
Cvision – 1

Introduction; Camera Geometry

João Paulo Silva Cunha

(jcunha@det.ua.pt)



IEETA / Universidade de Aveiro

Outline

- ‘Computer Vision’?
- Cameras & Cameras Geometry
- The Human Visual System

Acknowledgements: Most of this course is based on the excellent courses offered by Prof. Shree Nayar at Columbia University, USA and by Prof. Srinivasa Narasimhan at CMU, USA. This was also based on Prof. Miguel Coimbra's slides. Please acknowledge the original source when reusing these slides for academic purposes.



jcunha@det.ua.pt

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Topic: Computer Vision?

- 'Computer Vision'?
- Camera and Camera Geometry
- The Human Visual System



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A Picture is Worth 1000 Words



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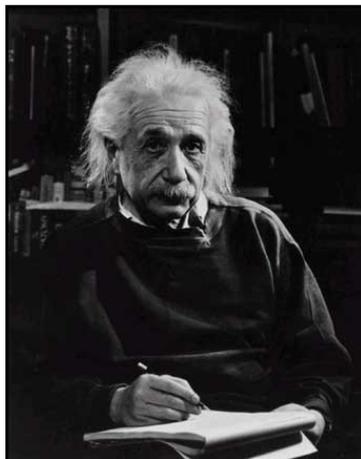
A Picture is Worth 100.000 Words



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A Picture is Worth a Million Words



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Human Vision

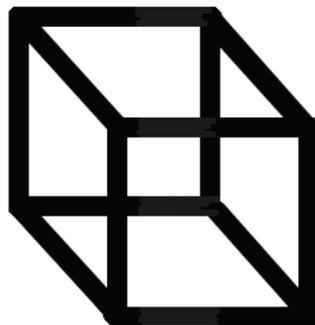
- Can do amazing things like:
 - Recognize people and objects
 - Navigate through obstacles
 - Understand mood in the scene
 - Imagine stories
- But:
 - Suffers from Illusions
 - Ignores many details
 - Ambiguous description of the world
 - Doesn't care about accuracy of world



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A Picture is Worth a ...?



Necker's Cube Reversal

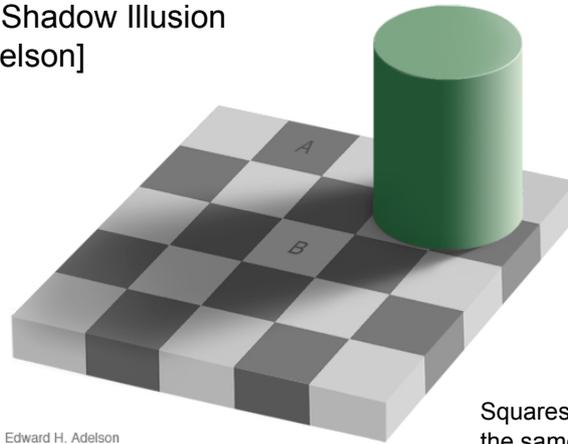


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A Picture is Worth a ...?

Checker Shadow Illusion
[E. H. Adelson]



Edward H. Adelson

Squares A&B are at
the same level of grey

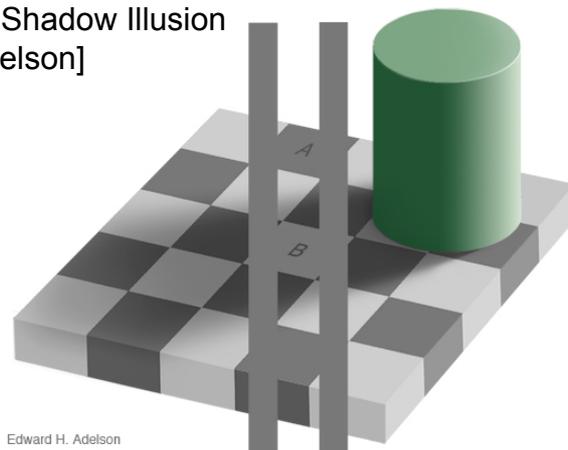


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A Picture is Worth a ...?

