MAPI – Computer Vision

Multiple View Geometry

In this module we intend to present several techniques in the domain of the 3D vision

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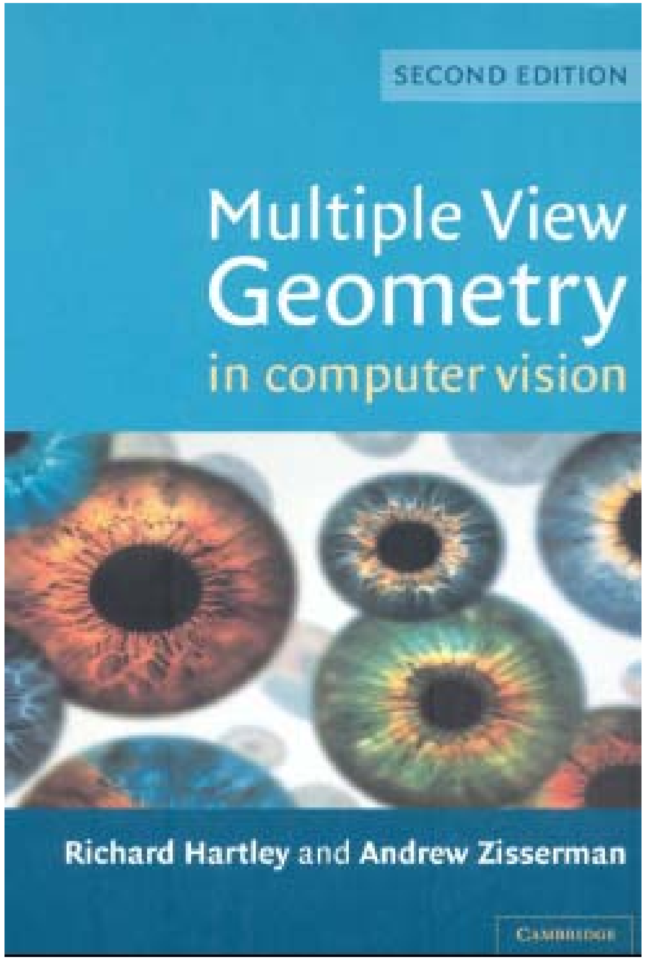
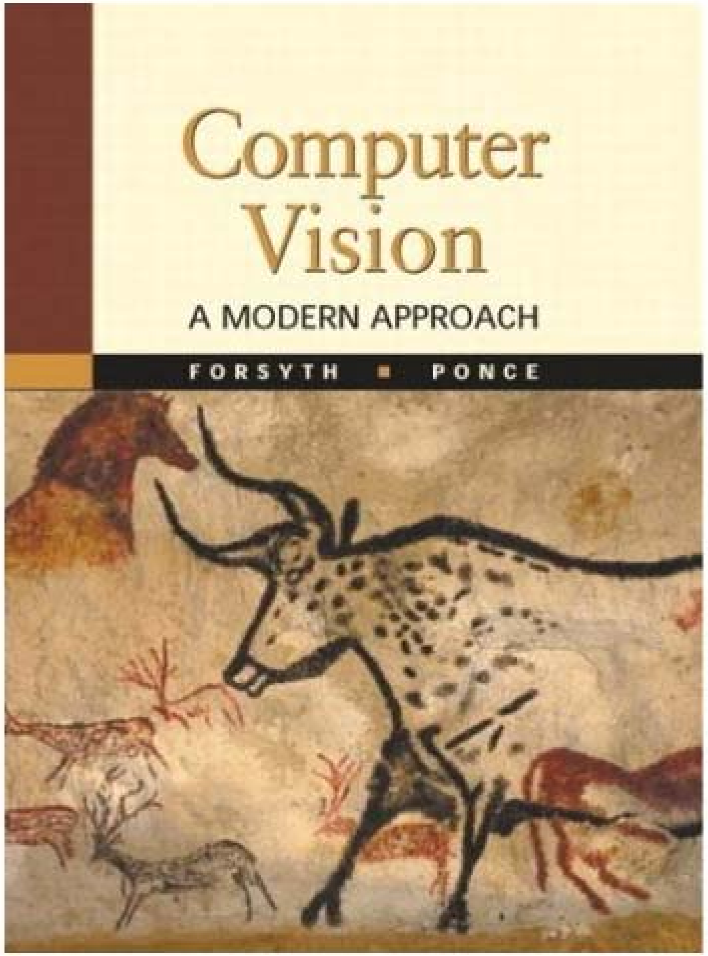
- Applications
- Camera Models
- Projective transformations
- Vanishing points and vanishing lines
- Calibration Methods
    - *Intrinsic* Camera Parameters
    - Extrinsic Parameters
- Geometry of Multiple Views
    - 2 Camera Geometry
        - Epipolar Geometry
        - Fundamental matrix F
        - Stereo Vision
    - 3 Camera Geometry
        - Epipolar Geometry
- Active Vision
- Background subtraction and optical flow

The material used is based on the following books:



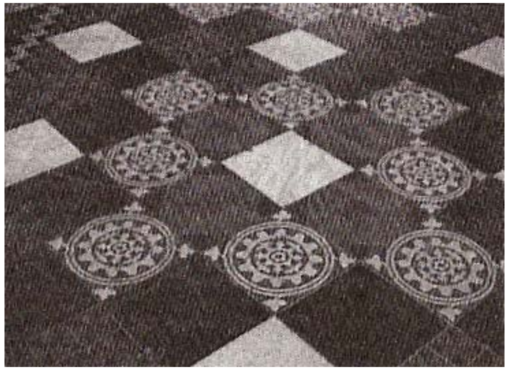


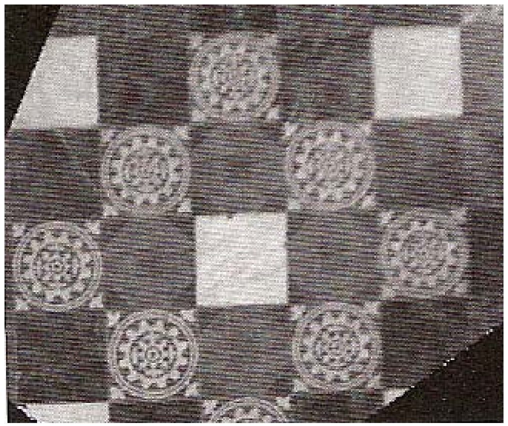
The first is especially dedicated to these specific issues, and the second presents a wide range of computer vision techniques

