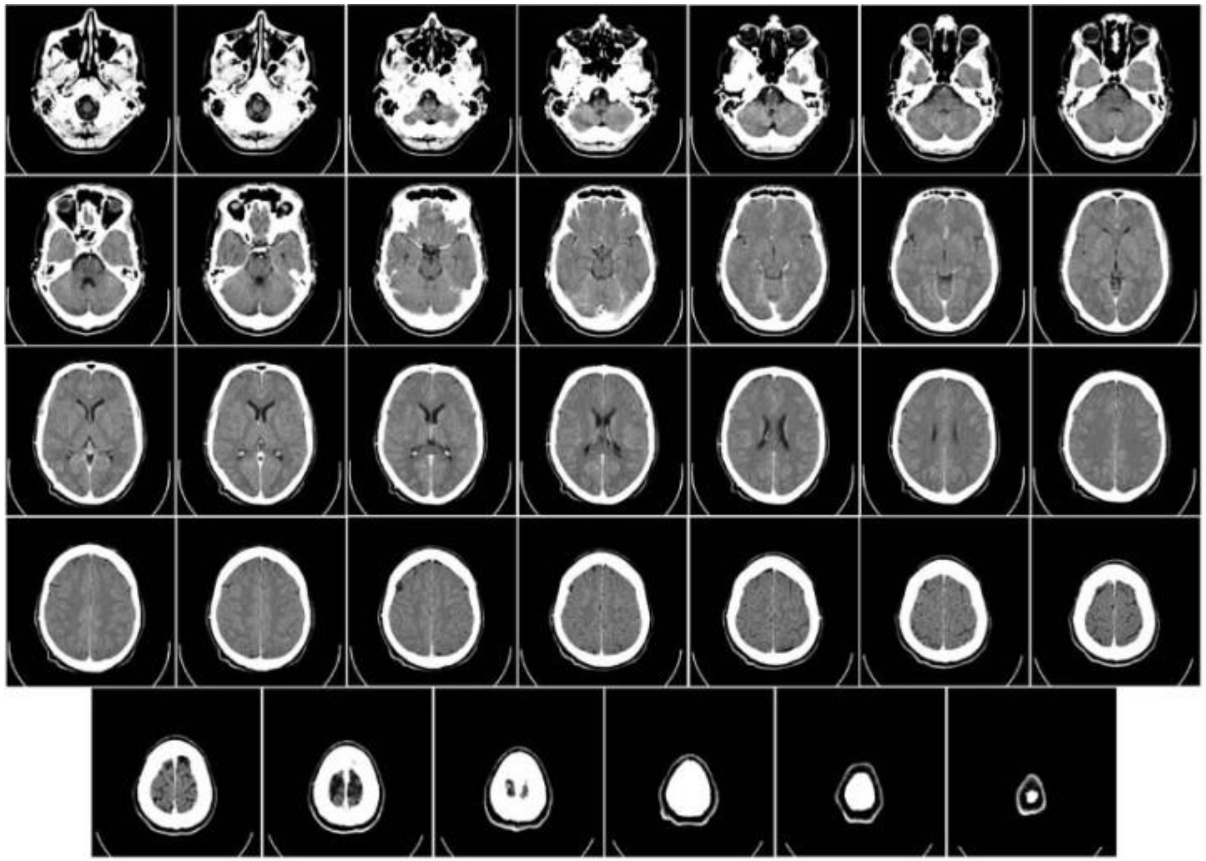
Scattering tomography with path integral

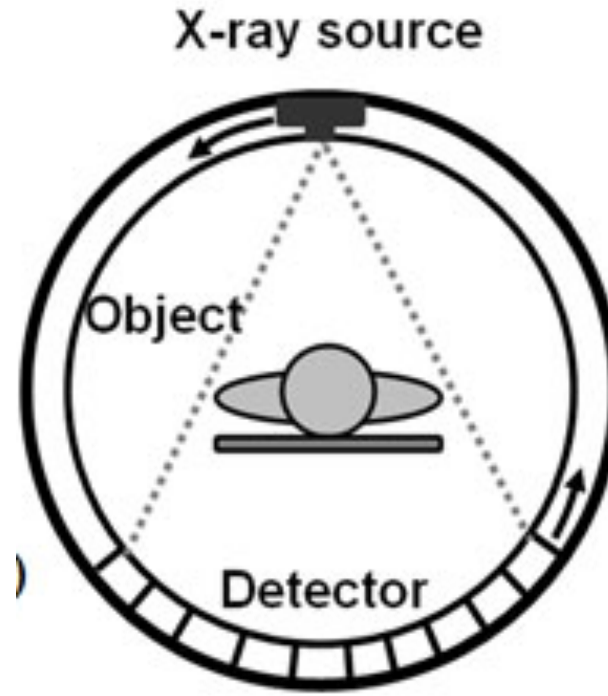
Toru Tamaki (Hiroshima University)

Joint work with
Bingzhi Yuan, Yasuhiro Mukaigawa,
and many others

HIROSHIMA UNIVERSITY

Computed Tomography





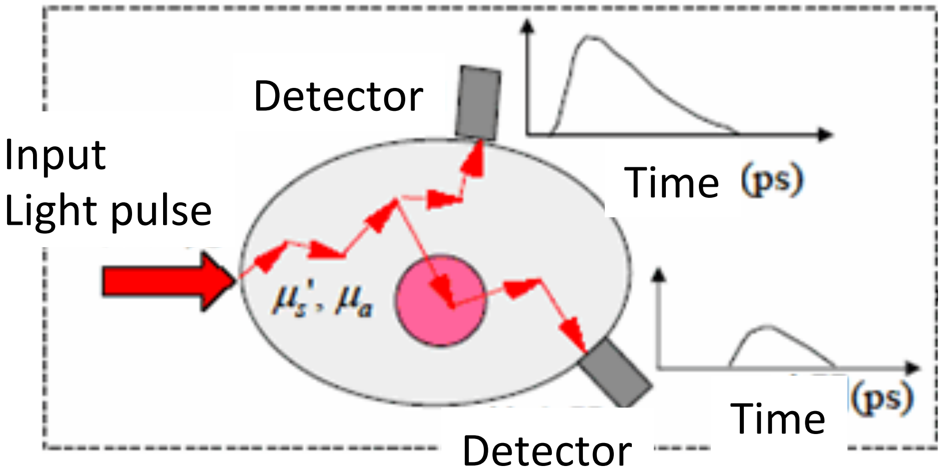
Multiplexing Radiography for Ultra-Fast Computed Tomography
J. Zhang, G. Yang, Y. Lee, S. Chang, J. Lu, O. Zhou
University of North Carolina at Chapel Hill
https://www.aapm.org/meetings/07AM/VirtualPressRoom/LayLanguage/IIMultiplexing.asp

