1. A company manufactures two products, 1 and 2. Each requires the quantities of raw material and labor, and is sold at indicated in the following table:

	Product 1	Product 2
Raw material	1 unit	2 units
Labor	2 hours	1 hours
Sale price	7 euros	8 euros

Up to 350 units of raw material can be purchased, at a price of 2 euros per unit, and up to 400 working hours can be contracted at 1.5 euros per hour.

(a) Formulate the problem.

(b) Convert the formulation to standard form.

(c) Represent the feasible region graphically.

(d) Show the correspondence between basic feasible solutions of the standard form and extreme points in the feasible region.

(e) Solve the problem using the simplex method, showing at each iteration to which point on the graphic its solution corresponds.

2. Consider the problem of a dairy company that produces milk, butter, and yogurt. Its profit is given by $z = 3x_1 + x_2 + 4x_3$, where x_1, x_2 and x_3 are the quantities in kiloliters of milk, butter and yogurt produced per day, respectively. There is a constraint (1) associated with the milk pasteurization machine, which implies that $6x_1 + 3x_2 + 5x_3 \leq 25$. Another constraint (2), associated with the packaging machine, is the following: $3x_1 + 4x_2 + 5x_3 \leq 20$.

(a) Write the complete formulation of this linear problem.

(b) Convert it to standard form.

(c) In an iteration of the simplex algorithm, the following system was obtained:

$$\begin{array}{lllllll}
z & & +2x_2 & & +s_1/5 & +3s_2/5 & = 17 \\
 & x_1 & -x_2/3 & & +s_1/3 & -s_2/5 & = 5/3 \\
 & & x_2 & +x_3 & -s_1/5 & +2s_2/5 & = 3
\end{array}$$

where s_1 and s_2 are the slack variables associated with constraints (1) and (2), respectively. Determine the optimal solution to the problem.

3. Breadco Bakeries bakes two kinds of bread: French and sourdough. Each loaf of French bread can be sold for 36¢, and each loaf of sourdough bread for 30¢. A loaf of French bread requires 1 yeast packet and 6 oz of flour; sourdough requires 1 yeast packet and 5 oz of flour. At present, Breadco has 5 yeast packets and 10 oz of flour. Additional yeast packets can be purchased at 3¢each, and additional flour at 4¢/oz. Formulate and solve an LP that can be used to maximize Breadco's profits (= revenues − costs). [source: Winston]

4. A company has 30 programmers and 5 optimization specialists. It currently has the possibility to participate in 7 projects, each yielding 30 thousand euros. Each project requires 5 programmers and 1 optimization specialist. The company can subcontract programmers, at 4 thousand euros per person, or assign its programmers to external companies for the same amount. It is assumed that both project participation and workers' time are perfectly divisible. Determine, using the simplex method, the maximum profit that the firm can earn.

5. **Capital budgeting.** Star Oil Company is considering five different investment opportunities. The cash outflows and net present values (NPV, in millions of dollars) are:

	Investment ($)				
	1	2	3	4	5
Time 0 cash outflow	11	53	5	5	29
Time 1 cash outflow	3	6	5	1	34
Net present value	13	16	16	14	39

Star Oil has $40 million available for investment now (time 0); it estimates that one year from now (time 1) $20 million will be available for investment. Star Oil may purchase any fraction of each investment. In this case, the cash outflows and NPV are adjusted accordingly. For example, if Star Oil purchases one-fifth of investment 3, then a cash outflow of $1/5 \cdot (5) = 1$ million would be required at time 0, and a cash outflow of $1/5 \cdot (5) = 1$ would be required at time 1. The one-fifth share of investment 3 would yield an NPV of $1/5 \cdot (16) = 3.2$ million. Star Oil wants to maximize the NPV that can be obtained by investing in investments 1—5. Formulate an LP that will help achieve this goal. Assume that any funds left over at time 0 cannot be used at time 1. [source: Winston]

6. Solve the following problems using the simplex method:

(a)

$$\begin{array}{llllll} \text{minimize } z = & 4x_1 & -x_2 & +2x_3 & \\ \text{subject to:} & x_1 & +x_2 & & \leq 5 \\ & 2x_1 & -x_2 & & \leq -1 \\ & & 2x_2 & +x_3 & \geq 6 \\ & x_1 & & +x_3 & = 4 \\ & & x_1, x_2, x_3 \geq 0 & & \end{array}$$

(b)

$$\begin{array}{llll} \text{minimize } z = & 2x_1 & +3x_2 & \\ \text{subject to:} & 1/2x_1 & +1/4x_2 & \leq 4 \\ & x_1 & +3x_2 & \geq 36 \\ & x_1 & +x_2 & = 20 \\ & & x_1, x_2 \geq 0 & \end{array}$$

7. For each of the following cases, check, using the simplex method, if the problem has a unique solution, if it has multiple solutions, if it is infeasible (*impossível*), or if it is unbounded (*ilimitado*).

(a)

$$\begin{array}{lll} \text{maximize } z = & x_1 + x_2 & \\ \text{subject to:} & x_1 + x_2 & \leq 4 \\ & x_1 - x_2 & \geq 5 \\ & x_1, x_2 & \geq 0 \end{array}$$

(b)

$$\begin{array}{lll} \text{maximize } z = & 4x_1 + x_2 & \\ \text{subject to:} & 8x_1 + 2x_2 & \leq 16 \\ & 5x_1 + 2x_2 & \leq 12 \\ & x_1, x_2 & \geq 0 \end{array}$$

(c)

$$\begin{array}{lll} \text{maximize } z = & -x_1 + 3x_2 & \\ \text{subject to:} & x_1 - x_2 & \leq 4 \\ & x_1 + 2x_2 & \geq 4 \\ & x_1, x_2 & \geq 0 \end{array}$$

(d)

$$\begin{array}{lll} \text{maximize } z = & 3x_1 + x_2 & \\ \text{subject to:} & 2x_1 + x_2 & \leq 6 \\ & x_1 + 3x_2 & \leq 9 \\ & x_1, x_2 & \geq 0 \end{array}$$