Checker Shadow Illusion
[E. H. Adelson]



Edward H. Adelson

Squares A&B are at
the same level of grey



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Topic: Camera Geometry

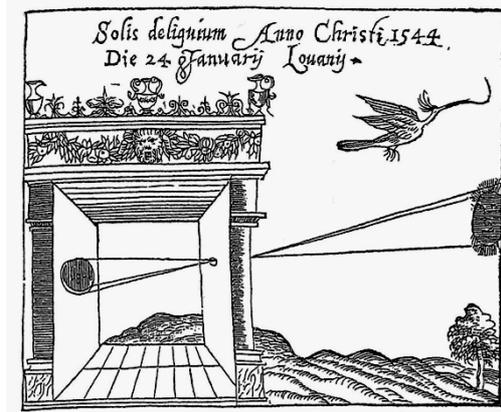
- 'Computer Vision'?
- Camera and Camera Geometry
- The Human Visual System



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A Brief History Cameras



Camera Obscura, Gemma Frisius, 1544

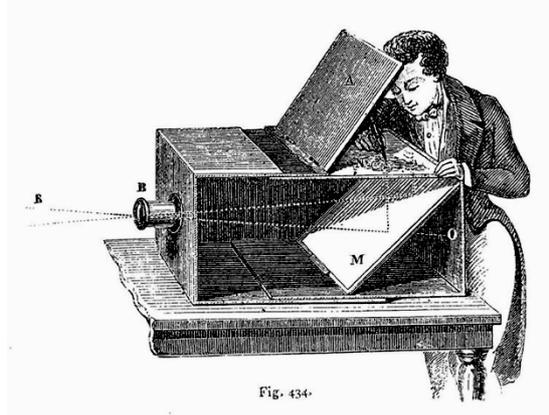
1544



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A Brief History of Cameras



Lens Based Camera Obscura, 1568

1544
1568



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A Brief History of Cameras



Still Life, Louis Jaques Mande Daguerre, 1837

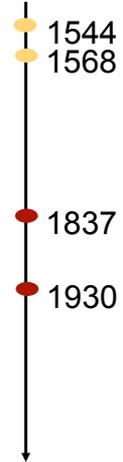
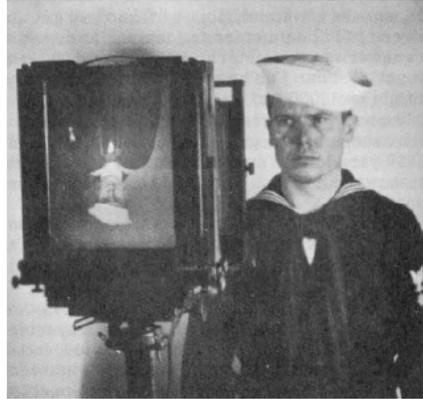
1544
1568
1837



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A Brief History of Cameras



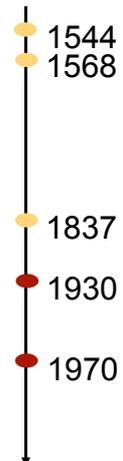
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A Brief History of Cameras



Silicon Image Detector, 1970



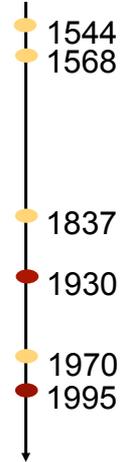
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A Brief History of Cameras



Digital Cameras



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Pinhole Camera Model

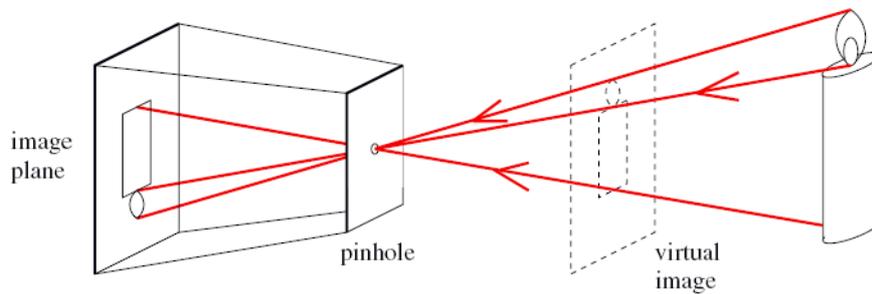


Figure 1.2. The pinhole imaging model.