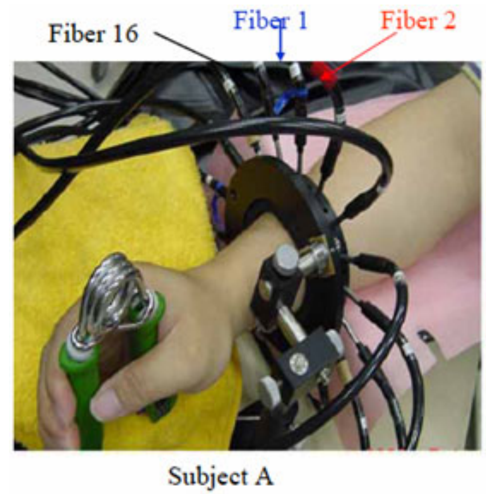
http://www.sciencekids.co.nz/pictures/humanbody/braintomography.html

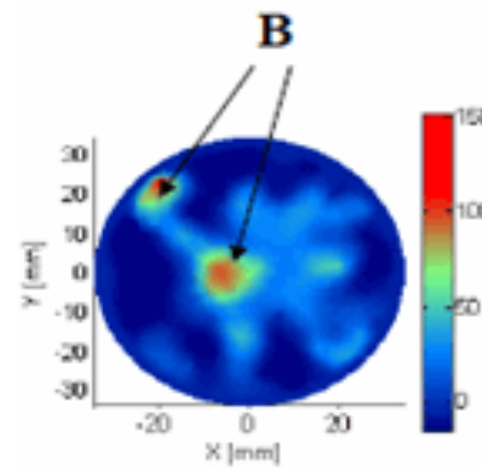
CT : http://www.hiroshima-u.ac.jp/news/show/id/13475/dir_id/19
PET : http://www.hiroshima-u.ac.jp/hosp/hitokuchi/p_vdo8u6.html
MRI : http://www.hiroshima-u.ac.jp/news/show/id/14427/dir_id/19

Different Modalities

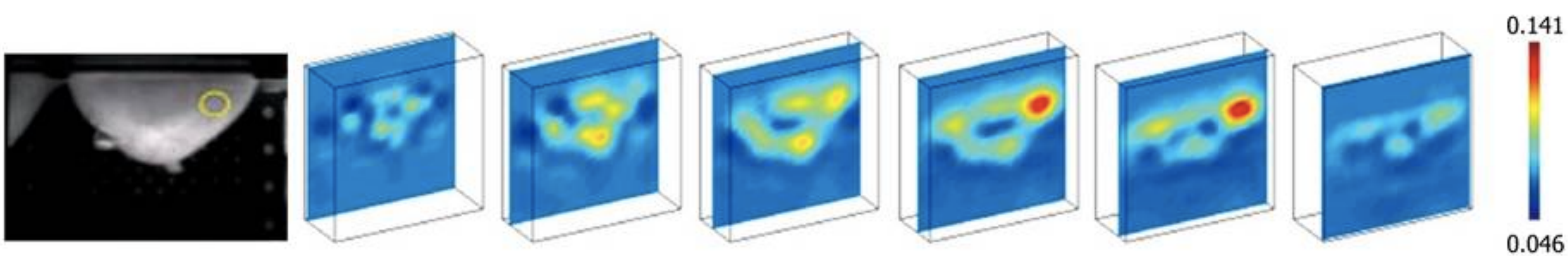
Diffuse Optical Tomography (DOT)





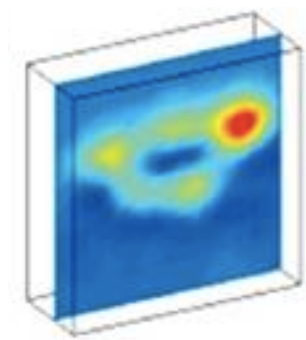


文部科学省科学技術・学術審議会・資源調査分科会報告書, 光資源を活用し、創造する科学技術の振興–持続可能な「光の世紀」に向けて
– 平成19年9月5日, 山田幸生, 第3章 健康なくらしに寄与する光, 3 光を用いた非侵襲生体診断,
http://www.mext.go.jp/b_menu/shingi/gijyutu/gijyutu3/toushin/07091111/008.htm



Lee, Kijoon. "Optical Mammography: Diffuse Optical Imaging of Breast Cancer." World Journal of Clinical Oncology 2.1 (2011): 64–72. PMC. Web. 8 June 2015. http://dx.doi.org/10.5306/wjco.v2.i1.64

Our research target

	X-ray CT	Our target	DOT
No Radiation	✗	✓	✓
Sharpness	✓ 	(✓)	✗ 
Scattering	No	Yes	So much
Method	Radon trans.	Ours	PDE by FEM

http://www.sciencekids.co.nz/pictures/humanbody/braintomography.html

Lee, Kijoon. “Optical Mammography: Diffuse Optical Imaging of Breast Cancer.” World Journal of Clinical Oncology 2.1 (2011): 64–72. PMC. Web. 8 June 2015. http://dx.doi.org/10.5306/wjco.v2.i1.64