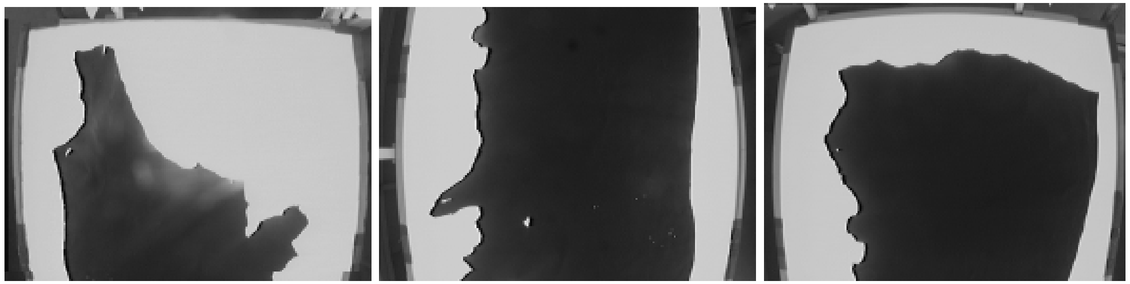
Multiple View Geometry
- applications -

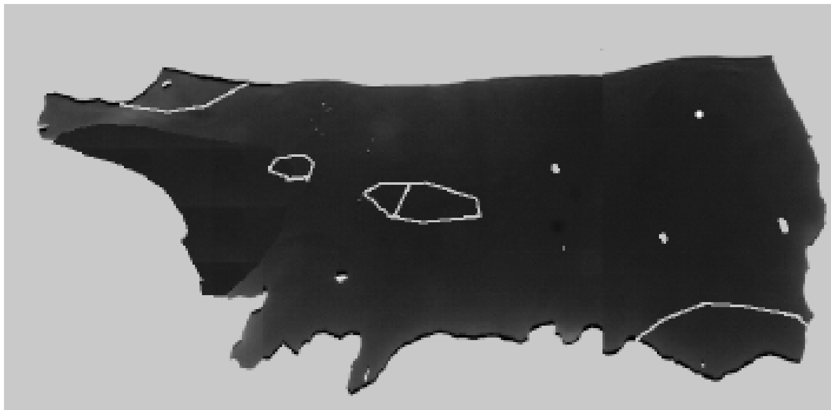
- Automation
  – Vision inspection systems

In several computer vision applications we have to transform the image coordinates to world coordinates,
even in applications using only 2D information.









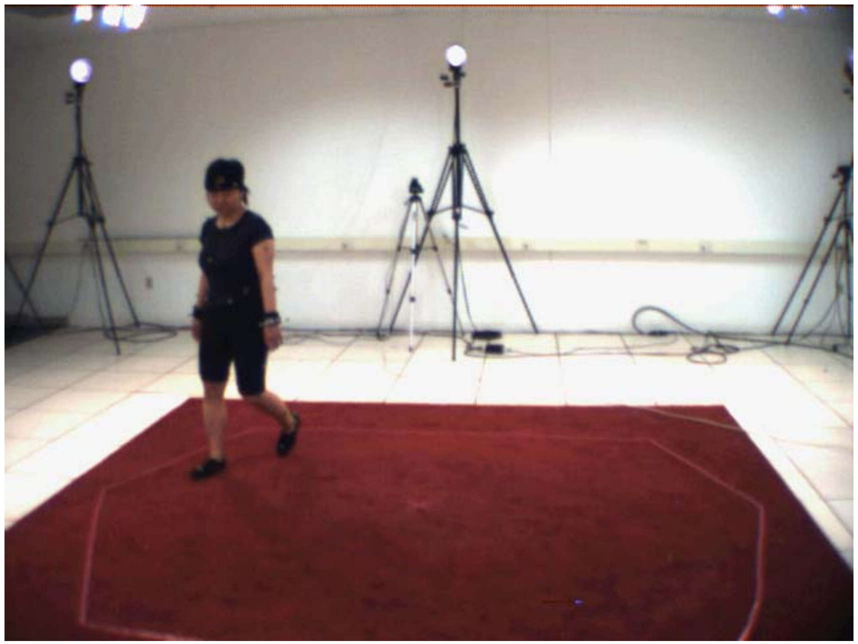
Multiple View Geometry
- applications -

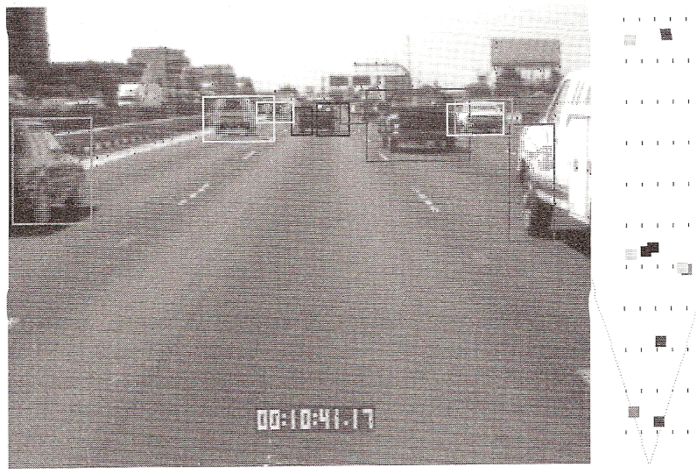
- Robotics
  - Manipulator - PUMA
  - Mobile robots

Robotics is another field where the transformation from image coordinates to world coordinates is also mandatory.
To perform grasping and tracking the robots must know the position of the objects, visualized by the vision system, in their own coordinates.
So a set of transformation between frames must be performed.

Multiple View Geometry
- applications -

- Surveillance
  - Human motion capture
    - Measurement
    - Evaluate the posture
  - Traffic applications



