.11ac) causes a *non-negligible* impact of roughly $T_{\text{DFT}}(512)/T_{\text{DFT}}(64) = 12\times$ in time complexity. Despite this increase, the DFT computation time $T_{\text{comp}N} = T_{\text{DFT}}(N)/\mathcal{I}_N$ (Eq. 9) ($\mathcal{I}_N$ denotes the number of instructions per second of a N-point DFT baseband processor) must not exceed the symbol duration T_{sym}. Otherwise, the implementation would compromise spectral efficiency by introducing idle periods between OFDM symbols [6]. Hence, $T_{\text{DFT}}(N)/\mathcal{I}_N \leq T_{\text{sym}} = 3.2$ must hold. Applying this constraint to Eq. (9), the *minimum* performance required by the baseband processors of the 512-point and 64-point signals are $\mathcal{I}_{512} = T_{\text{DFT}}(512)/3.2 = 1440$ and $\mathcal{I}_{64} = 120$ instructions/microsecond, respectively. This superior demand for computational resources of the 512-point signal is clearly illustrated in the die micrograph comparison of Fig. 2[1].

We argue that a fairer data rate comparison should account for the signal processing runtime overhead (since it is implicit to the throughput perceived by the upper layers) under

[1]Illustration art from [9]. Due to space constraints, we kindly request that the reader refer to the cited work for the technical references about the baseband signal processors.

equitable computational resources, i.e., $\mathcal{I}_{64} = \mathcal{I}_{512} = 1440$, in this case study. We accomplish this by comparing the SC throughput (Eq. 11) of the 512-point and 64-point signals that are $\mathsf{SC}_R(512) = 512/6.4 = 80$ bits/microsecond and $\mathsf{SC}_R(64) = 64/(0.26 + 3.2) = 18.46$ bits/microsecond, respectively. In this case, the gain $\mathsf{SC}_R(512)/\mathsf{SC}_R(64) \approx 4.3$ is nearly half of the claimed by $N = 512$ setup. *Therefore, computational complexity should not be neglected in the data rate analysis of signals constrained by different computational resources. In this case, the proposed SC framework can constitute a valuable tool to assist the comparison and design of novel waveforms.*

C. Is OFDM a Comp-Limited Signal?

Next, we present a step-by-step analysis of the SC capacity of OFDM to answer whether it classifies as a comp-limited signal.

1) Asymptotic model and Maximum Number of bits: In the fixed SNR regime of Shannon, capacity grows linearly with W. Translated to OFDM, $N \to \infty$ as $W \to \infty$ because Δf can be assumed constant within the OFDM bandwidth $W = N\Delta f$ Hz. Additionally, M remains constant as explained in section V-A. Consequently, the maximum number of bits in the OFDM frame increases linearly with N, represented as $B_{\text{max-OFDM}}(N \to \infty) = cN$ for some real constant $c > 0$.

2) Complexity of OFDM: The overall time complexity $T_{\text{OFDM}}(N)$ of the uncoded OFDM signal results from the sum of its individual procedures, including, (de)mapping, (I)DFT computation, addition/deletion of CP, signal detection, and equalization. Among these, DFT and signal detection are the most computationally expensive. However, considering that the input of the data signal detection typically involves a small fraction of N (i.e., the pilot subcarriers), and employing a low-complexity estimator (e.g., the least squares detector) with a complexity of $O(N)$, the DFT computation emerges as the most complex procedure of OFDM. Therefore, we equate the asymptotic complexity $T_{\text{OFDM}}(N)$ of OFDM to the complexity $T_{\text{DFT}}(N)$ of DFT. Consequently, as N grows, the constants and the complexities of all other procedures can be neglected for the sake of the asymptotic analysis. It's worth noting that one can proceed with the SC analysis of any OFDM algorithm since the only prerequisites are the waveform parameters (i.e., the number of bits and the symbol duration) and the complexity of the considered algorithm(s).

3) SC Capacity of OFDM $\mathsf{SC}_{C\text{-}OFDM}(N)$: Unfortunately, we are unable to define the SC capacity of OFDM due to the unresolved lower bound complexity of the DFT problem. Nevertheless, with consideration of the conjectures of $\Omega(N \log_2 N)$ and $\Omega(N)$ [3], Theorem 1 is established.

Theorem 1 (Comp-Limited OFDM Signal). *The uncoded OFDM signal is comp-limited unless the N-point DFT problem can be solved in linear time complexity.*

Proof. Suppose the $\Omega(N \log_2 N)$ conjecture holds for the DFT problem, indicating that the complexity of the FFT cannot be outperformed. Then, OFDM classifies as a comp-limited signal since its SC capacity is $\mathsf{SC}_{C\text{-OFDM}}(N \to \infty) = N/(N \log_2 N) = 0$. (Def. 1). By contrast, if $\Omega(N)$ conjecture verifies, OFDM is not comp-limited since, in this case, $\mathsf{SC}_{C\text{-OFDM}}(N \to \infty) = N/(c_1 N) > 0$ for some constant $c_1 > 0$. *Therefore, the uncoded OFDM signal is comp-limited unless the N-point DFT problem can be solved in linear time complexity.* □

Theorem 1 further motivates the research at answering whether the N-point DFT problem is $\Omega(N)$, which remains an open question in theory of computation [3].

VI. Conclusion and Future Work

In this letter, we proposed a mathematical framework to incorporate signal processing time complexity into performance indicators from information theory. We addressed why time complexity is often overlooked in classic formulas such as data rate, suggesting it is neglected in favour of non-temporal indicators like manufacturing cost and chip area. Ignoring the computational resources needed for faster signals can lead to unfair comparisons. In one case study, we showed that the expected data rate gain from widening bandwidth can nearly halve if the narrower signal receives the same computational resources as the wider one. Thus, our framework allows for fairer comparisons of signals with different computational resource requirements.

In another case study, we demonstrated that signal processing complexity can impose a tighter upper bound than channel capacity, making computational resources more critical. We termed this regime comp-limited signals. Specifically, the uncoded OFDM signal is comp-limited unless the lower-bound complexity of the N-point DFT problem is $\Omega(N)$. Future work may enhance our model to consider the interplay between time complexity and bit error rate in algorithms like error correction codes

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