Scattering

http://photosozai-database.com/modules/photo/photo.php?lid=664

Volumetric scattering in CG

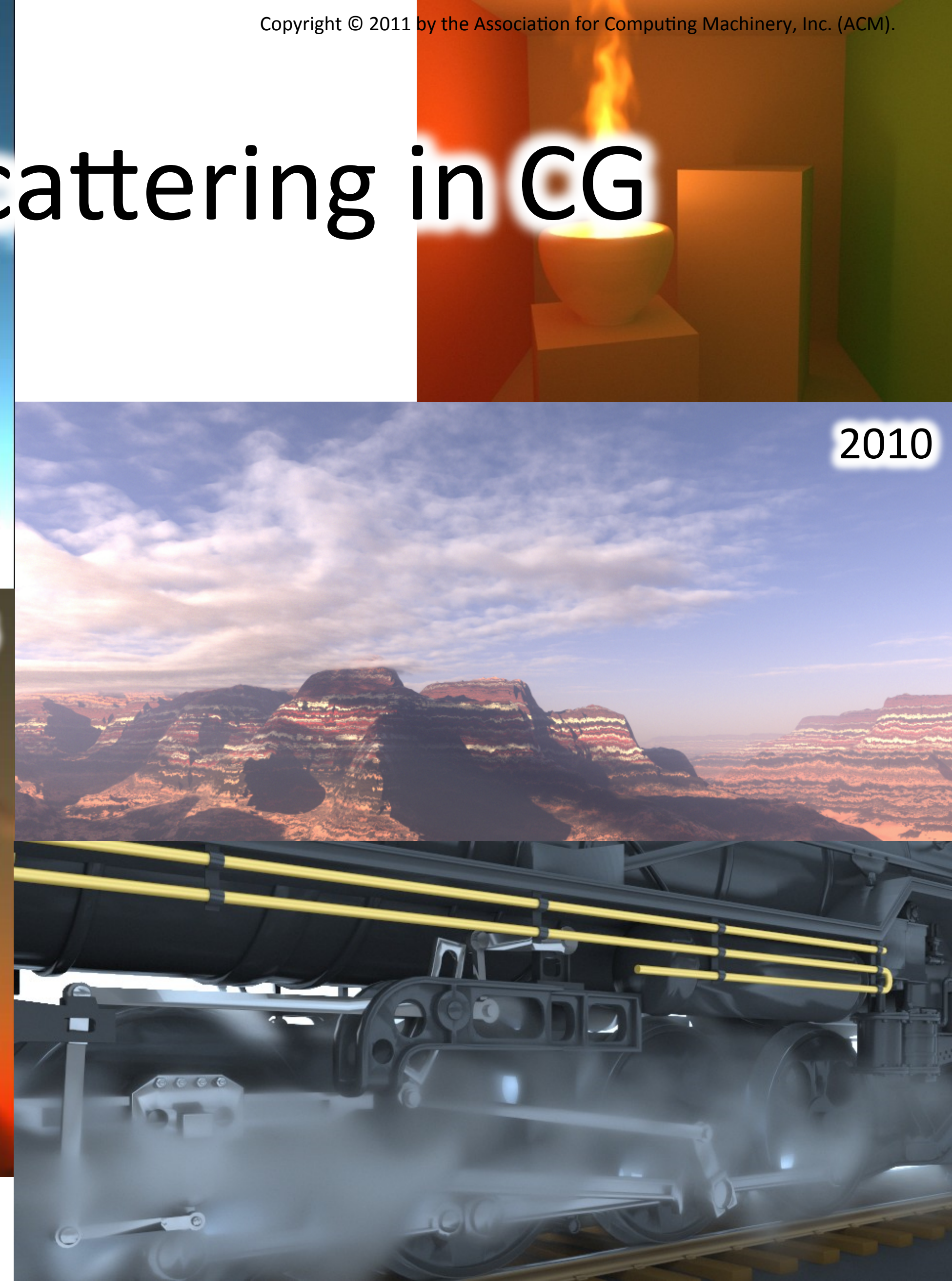
Tomoyuki Nishita, Yasuhiro Miyawaki, and Eihachiro Nakamae. 1987. A shading model for atmospheric scattering considering luminous intensity distribution of light sources. *SIGGRAPH Comput. Graph.* 21, 4 (August 1987), 303-310. DOI=10.1145/37402.37437 http://doi.acm.org/10.1145/37402.37437

Tomoyuki Nishita, Takao Sirai, Katsumi Tadamura, and Eihachiro Nakamae. 1993. Display of the earth taking into account atmospheric scattering. In *Proceedings of the 20th annual conference on Computer graphics and interactive techniques* (SIGGRAPH '93). ACM, New York, NY, USA, 175-182. DOI=10.1145/166117.166140 http://doi.acm.org/10.1145/166117.166140

Tomoyuki Nishita and Eihachiro Nakamae. 1994. Method of displaying optical effects within water using accumulation buffer. In *Proceedings of the 21st annual conference on Computer graphics and interactive techniques* (SIGGRAPH '94). ACM, New York, NY, USA, 373-379. DOI=10.1145/192161.192261 http://doi.acm.org/10.1145/192161.192261

Volumetric scattering in CG

Tomoyuki Nishita, Yoshinori Dobashi, and Eihachiro Nakamae. 1996. Display of clouds taking into account multiple anisotropic scattering and sky light. In *Proceedings of the 23rd annual conference on Computer graphics and interactive techniques* (SIGGRAPH '96). ACM, New York, NY, USA, 379-386. DOI=10.1145/237170.237277 http://doi.acm.org/10.1145/237170.237277



Yonghao Yue, Kei Iwasaki, Bing-Yu Chen, Yoshinori Dobashi, and Tomoyuki Nishita. 2010. Unbiased, adaptive stochastic sampling for rendering inhomogeneous participating media. *ACM Trans. Graph.* 29, 6, Article 177 (December 2010), 8 pages. In *ACM SIGGRAPH Asia 2010 papers* (SIGGRAPH ASIA '10). DOI=10.1145/1882261.1866199 http://doi.acm.org/10.1145/1882261.1866199