Pinhole or central perspective, Bruneleschi, XV century



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Basic Camera Geometry

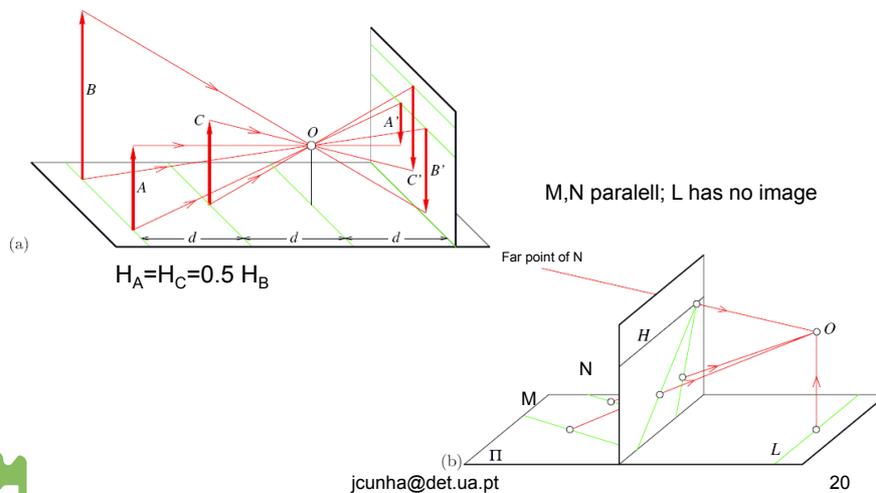
- Far objects appear smaller
- Lines project to lines
- Projection of parallel lines meet at a single “vanishing point”(VP)
- VPs of coplanar sets of lines are colinear: form the single vanishing line of the plane (Horizon)



jcunha@det.ua.pt

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Basic Camera Geometry



(Strong)Perspective projection

- These geometric properties are “common sense”
- Other properties can be inferred if we formalize the model using....
- Mathematics, of course...



jcunha@det.ua.pt

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(Strong)Perspective projection

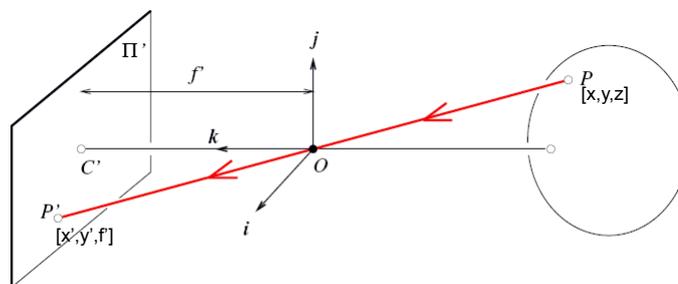


Figure 1.4. The perspective projection equations are derived in this section from the colinearity of the point P , its image P' and the pinhole O .

$$\begin{cases} x' = \lambda x \\ y' = \lambda y \\ f' = \lambda z \end{cases} \iff \lambda = \frac{x'}{x} = \frac{y'}{y} = \frac{f'}{z}, \quad \begin{cases} x' = f' \frac{x}{z}, \\ y' = f' \frac{y}{z}. \end{cases}$$



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Special case: Weak perspective (affine projection)

