1. Farmer Leary grows wheat and corn on his 45-acre farm. He can sell at most 140 bushels of wheat and 120 bushels of corn. Each acre planted with wheat yields 5 bushels, and each acre planted with corn yields 4 bushels. Wheat sells for \$30 per bushel, and corn sells for \$50 per bushel. To harvest an acre of wheat requires 6 hours of labor; 10 hours are needed to harvest an acre of corn. Up to 350 hours of labor can be purchased at \$10 per hour. *[source: Winston]*

   (a) Formulate Farmer Leary's optimization problem.

   (b) Solve the problem, determining the optimum profit.

   (c) Identify which areas should be planted with which product.

   (d) Identify the constraints that completely use the corresponding resource.

   (e) Determine the shadow price of each resource (area and labor available, demand, ...).

   (f) If only 40 acres of land were available, what would Leary's profit be?

   (g) If the price of wheat dropped to \$26, what would be the new optimal solution to Leary's problem?

   (h) Use the slack portion of the output to determine the allowable increase and allowable decrease for the amount of wheat that can be sold. If only 130 bushels of wheat could be sold, then would the answer to the problem change?

   (i) What is the most that Leary should pay for an additional hour of labor?

   (j) What is the most that Leary should pay for an additional acre of land?

   [source: W. Winston][1]

2. Tucker Inc. must produce 1,000 Tucker automobiles. The company has four production plants. The cost of producing a Tucker at each plant (in thousands of dollars), along with the raw material and labor needed, is:

Plant	Cost	Labor	Raw material
1	15	2	3
2	10	3	4
3	19	4	5
4	17	5	6

   The autoworkers' labor union requires that at least 400 cars be produced at plant 3. There are 3300 hours of labor and 4000 units of raw material available for allocation to the four plants.

   Formulate an LP whose solution will enable Tucker Inc. to minimize the cost of producing 1,000 cars. Use duality to answer the following questions.

   (a) What is the most that Tucker should pay for an extra hour of labor?

   (b) What is the most that Tucker should pay for an extra unit of raw material?

   (c) A new customer is willing to purchase 20 cars at a price of \$25,000 per vehicle. Should Tucker fill her order? *[source: W. Winston]*

[1] Wayne Winton, "Operations Research". ISBN: 9780534423629

3. An engineering factory can produce five types of product (A, B, C, D, E) by using two production processes: grinding and drilling. After deducting raw material costs, each unit of each product yields the contributions to profit indicated next. Each unit requires a certain time on each process, given below (in hours; a dash indicates when a process is not needed).

Product	Contribution to profit (euros)	Requirements (in hours)	
		Grinding	Drilling
A	550	12	10
B	600	20	8
C	350	–	16
D	400	25	–
D	200	15	–

In addition, the final assembly of each unit of each product uses 20 hours of an employee's time.

The factory has three grinding machines and two drilling machines and works a six-day week with two shifts of 8 hours on each day. Eight workers are employed in assembly, each working one shift a day.

The problem is to find how much of each product is to be manufactured so as to maximize the total profit contribution.

(a) Formulate the factory's mathematical optimization problem.

(b) Solve the problem, determining the optimum profit.

(c) Identify which products should be produced, and the optimal quantities.

(d) For the remaining products, determine how much more expensive they should be in order for it to be worth manufacturing them (hint: these are their reduced costs).

(e) Identify the constraints that completely use the corresponding resource.

(f) Determine the shadow price of each resource (grinding, drilling, labor), i.e., how much the company would be ready to pay for additional capacity (hint: this is the associated optimal dual variable).