Computer vision approach to the inverse problem

minimizing observations and model prediction

$$\min_{\theta} \sum_{i} |I_i - f_i(\theta)|^2$$

Model parameters

Observed image

Forward model
Generated by Computer Graphics

Analysis

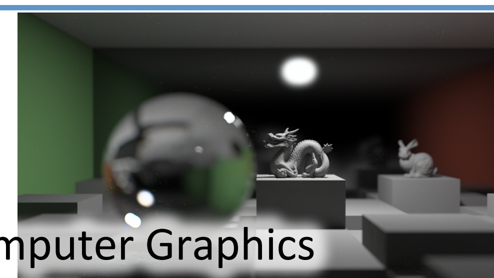
Generation

Image

Computer Vision

コンピュータビジョン

CV

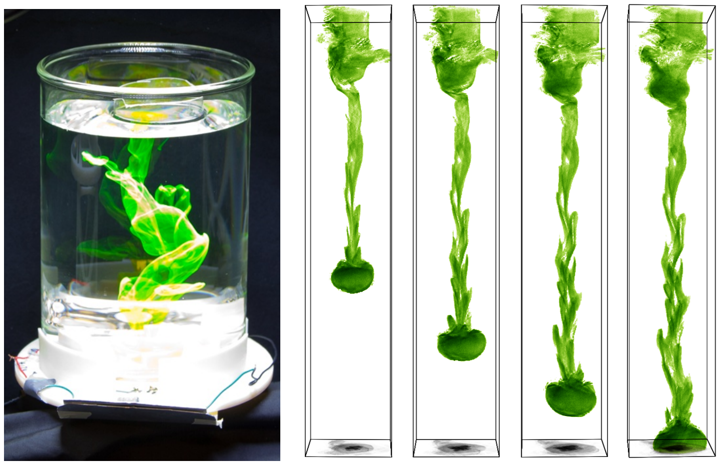


Computer Graphics

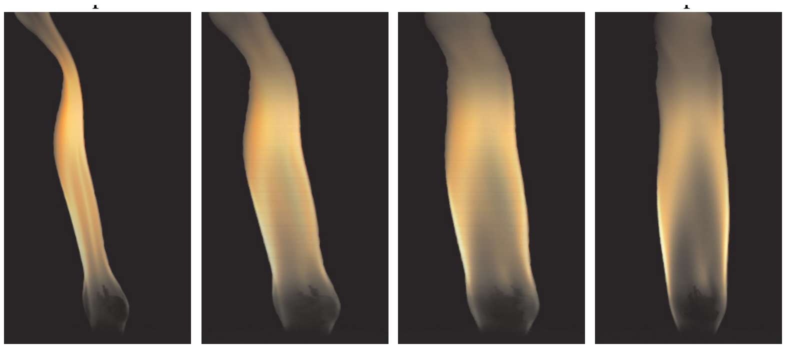
Vision / CG approaches

No scattering

Gregson+ 2012TOG



0° 30° 60° 90°

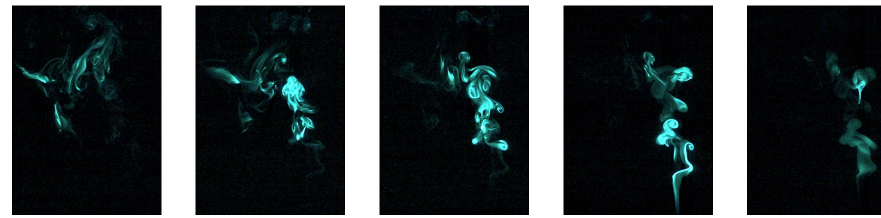