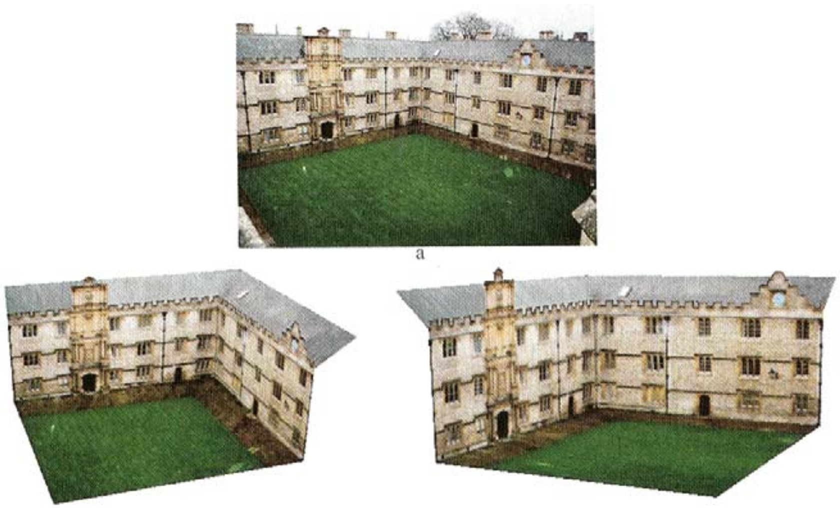


Multiple View Geometry - applications -

- Computer Graphics
  - Planar panoramic Mosaicing
  - 3D reconstruction

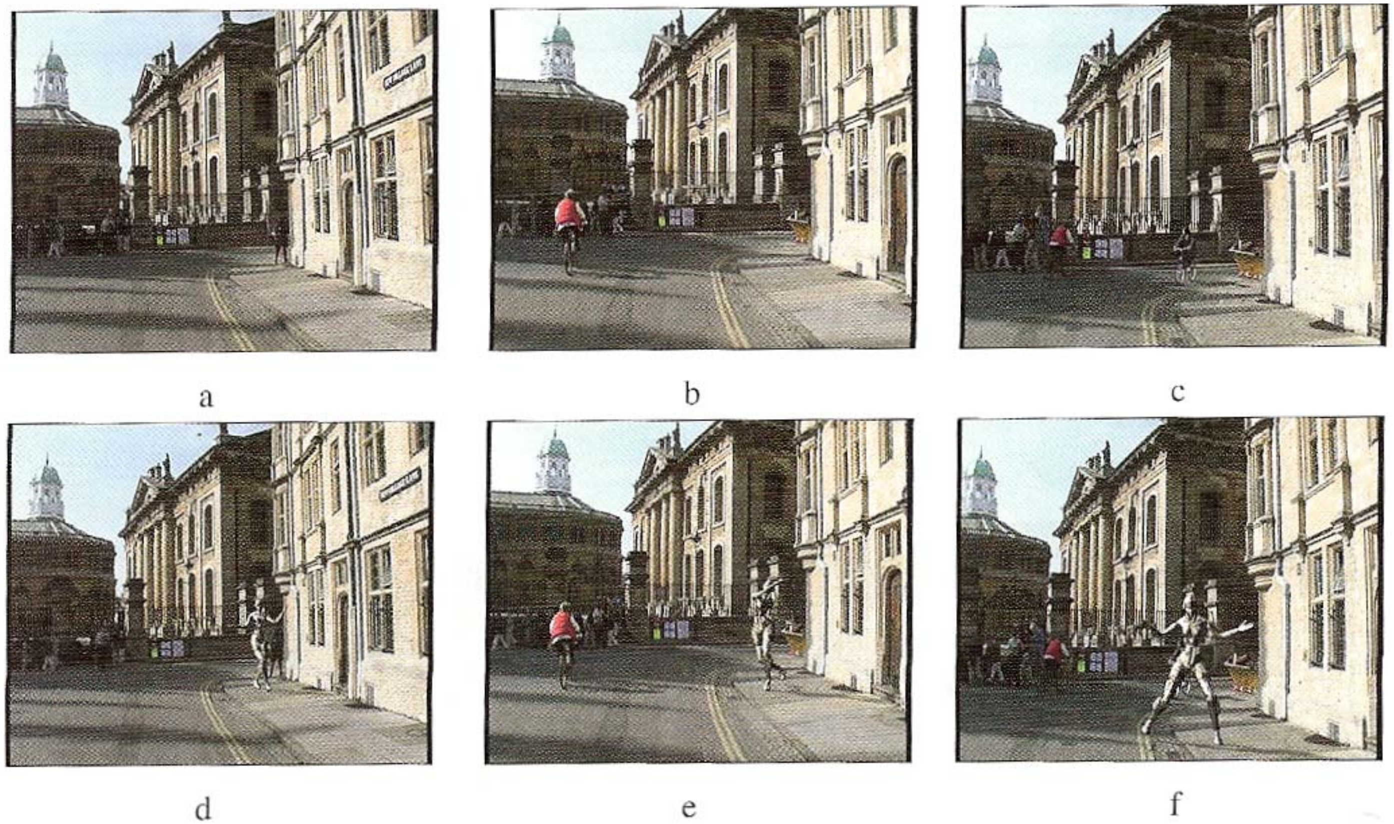
Also require information in world coordinates acquired through visual information

http://photosynth.net/Default.aspx



Multiple View Geometry
- applications -

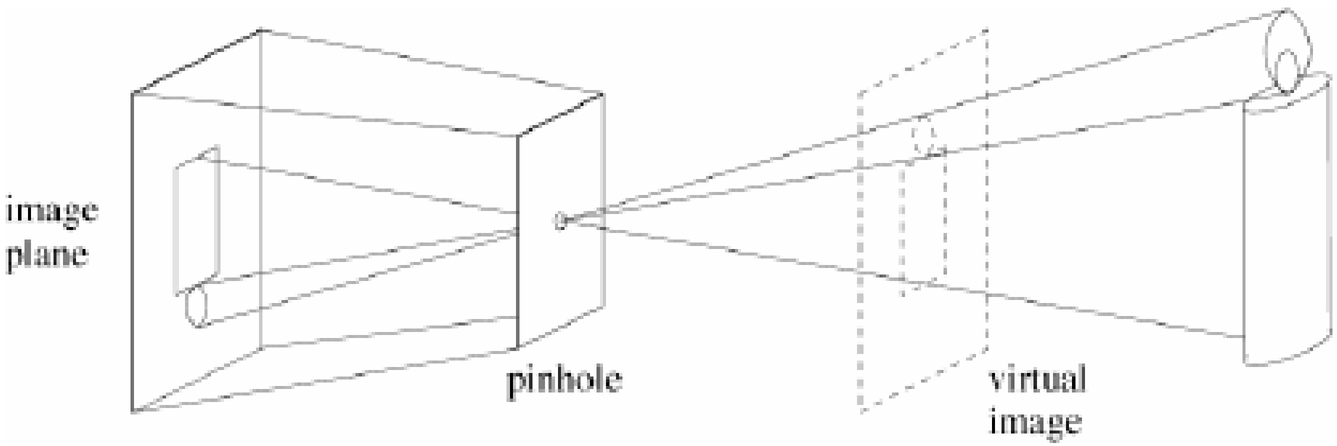
- Augmented Reality
  - ARToolKit



Pinhole cameras

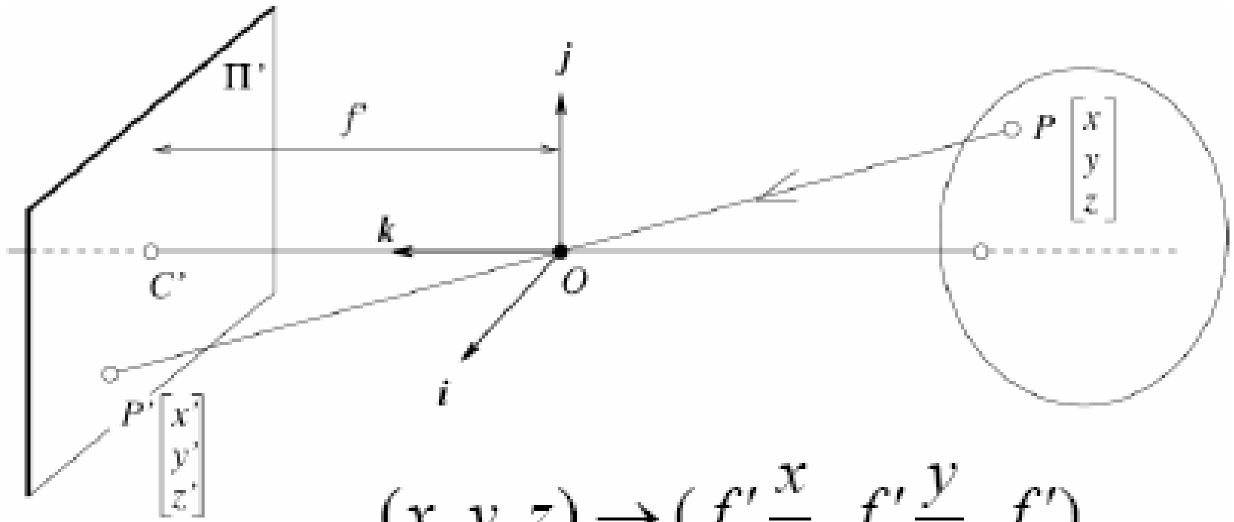
One of the first step to determine the relation between the image coordinates and the world coordinates is the definition of a camera model.