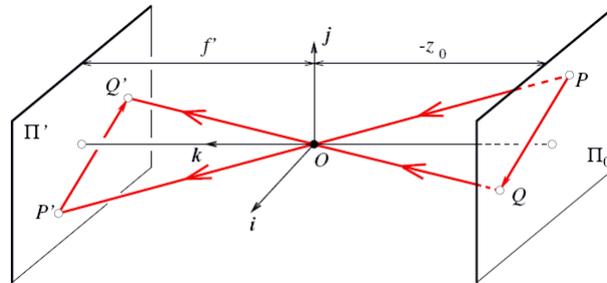


Figure 1.5. Weak-perspective projection: all line segments in the plane Π_0 are projected with the same magnification.

$$\begin{cases} x' = -mx \\ y' = -my \end{cases} \quad \text{where } m = -\frac{f'}{z_0}$$

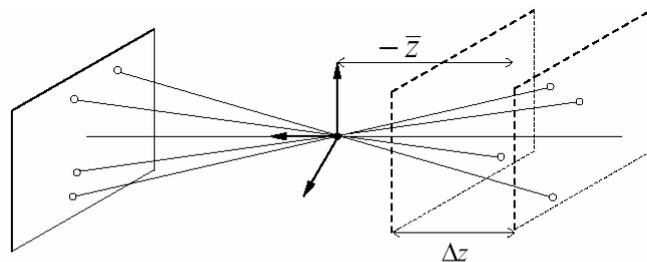
If scene points are in a plane, projections are simply magnified by m



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Special case: Weak perspective (affine projection)



$$\text{If } \Delta z \ll -\bar{z} : \begin{cases} x' \approx -mx \\ y' \approx -my \end{cases} \quad m = -\frac{f}{\bar{z}}$$

Justified if scene depth is small relative to average distance from camera



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Pinhole Photography



©Charlotte Murray Untitled, 4" x 5" pinhole photograph, 1992



Image Size inversely proportional to Distance

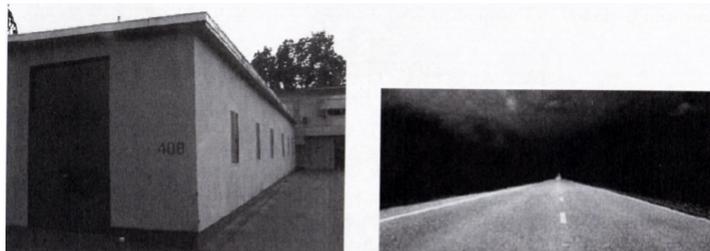
Reading: <http://www.pinholeresource.com/>



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Perspective models photos



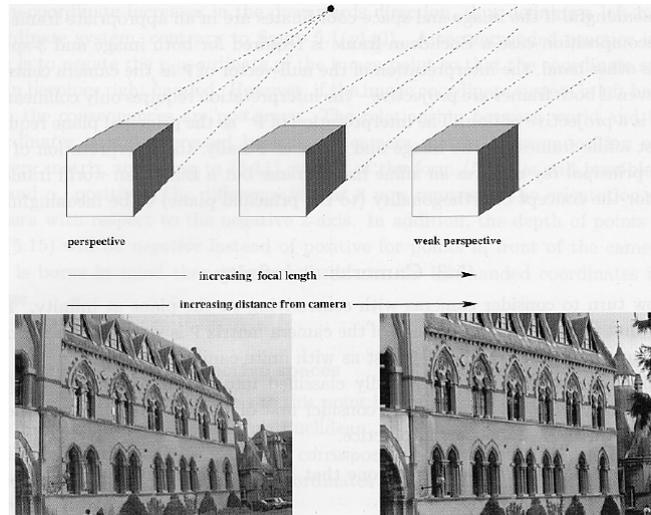
Strong perspective:
Angles are not preserved
The projections of parallel lines intersect at one point



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Perspective models photos



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Perspective models photos

Strong perspective:
Angles are not preserved
The projections of parallel
lines intersect at one point



Weak perspective:
Angles are better preserved
The projections of parallel
lines are (almost) parallel