input reconstructed input

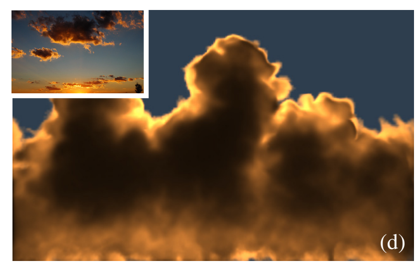
Hasinoff and Kutulakos 2007PAMI

Smoke only

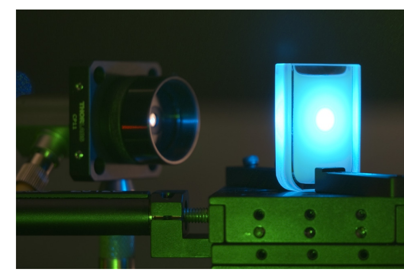
Hawkins+ 2005TOG



Uniform parameter estimation



Dobashi+ 2012TOG



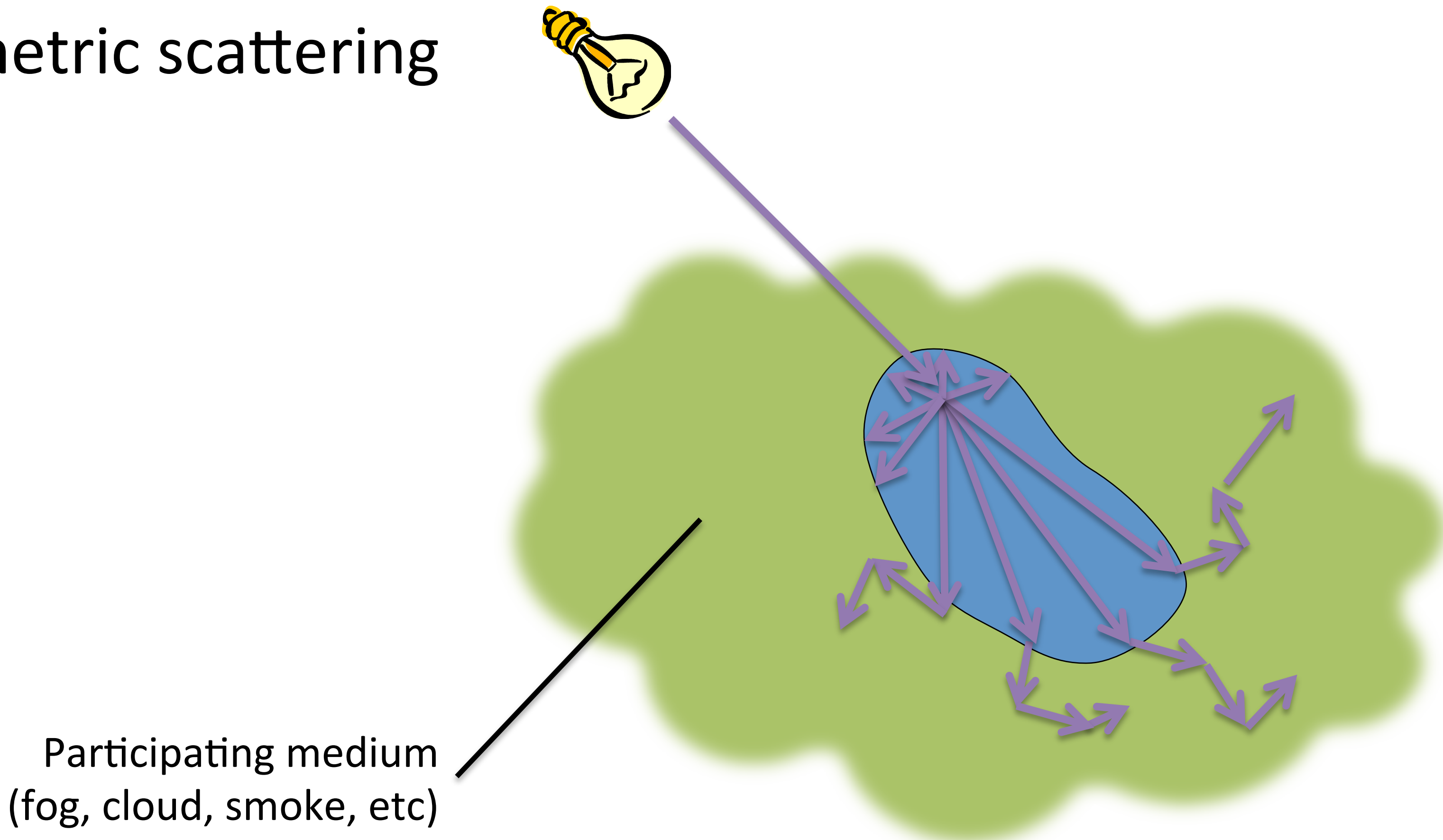
Gkioulekasj+ 2013SIGGRAPHAsia

Narasimhan+ 2006SIGGRAPH

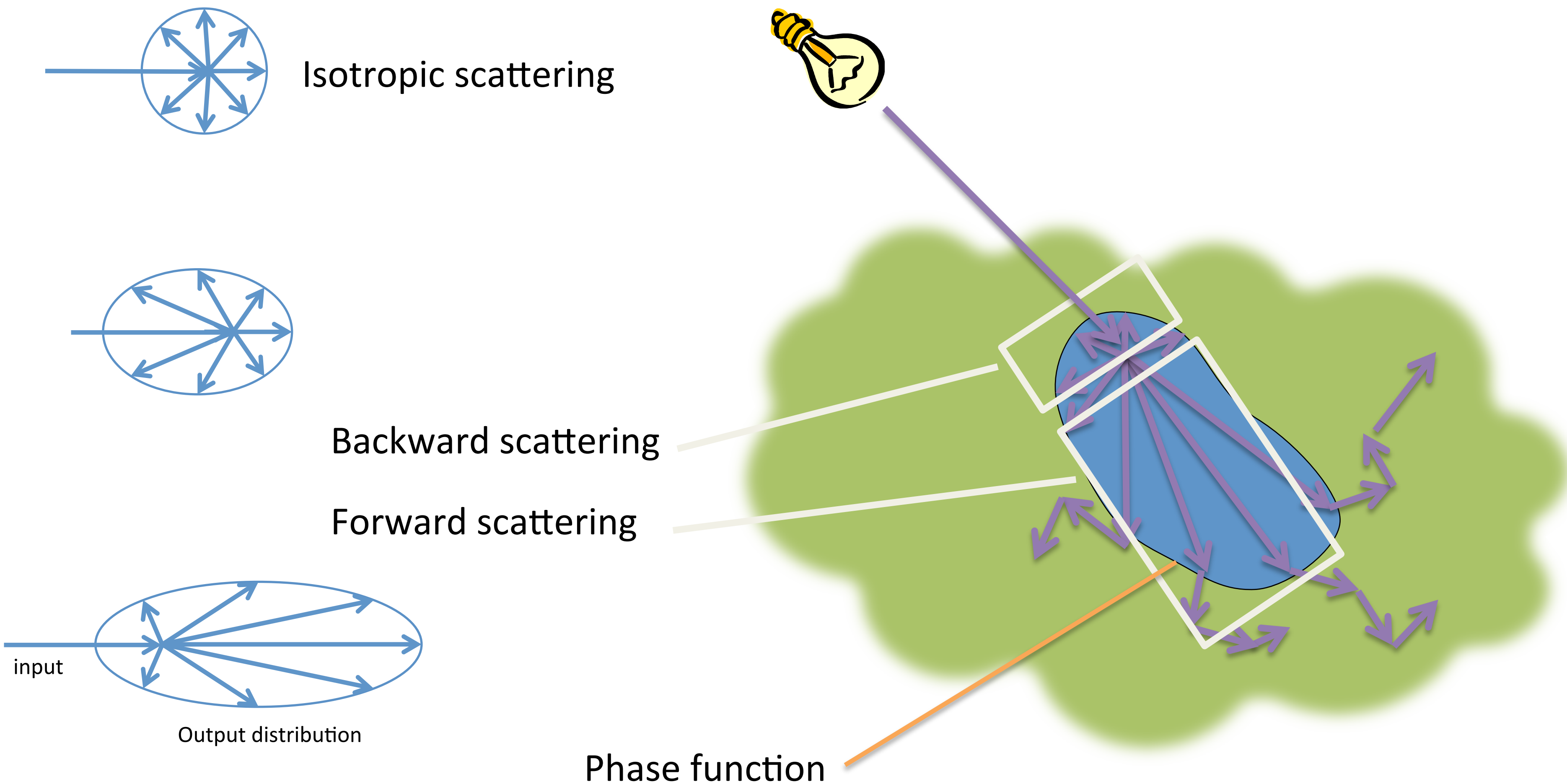
INTRODUCTION TO SCATTERING

Scattering model

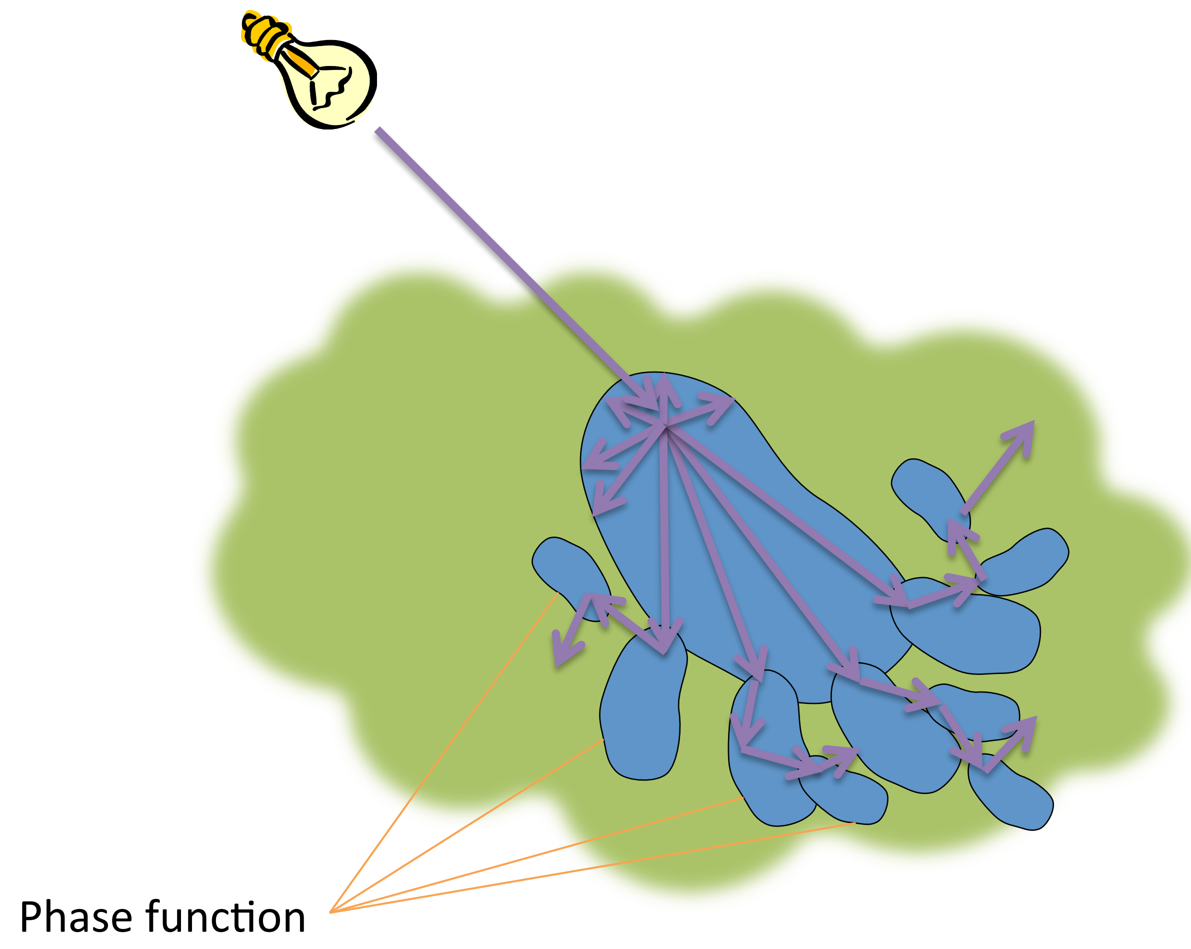
- Volumetric scattering



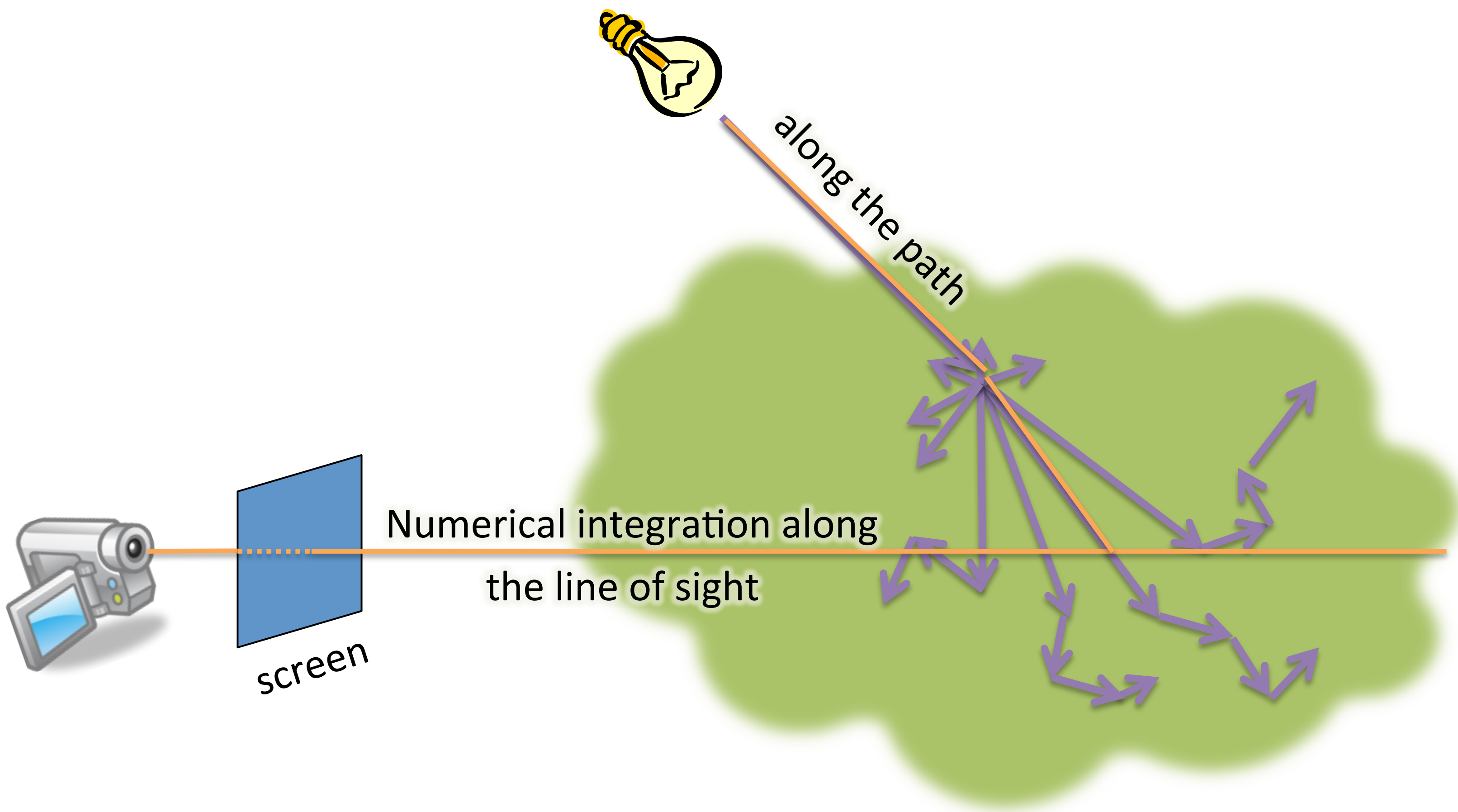