- Abstract camera model
  - Box with a small hole in it
  - Note inverted image



Equation of projection

This equation presents the relation between the pixels coordinates and the world coordinates based on the pinhole model



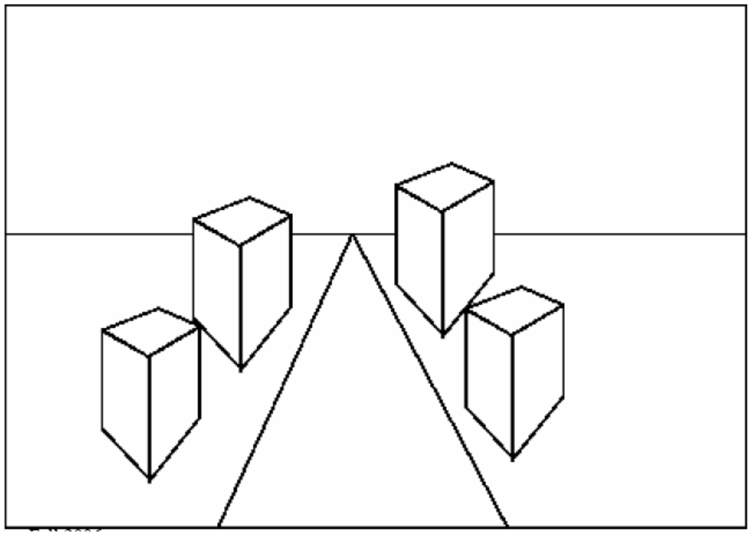
$$(x, y, z) \rightarrow (f' \frac{x}{z}, f' \frac{y}{z}, f')$$

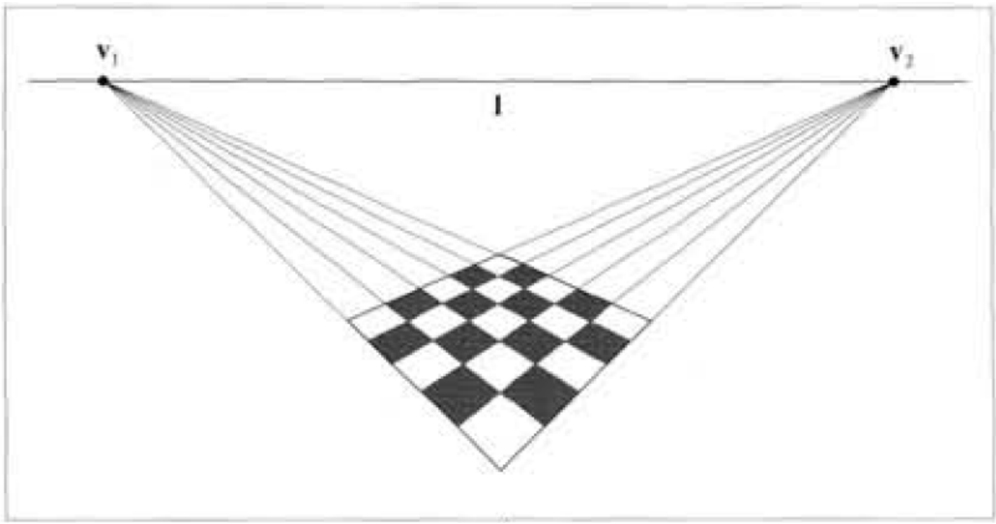
Some Qualitative Properties

- Forward projection
  - Can project curves by projecting their points
  - When project surfaces or solids, some points may not be visible.
  - Straight lines project to straight lines, circles project to ellipses
  - Angles are not preserved, in general
- Back projection
  - Given an image point, only an orientation for corresponding 3-D point can be determined
  - Given an image line, corresponding line must be on a specific plane

Some Qualitative Properties

- Projections of parallel lines are not parallel
  - They meet in a common point called the *vanishing point*
  - The vanishing point depends on the *direction* of the set of parallel lines
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane





Coordinate Systems

- Previous transformation matrix requires object coordinates to be expressed in the ***camera* coordinate system** (with origin at lens center)
  - This, in general, is not very convenient
- ***Object* coordinate system**
  - Aligned with some components of the object
    - the three sides of a rectangular solid

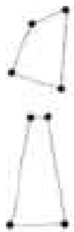
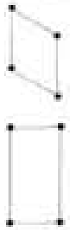
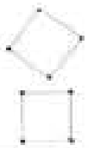
- ***World* coordinate system**
  - Chosen for global convenience, *e.g.* lines forming corner of a room , or earth coordinates (latitude, longitude, height)
- **Coordinate transformations** define relations between different coordinate systems

Projective transformations

- 2D Definition
  - A planar projective transformation is a linear transformation on homogeneous 3-vectors represented by a non-singular 3x3 matrix
  - *x'* = Hx

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Projective transformations

Group	Matrix	Distortion	Invariant properties
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, **order of contact**: intersection (1 pt contact); tangency (2 pt contact); inflections (3 pt contact with line); tangent discontinuities and cusps. cross ratio (ratio of ratio of lengths).
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines (e.g. midpoints), linear combinations of vectors (e.g. centroids). The line at infinity, $\mathbf{l}_\infty$.
Similarity 4 dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of lengths, angle. The circular points, **I**, **J** (see section 2.7.3).
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Length, area

Projective transformations

- A projectivity is an invertible mapping from points in $\mathbb{P}^2$ (that is homogeneous 3-vectors) to points in $\mathbb{P}^2$ that maps lines to lines.
- Projectivities form a group since the inverse of a projectivity is also a projectivity

- A projectivity is also called a *collineation* a *projective transformation* or a *homography*

- *A mapping* $h : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ is a *projectivity if and only if there exists a non-singular* 3x3 *matrix* H *such that for any point in* $\mathbb{P}^2$ *represented by a vector* x *it is true that* $h(x) = \mathrm{H}x$.

Projective transformations

- Removing the projective distortion from a perspective image of a plane
  - Shape is distorted under the perspective imaging
  - Parallel lines on a scene plane are not parallel in the image but instead converge to a finite point

Projective transformations

- Removing the projective distortion from a perspective image of a plane

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

  - This can be written in inhomogeneous form

$$x' = \frac{x_1'}{x_3'} = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

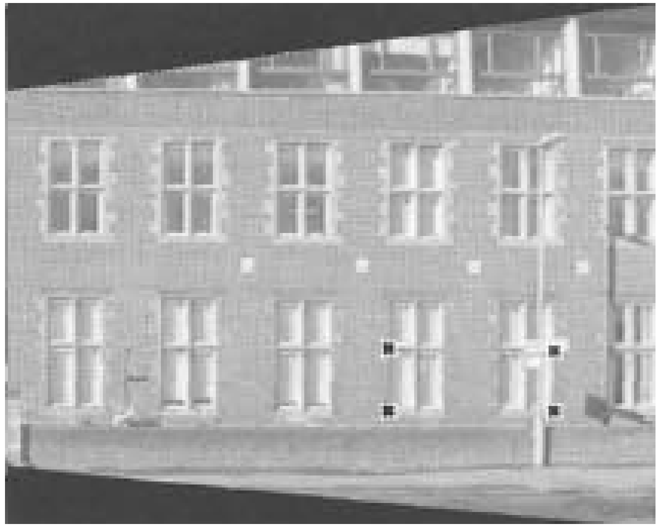
$$y' = \frac{x_2'}{x_3'} = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Projective transformations

- Removing the projective distortion from a perspective image of a plane
  - Each point correspondence generates two equations for the elements of H.
  - These equations are linear in the elements of H
  - Four points correspondence lead to eight such linear equations in the entries of H
  - Sufficient to solve H