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Problems with Pinholes

- Pinhole size (aperture) must be "very small" to obtain a clear image.
- However, as pinhole size is made smaller, less light is received by image plane.
- If pinhole is comparable to wavelength λ of incoming light, DIFFRACTION blurs the image!
- Sharpest image is obtained when:

$$\text{pinhole diameter } d = 2 \sqrt{f' \lambda}$$

Example: If $f' = 50\text{mm}$,
 $= 600\text{nm}$ (red),
 $d = 0.36\text{mm}$



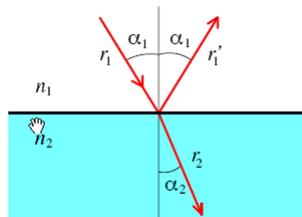
Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]



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Optics: Simple Lenses



Reflection and refraction – Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2.$$

First order or paraxial optics: for small angles

$$n_1 \alpha_1 \approx n_2 \alpha_2$$

Higher order optics: for real cases, use Taylor series

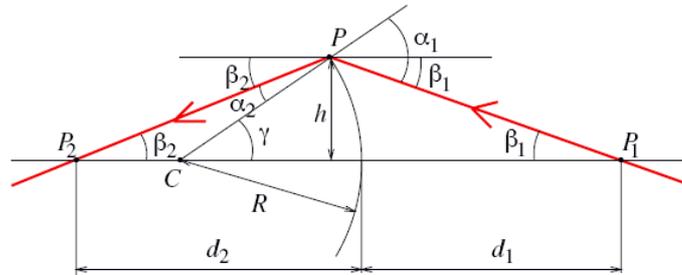
$$\sin \alpha = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots$$



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Paraxial refraction



$$\alpha_1 = \gamma + \beta_1 \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right) \quad \text{and} \quad \alpha_2 = \gamma - \beta_2 \approx h\left(\frac{1}{R} - \frac{1}{d_2}\right).$$

Paraxial refraction equation

$$n_1 \alpha_1 \approx n_2 \alpha_2 \iff \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}.$$



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Thin lenses model (paraxial optics)

- Lenses are used to avoid the problems of pinhole imaging
- They are based on the refraction optics laws

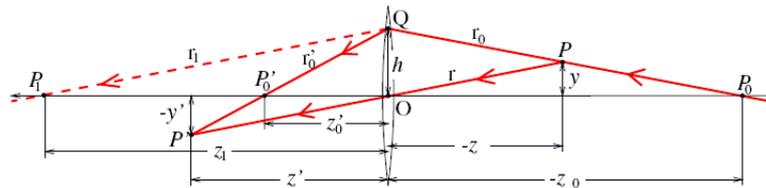


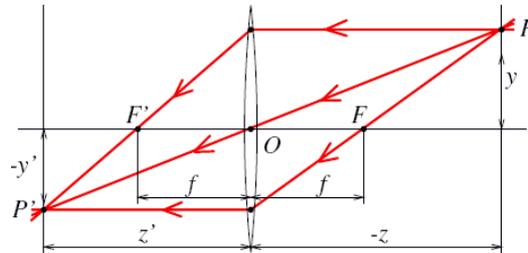
Figure 1.11. Image formation in the case of a thin lens. The ray r_1 is dashed to indicate that it will never actually reach the point P_1 since it will be refracted by the left boundary of the lens before that. Note that the z axis is oriented from right to left to ensure consistency with the previous figures, thus P and P_0 have negative depths z and z_0 .



jcunha@det.ua.pt

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Thin lenses model (paraxial optics)



All rays emanating from P converge to a single point P'

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

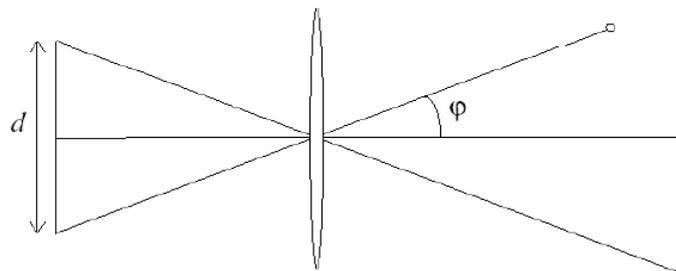
Points at infinity are focused on plane $z' = f$