Types of Phase function



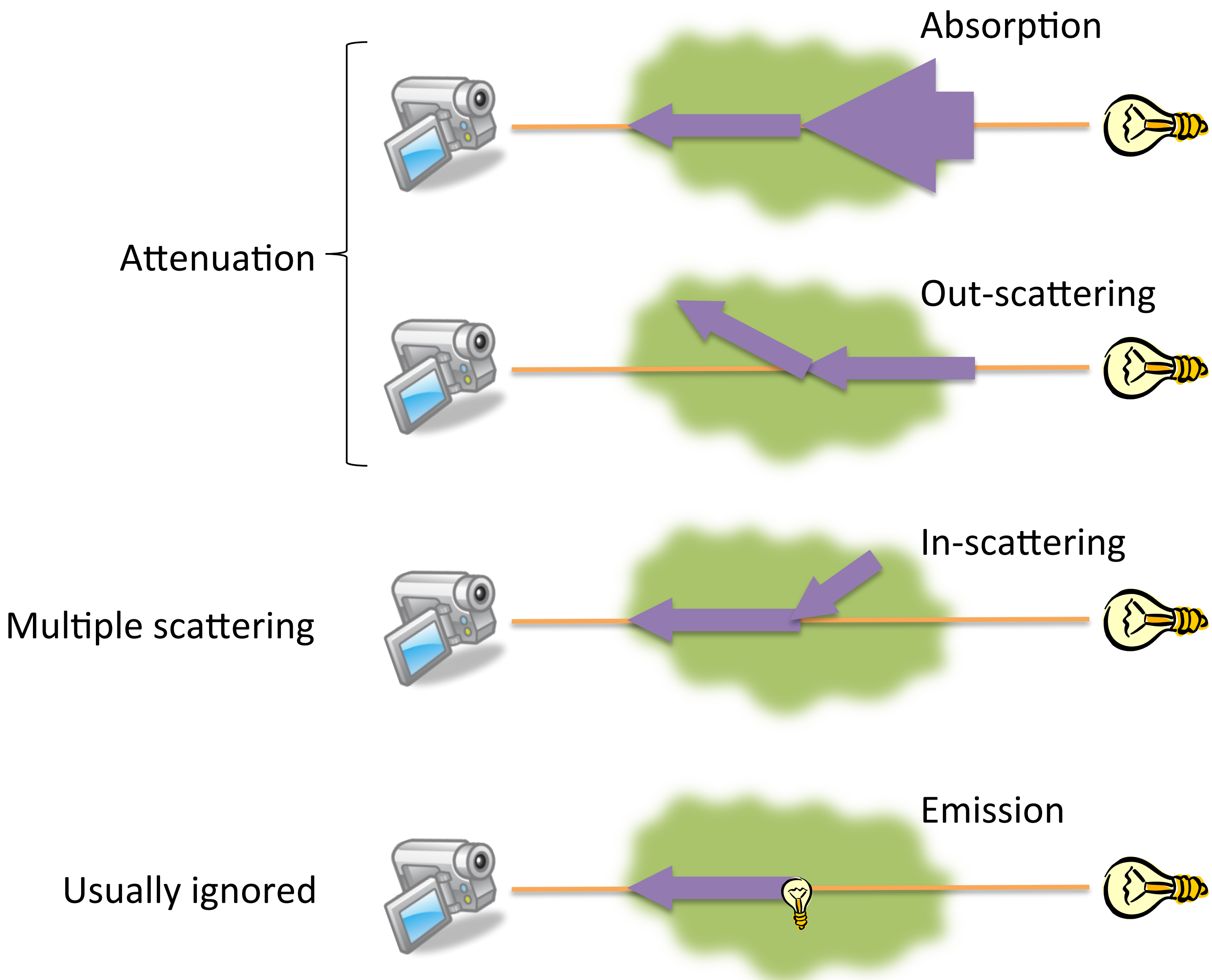
Scattering model



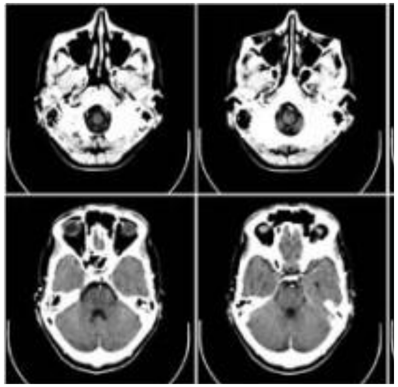
Rendering for CG



Light in participating medium

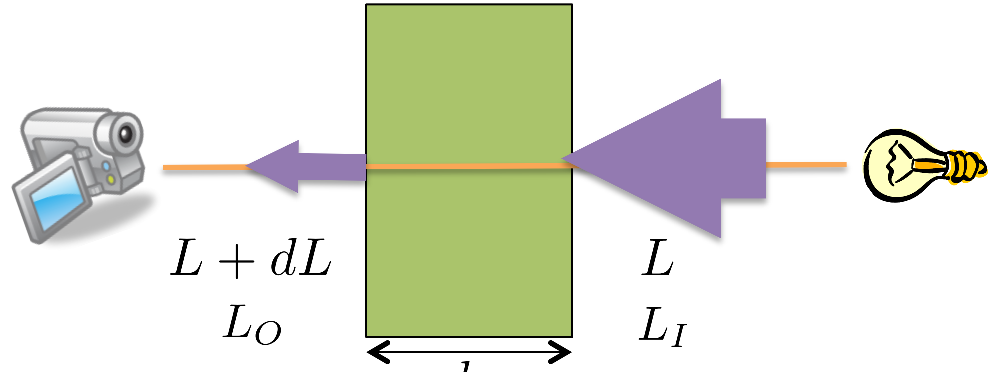


http://www.sciencekids.co.nz/pictures/humanbody/braintomography.html



CT: estimating X-Ray absorption

Absorption



(Microscopic) Beer-Lambert Law

$$dL = -\sigma_a \, L \, ds$$

Absorption coefficient

Infinitesimal path length

Integration from 0 to d

(Macroscopic) Beer-Lambert Law