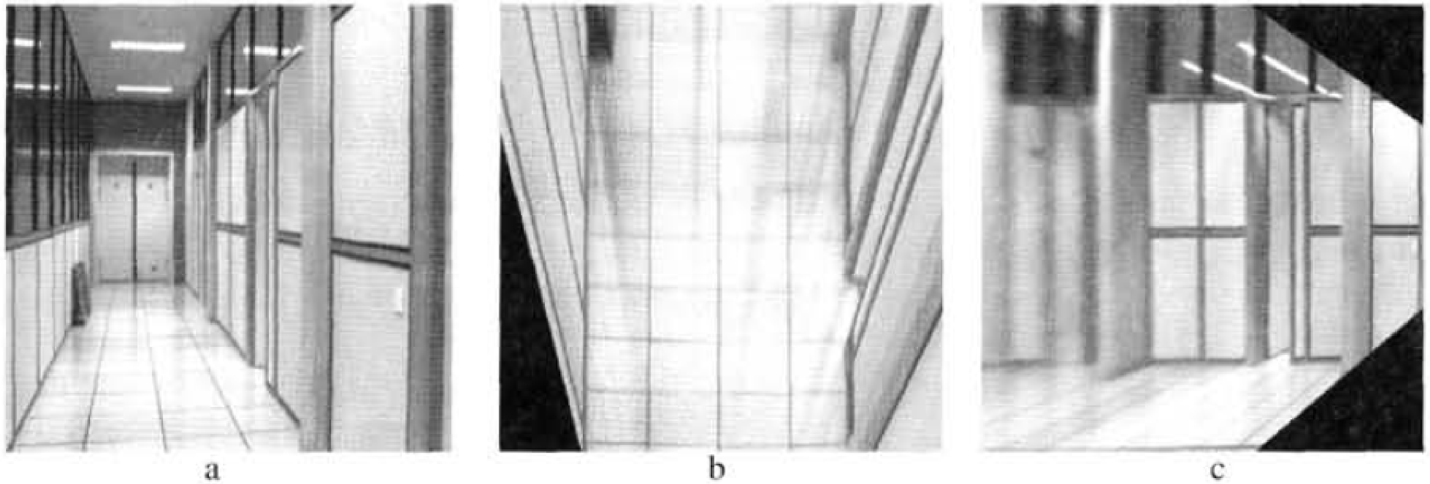
Projective transformations

- Removing the projective distortion from a perspective image of a plane



Projective transformations

- Homographic relation between images
  - Applications examples
    - 4 points



Synthetic views
b) Fronto-parallel view of the corridor floor generated from **a)** using the four corners of the floor tile to compute the homography
c) Fronto-parallel view of the corridor wall generated from a) using the four corners of the door frame to compute the homography

Projective transformations

- Removing the projective distortion from a perspective image of a plane
  - The computation of the rectifying transformation H in **this way does not require knowledge of *any* of the camera's parameters or the pose of the plane**
  - If **exactly four correspondences are** given, then an exact solution for the matrix H is possible. This is **the *minimal* solution**.
  - Points are measured inexactly ("noise"), **if more than four such correspondences are given,** then these correspondences may not be fully compatible with any projective transformation,
    - **determining the "best" transformation given the data**. This will generally be done by finding **the transformation H that minimizes** some cost function.
      - robust estimation algorithms
        - » RANSAC – RANdom SAmple Consensus

Projective transformations

RANSAC - RANdom SAmple Consensus

- Objective
  - Compute the 2D homography between two images.

- Algorithm
  - (i) **Interest points:** Compute interest points in each image.
  - (ii) **Putative correspondences:** Compute a set of interest point matches based on proximity and similarity of their intensity neighbourhood.
  - (iii) **RANSAC robust estimation:** Repeat for *N* samples
    - (a) Select a random sample of 4 correspondences and compute the homography H.
    - (b) Calculate the distance *d* for each putative correspondence.
    - (c) Compute the number of inliers consistent with H by the number of correspondences for which
      - $d < t = sqrt(5.99)\sigma$ pixels.
    - Choose the H with the largest number of inliers. In the case of ties choose the solution that has the lowest standard deviation of inliers.
  - (iv) **Optimal estimation:** re-estimate H from all correspondences classified as inliers, by minimizing the ML (Maximum Likelihood) cost function
  - (v) **Guided matching:** Further interest point correspondences are now determined using the estimated H to define a search region about the transferred point position.
- The last two steps can be iterated until the number of correspondences is stable.

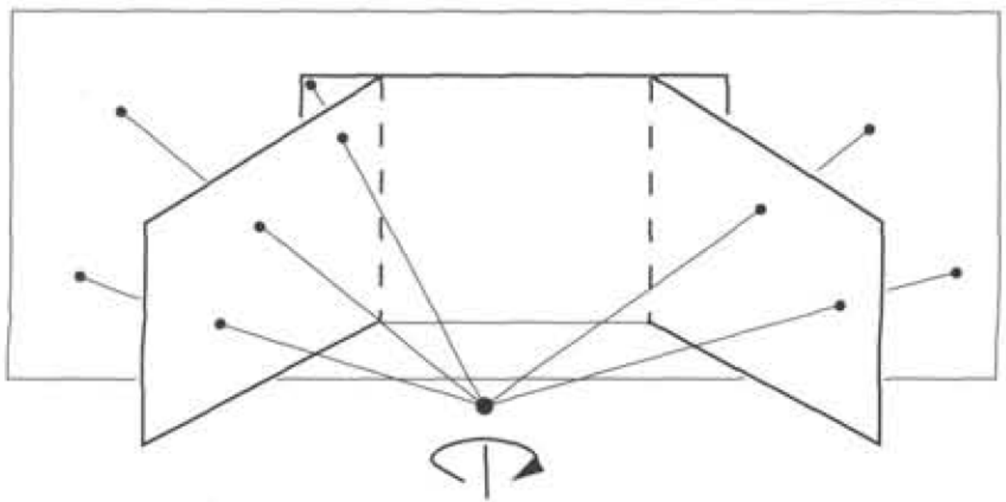
Projective transformations

RANSAC

- Application domain
  - Scenes should be lightly textured
  - The search window proximity constraint places an upper limit on the image motion of corners (the disparity) between views
    - Smaller search window means that fewer corner matches must be evaluated
  - Robust estimation
    - Moderate immunity
      - Independent motion
      - Changes in shadows
      - Partial occlusions,…

Projective transformations

- Homographic relation between images
  - Applications examples
    - Planar panoramic mosaicing

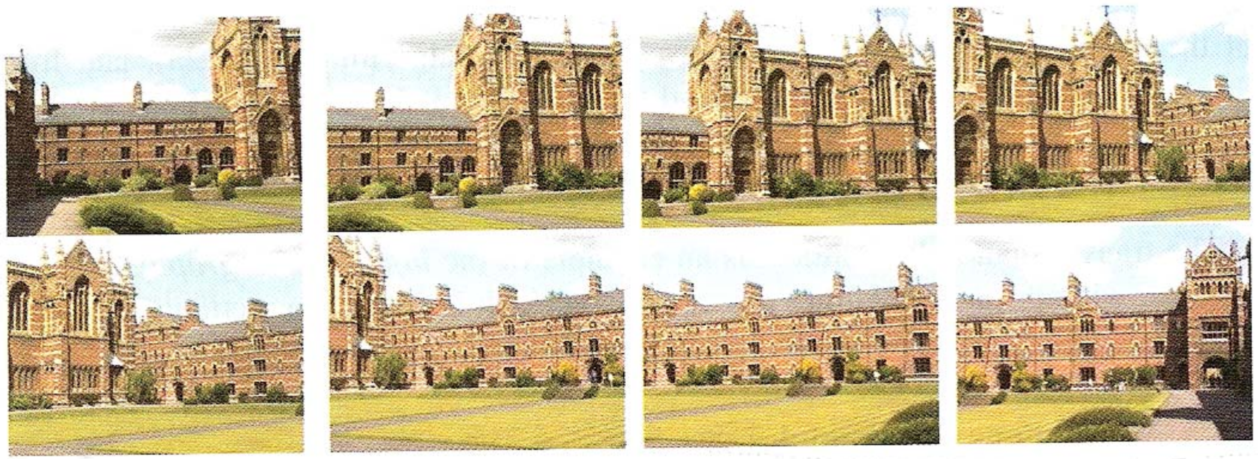


  - Three images acquired by a rotating camera may be registered to the frame of the middle one by projectively warping the outer images to align with the middle one.
    - » Given 4 points
    - » More points - ransac