Ideal because: infinite aperture
infinite field of view
infinitely small distance between surfaces



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Finite Field of View



Size of field of view governed by size of the camera retina:

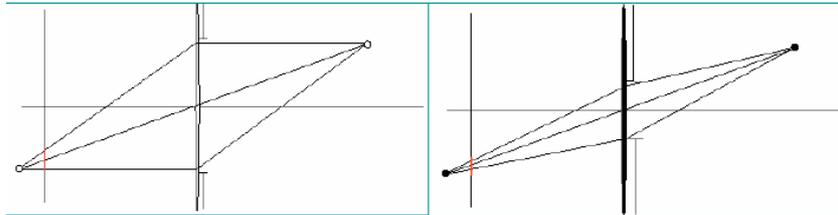
$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$



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Finite Aperture



Ideal case: Only the points on one plane are in perfect focus
Finite aperture: points within a region of depth D (depth of field) are in focus.
For a given f , the larger the aperture, the smaller D
Depth of field controlled by f/a



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Depth of Field

- Range of object distances over which image is sufficiently well focused.
- Range for which *blur circle* is less than the resolution of the sensor.



http://images.dpchallenge.com/images_portfolio/27920/print_preview/116336.jpg



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Real Lenses

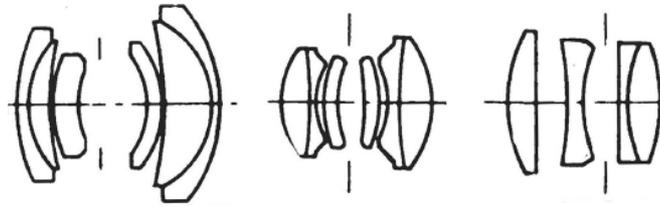


Figure 1.17. Photographics lenses. Reprinted from [Montel, 1972], p. 54.

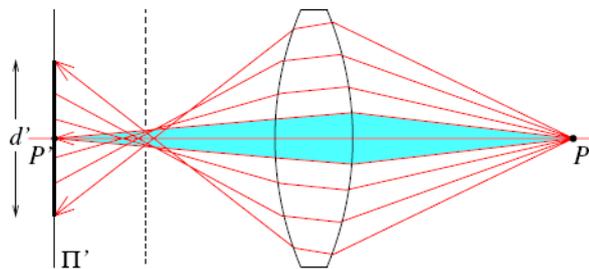
- Previous approximation is incorrect
 - Aberrations and distortions
 - Blurring and incorrect shape in the image



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Spherical aberrations



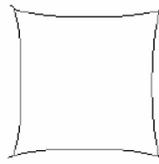
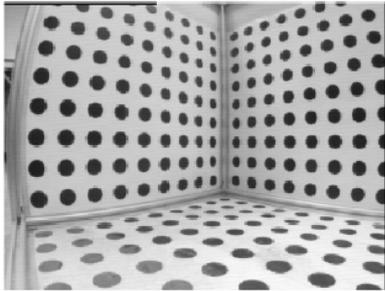
Rays further from the optical axis are focused closer to the lens



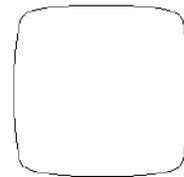
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Geometric distortion