$$\log \frac{L_I}{L_O} = \sigma_s d$$ or $$L_O = L_I e^{-\sigma_t d}$$

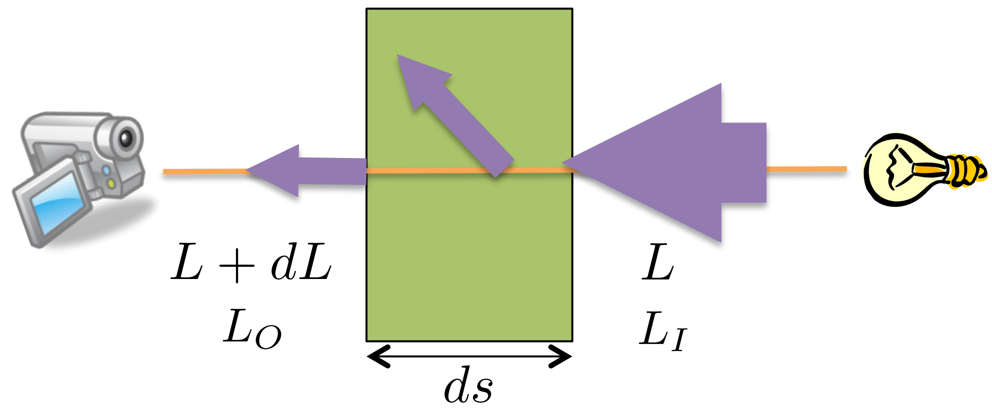
Exponential attenuation

optical depth

$$L_O = L_I e^{-\int_0^d \sigma_a(s) ds}$$

When σ_a is not constant

Out-scattering



(Microscopic) Beer-Lambert Law

$$dL = -\sigma_s \, L \, ds$$

Scattering coefficient

Infinitesimal path length

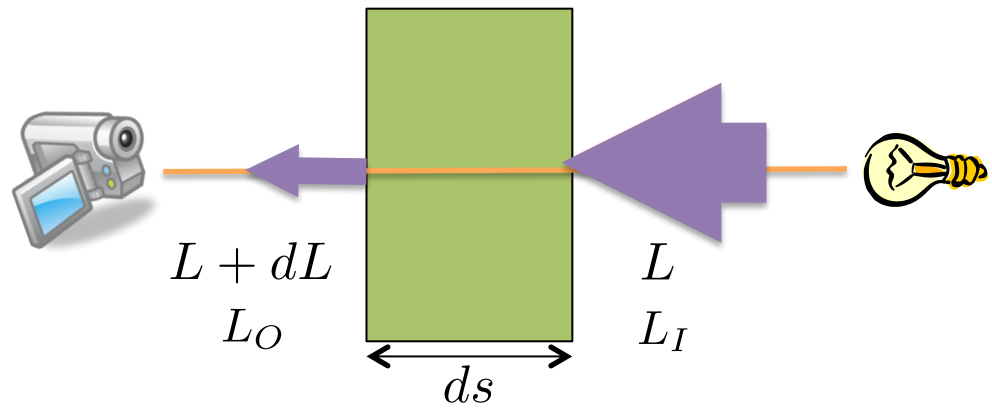
Integration from 0 to d

(Macroscopic) Beer-Lambert Law

$$\log \frac{L_I}{L_O} = \sigma_s d \quad \text{or} \quad L_O = L_I e^{-\sigma_t d}$$

Exponential attenuation