Projective transformations

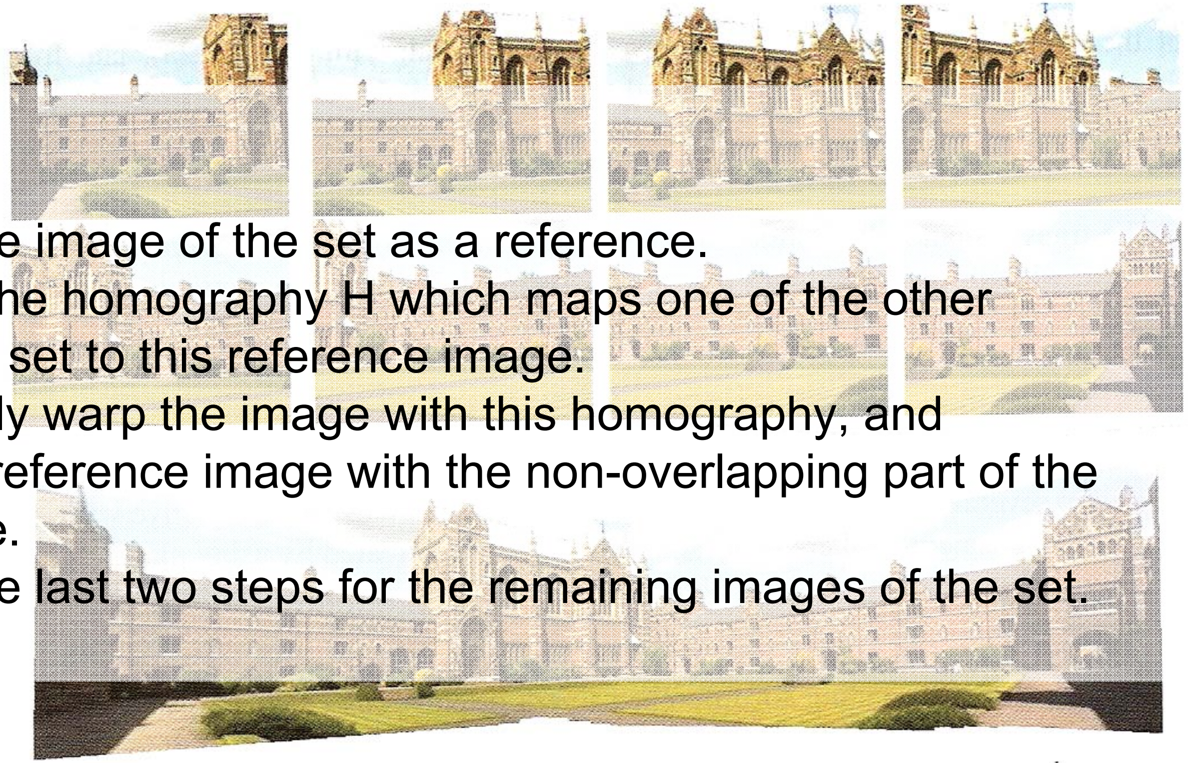
- Homographic relation between images
  - Applications examples
    - Planar panoramic mosaicing

Eight images acquired by rotating a camcorder about its centre



Projective transformations

- Homographic relation between images
  - Applications examples
    - Planar panoramic mosaicing



Algorithm:

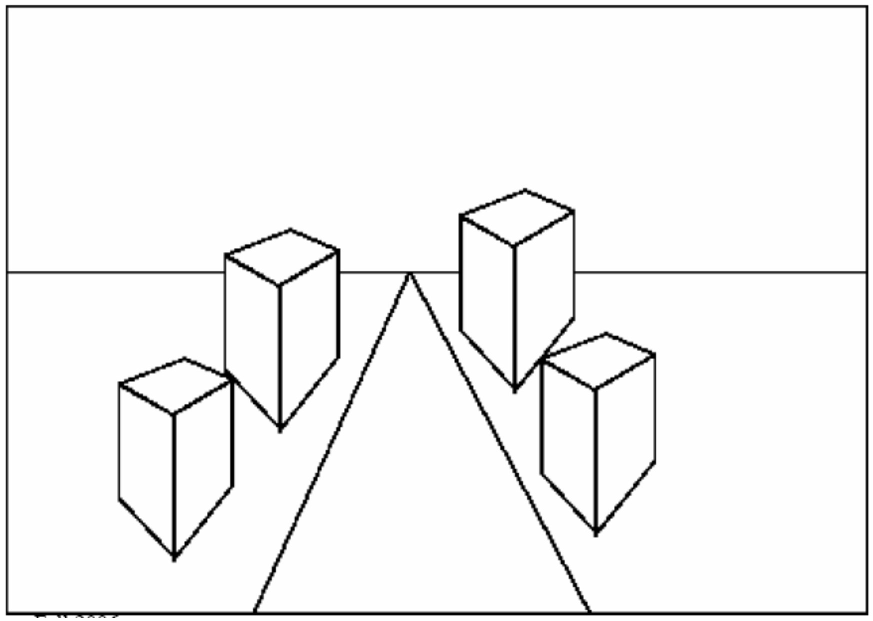
(i) Choose one image of the set as a reference.
(ii) Compute the homography H which maps one of the other images of the set to this reference image.
(iii) Projectively warp the image with this homography, and augment the reference image with the non-overlapping part of the warped image.
(iv) Repeat the last two steps for the remaining images of the set.

Vanishing points and vanishing lines

- One of the distinguishing features of perspective projection is that the image of an object that stretches off to infinity can have finite extent.

- An infinite scene line is imaged as a line terminating in a *vanishing point.*

- Similarly, parallel world lines, such as railway lines, are imaged as converging lines, and their image intersection is the vanishing point for the direction of the railway.

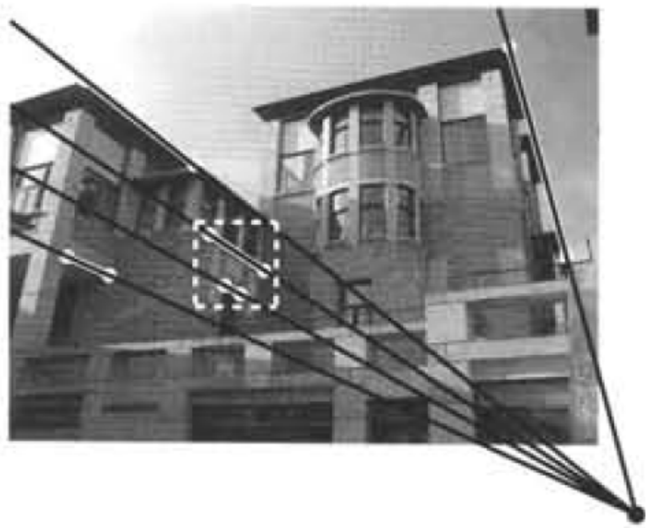


EXE

Vanishing points and vanishing lines

- **Vanishing point estimation**
  - *involves fitting a line (shown thin here) through* v *to each measured line (shown thick here).*
  - maximum likelihood (ML) *estimate*

