Data-driven Decision Making

Graph problems: Kidney Exchange Programs

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Last class

- matching problems
- kidney exchange problems

Today's class:

- more on kidney exchange

KEP: cycle formulation

Kidney Exchange Model

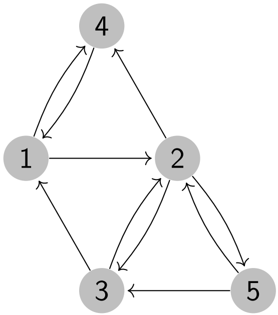
- Given:
  - a pool of n incompatible donor-patient pairs
  - the compatibility between all donors and all patients
- find the maximum number of kidney exchanges with cycles of size at most k

Complexity

- Is this problem easy to solve?
  - YES, if $k = 2$ or no limit is imposed on the size of the cycles
  - NO, if $k = 3, 4, 5, \ldots$
- If $k = 2$ the problem reduces to finding a maximum matching in a undirected graph, which can be solved efficiently (Edmonds 1965)
- If no limit is imposed on the size of the cycles the problem can be formulated as an assignment problem (can be solved efficiently by hungarian algorithm)
- The problem is NP-hard for $k = 3, 4, 5, \ldots$ (hence, no polynomial algorithms are known to solve it)

Mathematical programming formulations

- There are several possibilities for modeling the problem in mathematical programming
- One of the most successful is the cycle formulation:
  - enumerate all cycles in the graph with length at most K
  - for each cycle c, let variable x_c be 1 if c is chosen, 0 otherwise
  - every feasible solution corresponds to a set of vertex-disjoint cycles



Cycles for $K = 3$:

1. [1,2,3]
2. [1,2,4]
3. [1,4]
4. [2,3]
5. [2,5]
6. [2,5,3]
7. ~~[1,2,5,3]~~

Cycle formulation

$$
\begin{aligned}
\text{maximize} \quad & \sum_{c} w_c x_c & (1)\\
\text{subject to} \quad & \sum_{c:i \in c} x_c \leq 1 \quad \forall i \in V & (2)\\
& x_c \in \{0,1\} \quad \forall c
\end{aligned}
$$

- case of $0-1$ weights: $w_c = |c|$, (length of cycle c)
- objective: maximize the weight of the exchange
- constraints: every vertex is at most in one cycle
    - (*i.e.*, donate/receive at most one kidney)
- difficulty: exponential number of variables
    - in our experience: not an issue in practice

Objective

- Ethically, a sensitive subject
- Most common: maximize total number of transplant
  - easy to explain/justify
  - difficult to dispute
- Our claim: this is not adequate
  - KEPs are typically run periodically
  - most of the pairs will eventually be matched, after waiting a few periods
  - hence, we should look for alternatives
    $\rightarrow$ somehow access the quality of the transplants

KEP: possible objectives

Objective: our proposal

- For each pair i
  - for each possible matching j for that pair
    - i's patient may receive kidney of j's donor
    - determine expected survival time for patient s_{ji}
- Then, maximize $\sum s_{ji}$, for all arcs in the solution
- This can be done by assigning the weight w_c for each cycle:

$$w_c = \sum_{ji \in c} s_{ji}$$

- Problem:
  - n^2 weights to determine $\rightarrow$ *not practical, if done manually*
  - solution?

Predicting survival time

- There is a considerable amount of historical data
- Idea: use *historical data* to train a machine learning model
- This has been done in the past, but to our knowledge not to parameterize a KEP optimization model
- See, eg., Living Kidney Donor Risk Index (LKDPI) `http://www.transplantmodels.com/lkdpi/`

LKDPI Score:

9

This model calculates a risk score for a recipient of a potential live donor kidney.

