Data-Driven Decision Making
Assignment 1: Matching

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In this assignment there will be some questions based on the following exercises. In the assignment's class there will be a set of questions in *Codex*, with the computers set up as in previous classes. The AMPL book and the slides will be available for consulting.

Exercise 1

Your company is dealing with human resources in hospitals. Your first project is to build teams for surgeries; each team is composed of two surgeons. Some surgeons may be incompatible other some others; consider the following compatibility table, where 1 means compatible and "." means incompatible.

	1	2	3	4	5	6	7
1	.	1	1	1	1	1	1
2	1	.	1	1	1	1	1
3	1	1	.	1	1	1	1
4	1	1	1	.	1	1	1
5	.	1	.	1	.	1	1
6	1	.	1	1	1	.	1
7	1	1	1	1	1	1	.

For example, surgeons 3 and 5 cannot form a team, because 5 does not accept 3 (even though the inverse is not true).

Determine a matching that maximizes the number of teams (you may consider the following AMPL program as a starting point).

```
param m integer > 0;   # number of physicians
set P := {1..m};       # set of physicians
set C within {P,P};    # set of links to compatible physicians
set S;                 # medical specialties to consider
param a{(i,j) in C};   # compatibility matrix: dots means incompatibility
# determine set of pairs with compatibility in both directions:
set L := {i in P, j in P: (i,j) in C and (j,i) in C and i < j};

var x {i in P, j in P : i < j} binary;  # 1 if 'i' and 'j' are in the same team
var cardinality >= 0;  # objective

subject to
PACK {j in P}:
        sum {i in P : (i,j) in L} x[i,j] + sum {k in P : (j,k) in L} x[j,k] <= 1;
INCOMPAT {i in P, j in P: i < j and (i,j) not in L}:
        x[i,j] = 0;
CARD:   cardinality = sum {(i,j) in L} x[i,j];

maximize total_cardinality: cardinality;
```

Data:

```
param m := 7;
param: C: a:    1  2  3  4  5  6  7:=
           1    .  1  1  1  1  1  1
           2    1  .  1  1  1  1  1
           3    1  1  .  1  1  1  1
```

```
4    1  1  1  .  1  1  1
5    .  1  .  1  .  1  1
6    1  .  1  1  1  .  1
7    1  1  1  1  1  1  .
```
;

Exercise 2

Unfortunately, many surgeons were not happy with your solution. There were many complaints at the hospital's direction, as often surgeons were in teams with someone that was not they favorite partner. Your company was called, and you proposed to ask surgeons to fill a form with their preferences (values from 1 to 10, where 1 is the best and 10 is near-incompatible). An example of the information gathered is the following (again, a dot means incompatibility).

	1	2	3	4	5	6	7
1	.	1	2	5	3	2	9
2	3	.	3	9	7	1	2
3	7	9	.	5	9	1	2
4	6	9	9	.	8	1	2
5	.	1	.	8	.	4	8
6	1	.	7	6	8	.	10
7	5	3	10	7	10	8	.

You suggested as an approach to finding a solution keeping the number of teams found in Exercise 1, but which minimizes the *total weights* for the selected teams; for example, a solution with teams $\{(2,5),\ (3,4),\ (6,7)\}$ would have a weight of $7+1+5+9+10+8=40$. Determine a solution to this problem.

Exercise 3

After the success of your previous solution, the hospital called your company for a new challenge. Teams with two surgeons are no longer enough for most of the surgeries; besides, surgeries are getting more and more specialized, and not all surgeons are able to do all the surgeries. Currently, the set of surgeries under consideration is $\{A,B,C,D\}$, and the number of required specialized surgeons for each of them is

	A	B	C	D
Surgeons necessary	2	2	3	2

The compatibilities between surgeons and each of these specialized surgeries are the following:

	A	B	C	D
1	0	0	1	0
2	1	0	0	1
3	0	1	1	1
4	1	0	1	1
5	0	1	0	1
6	0	1	1	1
7	0	0	1	0

Determine a solution where teams (now with possibly more than two elements) cover as many surgeries as possible. For example, if you assign a team with surgeons $\{2,4\}$ to surgeries A, $\{5,6\}$ to surgeries B, and $\{1,3,7\}$ to surgeries C, there will be 3 specialties (A, B, C) covered; in that case, the hospital will not provide surgeries of specialty D. Notice that each surgeon may belong to at most one team.

As a final request, the hospital asked your company to determine a solution that maximizes the number of specialties covered with minimum total weight of surgeons preferences, summed for all elements of each team. For example, the previous solution would have weight $(9+9)$ *(team A)* plus $(4+8)$ *(team B)* plus $(2+7+9+5+2+10)$ *(team C)* $=65$.

Note 1: *For pedagogic reasons, it was decided to make the first assignment similar to that of last year. The specific questions in Codex will be slight variations.*

Note 2: *You will be able to use the commercial software AMPL (`https://ampl.com`); a version with a license for this course is available in `https://www.dcc.fc.up.pt/~jpp/AMPL`. A well-known solver for dealing well with integer optimization problems is `gurobi`.*

Note 3: *Please bring scratch paper, and do not use any other materials or electronic devices during the class.*