pincushion



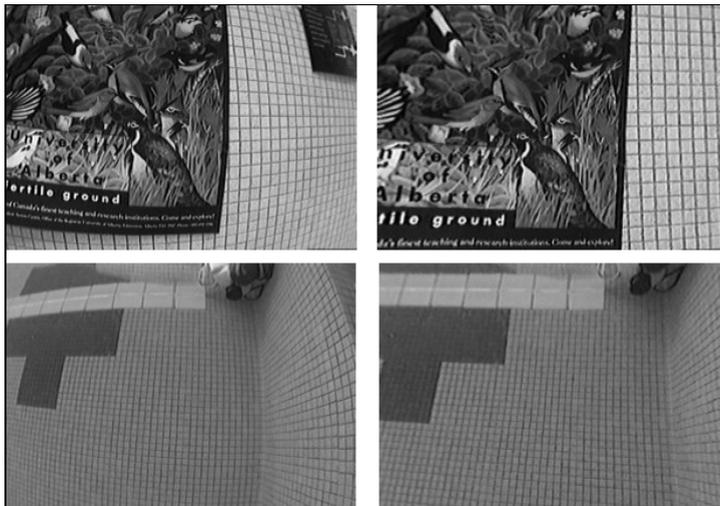
barrel



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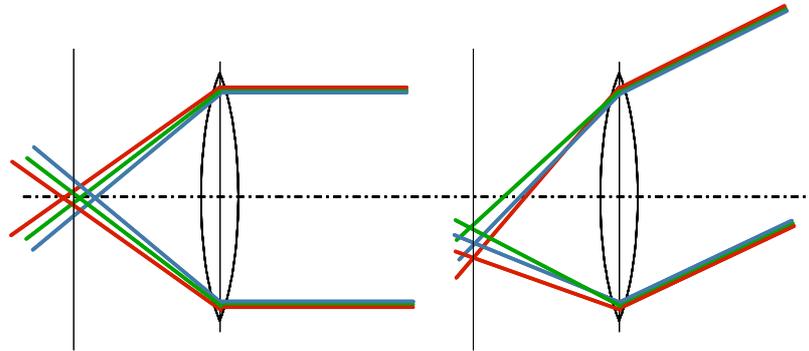
Geometric distortion



From Mark Fiala, Univ. Alberta

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Chromatic Aberration



longitudinal chromatic aberration
(axial)

transverse chromatic aberration
(lateral)



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Topic: The Human Visual System

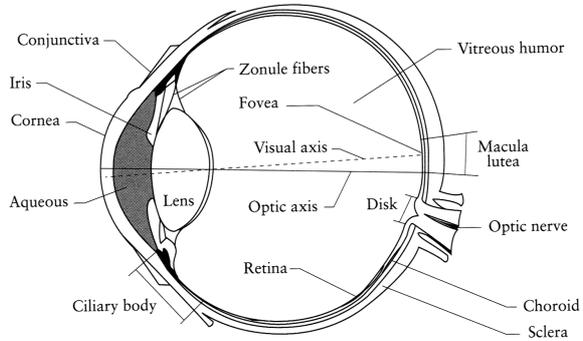
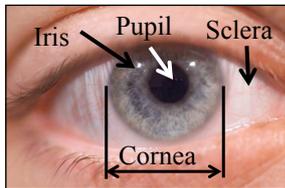
- 'Computer Vision'?
- Camera and Camera Geometry
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Our Eyes



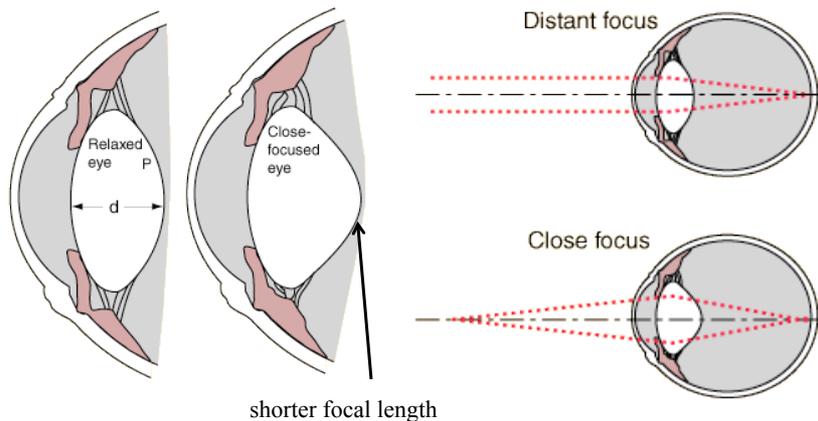
- Iris is the diaphragm that changes the aperture (pupil)
- Retina is the sensor where the fovea has the highest resolution



