Data-Driven Decision Making
Assignment 2: Kidney Exchange Optimization

João Pedro PEDROSO

DCC - FCUP, April 2025

In this assignment there will be some questions based on the following exercises. In the assignment's class there will be a set of questions in Codex*, with the computers set up as in previous classes. The AMPL book and the classes' slides will be available for consulting.*

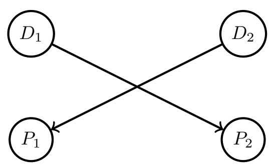
Note 1: *You will be able to use the commercial software AMPL (`https://ampl.com`), independently or via its Python API. A version with a license for this course is available in `https://www.dcc.fc.up.pt/~jpp/AMPL`. A well-known solver for dealing well with integer optimization problems is `gurobi`.*

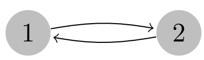
Note 2: *Please bring scratch paper, and do not use any other materials or electronic devices during the class.*

Note 3: *Even though students are encouraged to work in collaboration with a colleague in the solution of this assignment, each student will have to submit his answers separately.*

Renal diseases affect thousands of patients, who, to survive, must incur in dialysis — a costly treatment with many negative implications on their quality of life. As an alternative, patients may enter a waiting list for receiving a kidney from a deceased donor; however, waiting times are typically very long.

For reducing the waiting time, another alternative in some countries is to find a healthy living donor — usually, a relative of a person emotionally connected — who volunteers to cede one of his kidneys. However, in some situations transplantation is not possible due to blood, or tissue-level incompatibility. In these cases, a donor-patient (D_1, P_1) may enter a pool of pairs in the same situation; if in the list there is another pair (D_2, P_2) such that D_2 is compatible with P_1, and D_1 is compatible with P_2, they may do crossed transplants; this can be represented as a cycle in a graph, as shown in the right-hand figure.





Longer cycles may be allowed. In this particular program, compatible pairs may enter the program, in the hope to obtain a better exchange for the patient.

Compatibilities among blood groups are as follows.

	Patient			
Donor	O	A	B	AB
O	✓	✓	✓	✓
A	✗	✓	✗	✓
B	✗	✗	✓	✓
AB	✗	✗	✗	✓

Blood types of patients and donors in the current pool are the following:

Index	Patient	Donor
1	AB	A
2	A	AB
3	AB	O
4	B	O
5	A	A
6	O	B
7	A	O
8	A	O
9	A	O
10	A	AB

Note that in the pool some pairs are compatible.

Your task is to find the best donor-patient matching in this pool, i.e., the matching that maximizes the number of transplants.

1. Do not consider exchanges within compatible pairs; take the cases where maximum cycle sizes are 2 and 3.

2. Consider exchanges within compatible pairs; take the cases where maximum cycle sizes are 2 and 3.

3. For the previous case, minimize the number of cycles of size 3, subject to considering that the total number of transplants must be maximum.

4. Consider now that the remaining life expectation was assessed for each donor-patient assignment (after transplantation, if positive), as given in the following table.

	Patient									
Donor	1	2	3	4	5	6	7	8	9	10
1	3	.	6	.	9	.	15	19	15	9
2	27	.	30	.	.	.	.	.	.	.
3	22	10	21	21	21	14	7	10	7	13
4	9	9	20	20	20	1	9	9	9	8
5	3	4	13	.	15	.	23	17	22	17
6	20	.	11	14	.	.	.	.	.	.
7	1	.	10	11	10	.	10	11	8	4
8	11	.	10	10	10	5	.	.	.	2
9	14	.	4	5	3	5	19	13	18	25
10	23	.	34	.	.	.	.	.	.	.

Assume that transplantation is accepted, for patients that are blood-type compatible with the donor, only if 5 or more years of remaining life are expected. Compute the solution with maximum total survival time.