1. Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programs: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy ad costs \$50,000, and a 1-minute football ad costs \$100,000. Dorian would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men. [source: Winston]

   (a) Use linear programming to determine how Dorian Auto can meet its advertising requirements at minimum cost.

   (b) Find the range of values on the cost of a comedy ad for which the current basis remains optimal.

   (c) Find the range of values for required HIW exposures for which the current basis remains optimal. Determine the new optimal solution if $28 + \Delta$ million HIW exposures are required.

   (d) Find the shadow price of each constraint.

   (e) If 26 million HIW exposures are required, determine the new optimal objective value.

2. Write the dual of each of the following problems:

   (a)
$$\begin{array}{llllll} \text{maximize } z = & 4x_1 & -x_2 & +2x_3 & & \\ \text{subject to:} & x_1 & +x_2 & & \leq 5 \\ & 2x_1 & +x_2 & & \leq 7 \\ & & 2x_2 & +x_3 & \geq 6 \\ & x_1 & & +x_3 & = 4 \\ & \multicolumn{4}{l}{x_1 \geq 0, \quad x_2, x_3 \in \mathbb{R}} \end{array}$$

   (b)
$$\begin{array}{llllll} \text{minimize } z = & 4x_1 & -x_2 & +2x_3 & & \\ \text{subject to:} & x_1 & +x_2 & & \leq 5 \\ & 2x_1 & +x_2 & & \leq 7 \\ & & 2x_2 & +x_3 & \geq 6 \\ & x_1 & & +x_3 & = 4 \\ & \multicolumn{4}{l}{x_1 \geq 0, \quad x_2, x_3 \in \mathbb{R}} \end{array}$$

   (c)
$$\begin{array}{llllll} \text{minimize } z = & 4x_1 & +2x_2 & -x_3 & & \\ \text{subject to:} & x_1 & +2x_2 & & \leq 6 \\ & 2x_1 & +x_2 & & \leq 7 \\ & & 2x_2 & +x_3 & \geq 6 \\ & x_1 & & +x_3 & = 4 \\ & \multicolumn{4}{l}{x_1 \geq 0, \quad x_2, x_3 \in \mathbb{R}} \end{array}$$

3. My diet requires that all the food I eat come from one of the four “basic food groups” (chocolate cake, ice cream, soda, and cheesecake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola, and pineapple cheese-cake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional content per unit of each food is:

	Calories	Chocolate (Ounces)	Sugar (Ounces)	Fat (Ounces)
Brownie	400	3	2	2
Chocolate ice cream	200	2	2	4
Cola	150	0	4	1
Pineapple cheesecake	500	0	4	5

[Adapted from Winston]

(a) Formulate a linear programming model that can be used to satisfy my daily nutritional requirements at minimum cost.

(b) Solve the problem using GLPK (or other software).

(c) Determine the optimal values of variables, dual variables, reduced costs, and slack variables.

(d) Formulate the dual problem.

(e) Solve the dual problem and determine the optimal values of variables, dual variables, reduced costs, and deviation variables.

(f) Check that the complementary deviation theorem applies, and give it an economic interpretation.

4. Glassco manufactures glasses: wine, beer, champagne, and whiskey. Each type of glass requires time in the molding shop, time in the packaging shop, and a certain amount of glass. The resources required to make each type of glass are:

	(Wine) (x_1)	(Beer) (x_2)	(Champagne) (x_3)	(Whiskey) (x_4)
Molding time (min)	4	9	7	10
Packaging time (min)	1	1	3	40
Glass (oz)	3	4	2	1
Selling price ($)	6	10	9	20

Currently, 600 minutes of molding time, 400 minutes of packaging time, and 500 oz of glass are available. Assuming that Glassco wants to maximize revenue, the following LP should be solved:

$$\begin{array}{lllllll} \max z = & 6x_1 & +10x_2 & +9x_3 & +20x_4 & \\ \text{subject to:} & 4x_1 & +9x_2 & +7x_3 & +10x_4 & \leq 600 \\ & x_1 & +x_2 & +3x_3 & +40x_4 & \leq 400 \\ & 3x_1 & +4x_2 & +2x_3 & +x_4 & \leq 500 \\ & & & x_1, x_2, x_3, x_4 \geq 0 & & \end{array}$$

It can be shown that the optimal solution to this LP is $z = \frac{2800}{3}, x_1 = \frac{400}{3}, x_2 = 0, x_3 = 0, x_4 = \frac{20}{3}, s_1 = 0, s_2 = 0, s_3 = \frac{280}{3}$.

(a) Find the dual of the Glassco problem.

(b) Using the given optimal primal solution and the Theorem of Complementary Slackness, find the optimal solution to the dual of the Glassco problem.

(c) Find an example of each of the complementary slackness conditions (e.g., a positive slack in a constraint implies a corresponding dual variable equal to zero). Interpret each example in terms of shadow prices.