Live Donor Characteristics:

Donor age:	**43**
Donor sex:	**male**
Recipient sex:	**female**
Donor eGFR:	**95**
Donor SBP:	**130**
Donor BMI:	**24**
Donor is African-American:	**No**
Donor history of cigarette use:	**No**
Donor and recipient biologically related:	**Yes**
Donor and recipient are ABO incompatible:	**No**
Donor Weight:	**70 kg/155 lb**
Recipient Weight:	**80 kg/178 lb**
Donor and recipient HLA-B mismatches:	**1**
Donor and recipient HLA-DR	

Proposed flow:

1. Use past data to train regression model
   - features: $\rightarrow$ patients' data
   - output: $\rightarrow$ observed survival times
2. Gather information concerning patients in current pool
3. Parameterize KEP optimization model using
   pool information + predicted survival times

Our experiment

- Data: generated artificially, loosely based on LKDPI
  - *past data* $\rightarrow$ used for training regression model
    - features + survival time
  - *current pool data* (100 patient/donor pairs)
    - patients/donors features
      $\rightarrow$ used in regression model
      $\rightarrow$ predict survival time
    - "true" survival time $\rightarrow$ *"ground truth"*
      used for assessing performance/testing

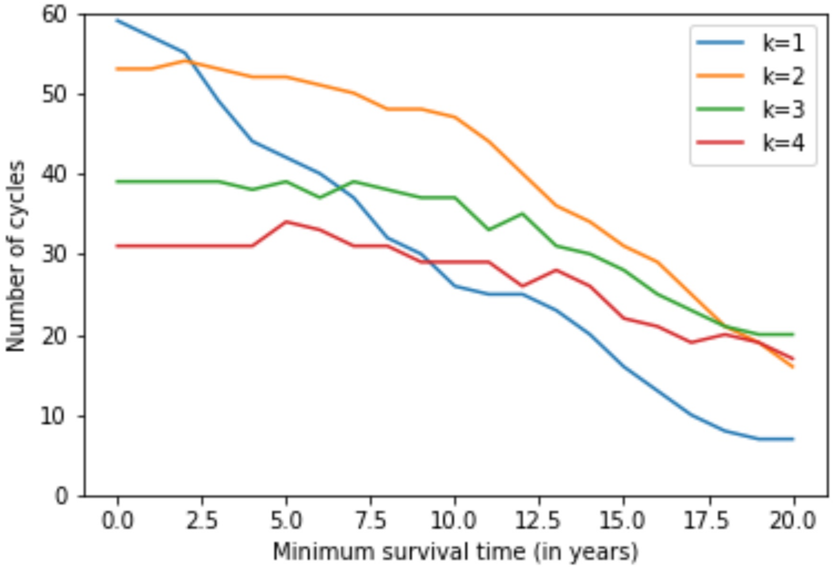
Our experiment

- Optimization model:
  - cycle model
  - each cycle's weight w_c : sum of patients' survival times
    - obtained by regression, using donor's and patient's *features*
  - objective: maximize survival time summed for all selected cycles
  - compatible pairs allowed
  - *rationality constraint:*
    compatible pairs only accept donors better than their own
    - i.e., leading to longer survival times

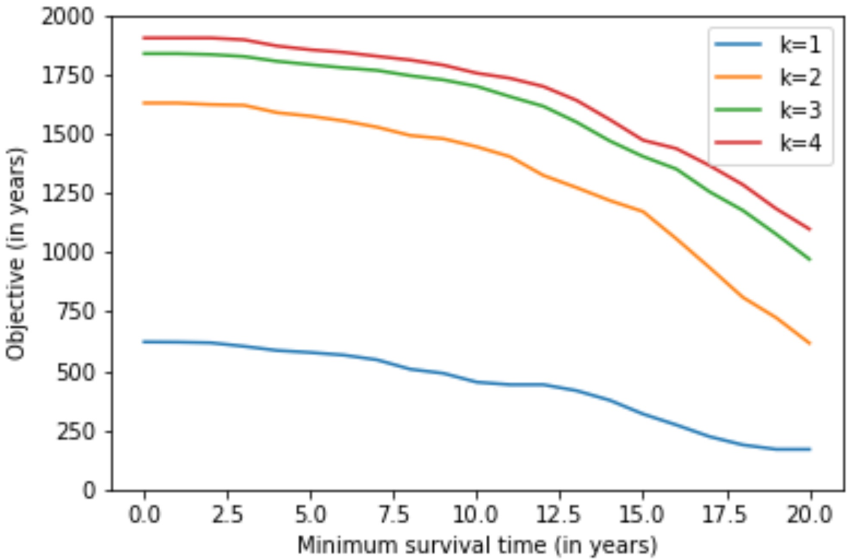
Our experiment

- Assessment:
  - compare "ground truth" survival times on:
    - solution with ML + KEP *vs.* "ground truth" + KEP
    - solution with ML + KEP *vs.* max-transplants KEP
- Parameter/Ethics:
  - we must decide the minimum patient's survival time allowed in the program...