Ex. Hough transform



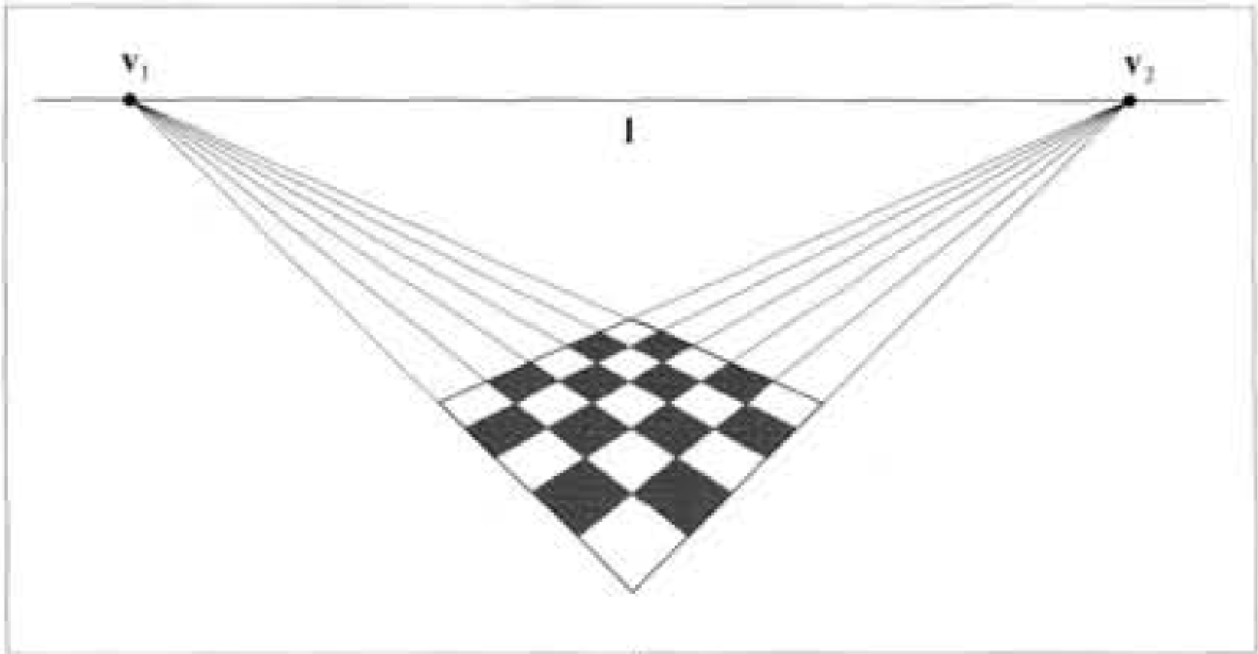
The maximum likelihood (ML) *estimate of* v *is the point which minimizes the sum of squared orthogonal distances between the fitted lines and the measured lines' end points*

Vanishing points and vanishing lines

- **Vanishing lines**
  - Parallel planes in 3-space intersect π_∞ in a common line, and the image of this line is the vanishing line of the plane.

It is clear that a vanishing line depends only on the *orientation* of the scene plane;

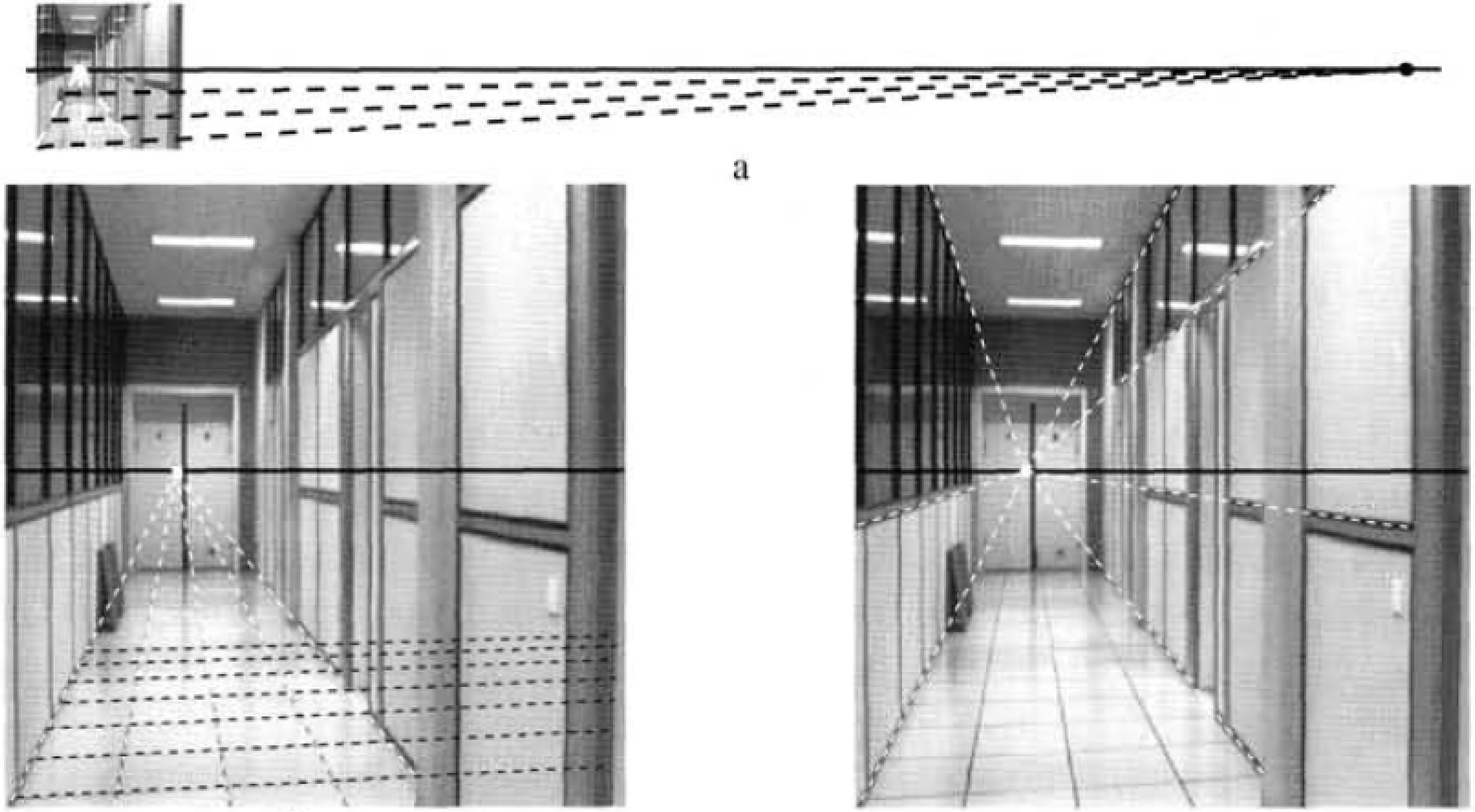
it does not depend on its position.



Since lines parallel to a plane intersect the plane at π_∞, it is easily seen that **the vanishing point** of a **line parallel to a plane lies on the vanishing line of the plane**.