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Focusing



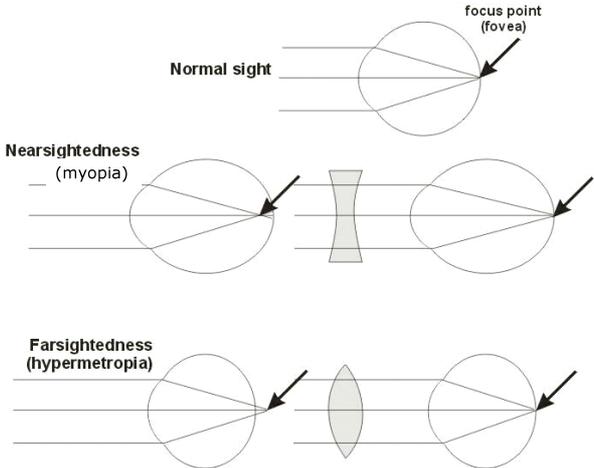
Changes the focal length of the lens



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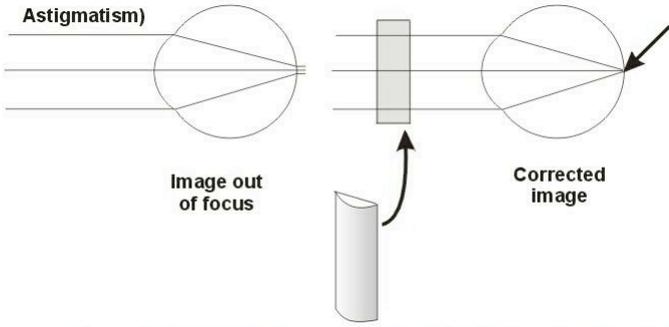
Myopia and Hyperopia



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Astigmatism



The cornea is distorted causing images to be un-focused on the retina.



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Blind Spot in the Eye



Close your right eye and look directly at the “+”

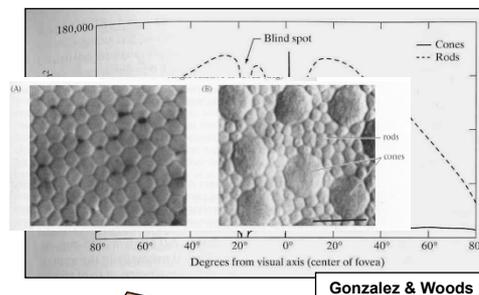


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Colour

- Our retina has:
 - **Cones** – Measure the frequency of light (colour)
 - 6 to 7 millions
 - High-definition
 - Need high luminosity
 - **Rods** – Measure the intensity of light (luminance)
 - 75 to 150 millions
 - Low-definition
 - Function with low luminosity



We only see colour in the center of our retina!



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Resources

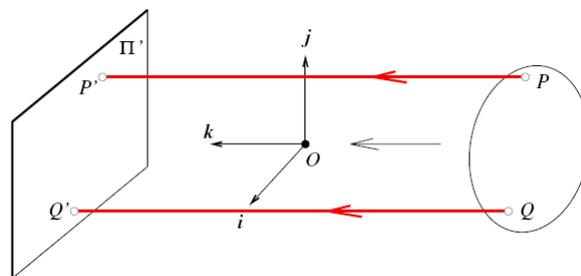
- J.C. Russ – Chapters 1 and 2
- Gonzalez & Woods – Chapter 1
- L. Shapiro, and G. Stockman – Chapter 1
- “Color Vision: One of Nature's Wonders” in <http://www.diycalculator.com/sp-cvision.shtml>



jcunha@det.ua.pt

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Orthographic projection



Justified if scene depth is small compared to distance from camera *and* camera remains at approximately constant distance



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