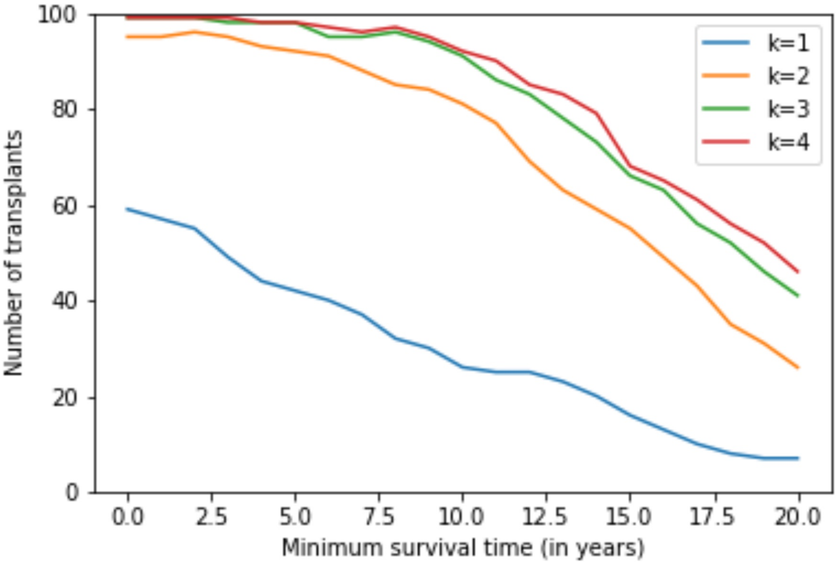
Number of cycles *vs.* minimum patient's survival time

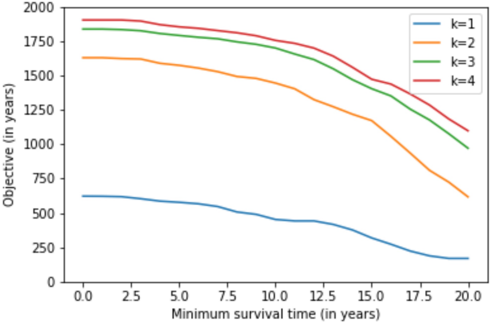


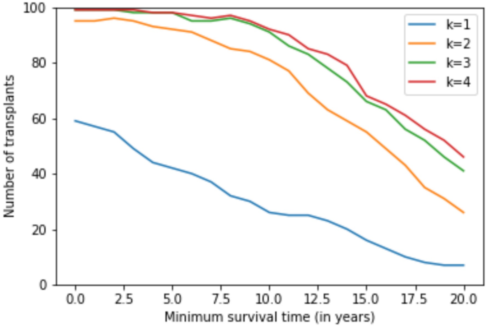
Results: total survival time *vs.* minimum survival time



Results: number of transplants *vs.* minimum survival time




2000
1750
1500
1250
1000
750
500
250
0
Objective (in years)
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Minimum survival time (in years)
k=1
k=2
k=3
k=4


100
80
60
40
20
0
Number of transplants
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Minimum survival time (in years)
k=1
k=2
k=3
k=4

Conclusion

Proposed flow: summary

1. Use past data to train regression model $\rightarrow$ survival times
2. Gather information concerning patients
   $\rightarrow$ predict survival time for each possible donor-patient assignment
3. Find all cycles of desired length
   $\rightarrow$ filter cycles with unacceptably low-survival patients
4. Prepare KEP optimization model using this information
5. Execute optimization model, retrieve and analyse solution
6. In the long run: follow transplanted patients, use survival times to improve regression model

Conclusions

- Are current kidney exchange programs wrong?
  - is the objective correct?
- There seems to be room for improvement

This lesson

- matching problems
- kidney exchange problems

Next lessons:

- kidney exchange problems
- machine learning $\rightarrow$ data-driven components