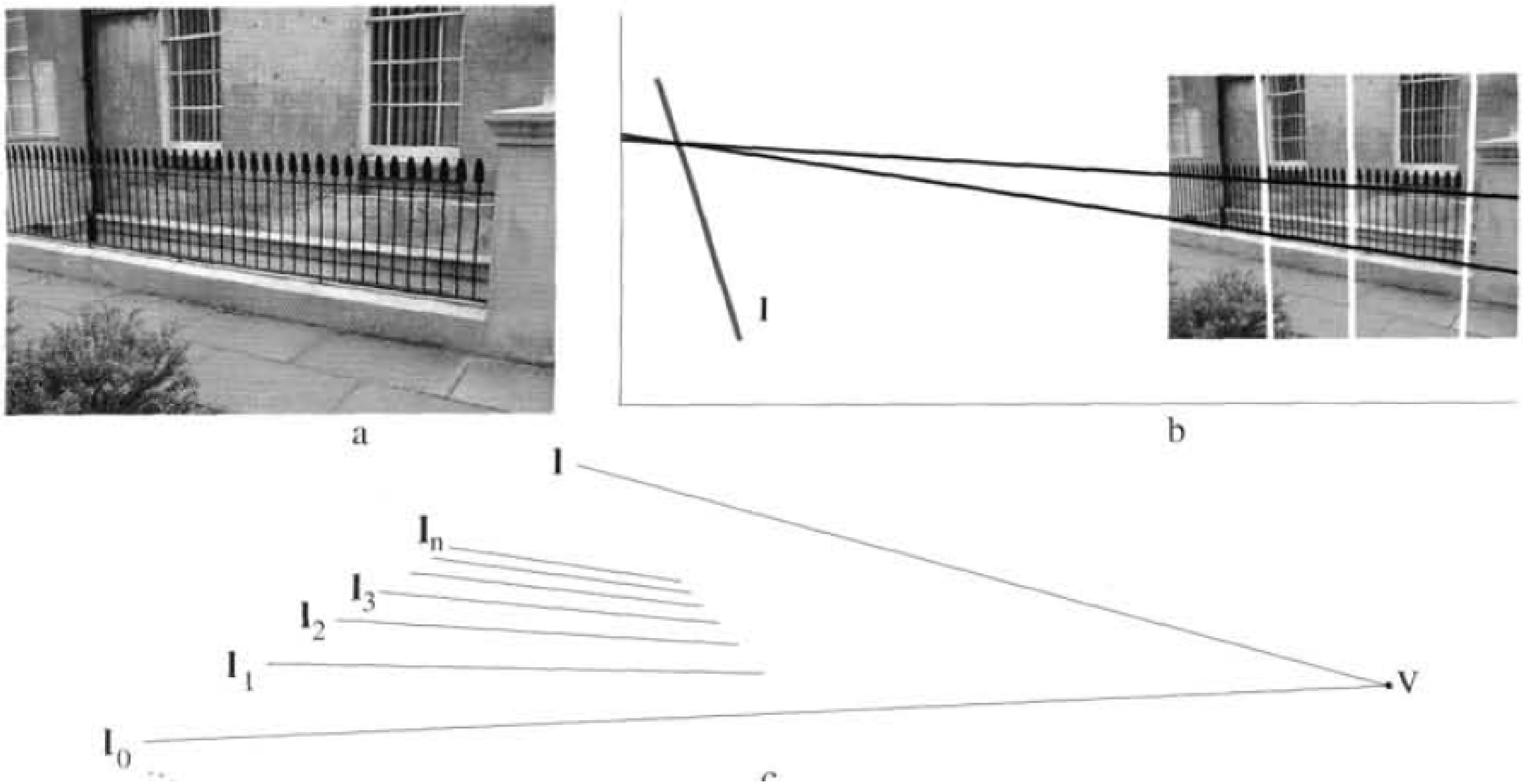
Vanishing points and vanishing lines

- **Vanishing lines**



Vanishing points and vanishing lines

- **Vanishing lines**



Vanishing points and vanishing lines

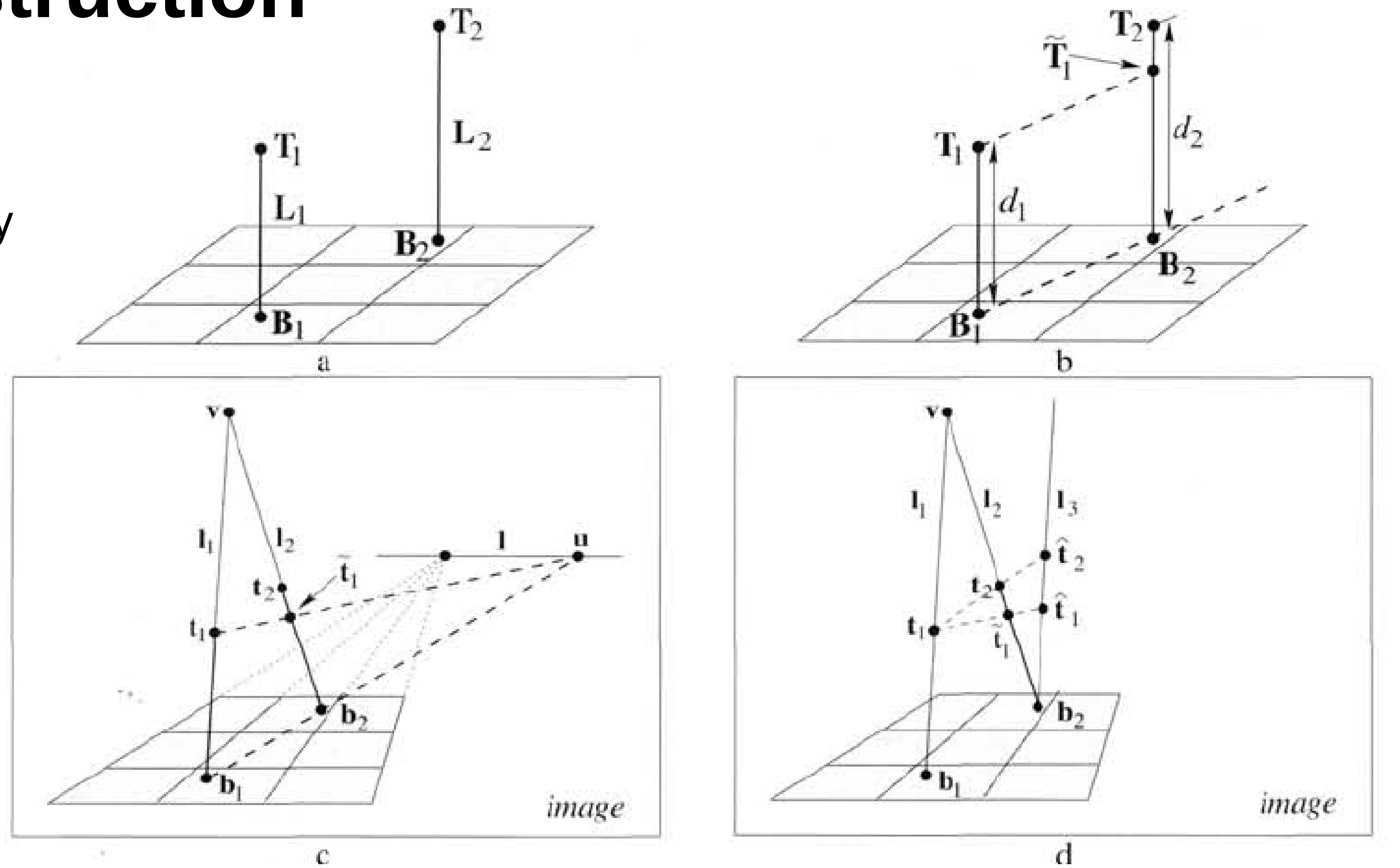
- **Affine 3D measurements and reconstruction**
  - *Given:*
    - *the vanishing line of the ground plane* **l**
    - *the vertical vanishing point* v,
    - *then the relative length of vertical line segments can be measured*
      - *provided their end point lies on the ground plane.*

Vanishing points and vanishing lines

- **Affine 3D measurements and reconstruction**

world geometry

image geometry



Vanishing points and vanishing lines

- **Affine 3D measurements and reconstruction**