Attenuation = absorption + out-scattering



Absorption $dL = -\sigma_a \, L \, ds$

Out-scattering $dL = -\sigma_s \, L \, ds$

Attenuation $dL = -(\sigma_a + \sigma_s) \, L \, ds$

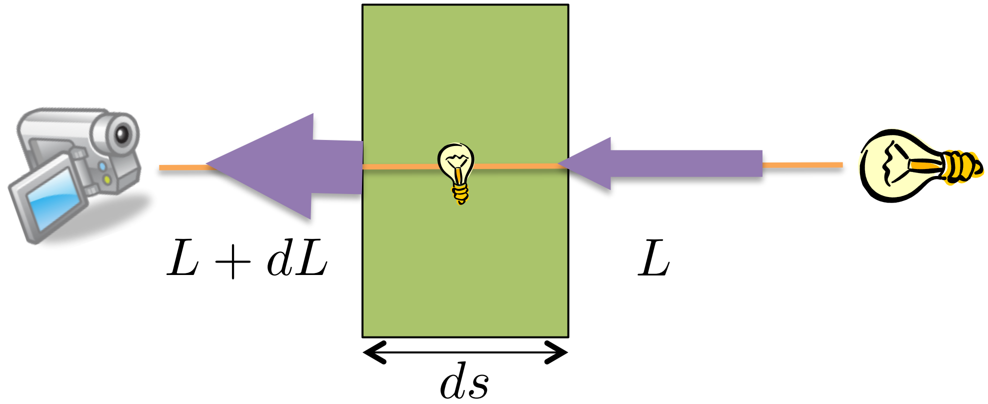
$$dL = -\sigma_t \, L \, ds$$

Extinction coefficient

Exponential attenuation

$$L_O = L_I e^{-\sigma_t d}$$

Emission



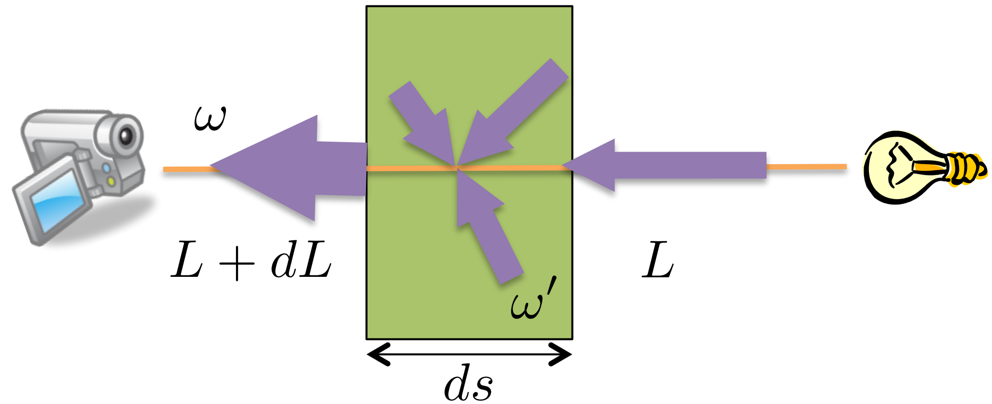
Emission $dL = \sigma_a \, L_e \, ds$

Absorption coefficient
(Emission of absorbed energy)

Emitted light

Usually ignored

In-scattering



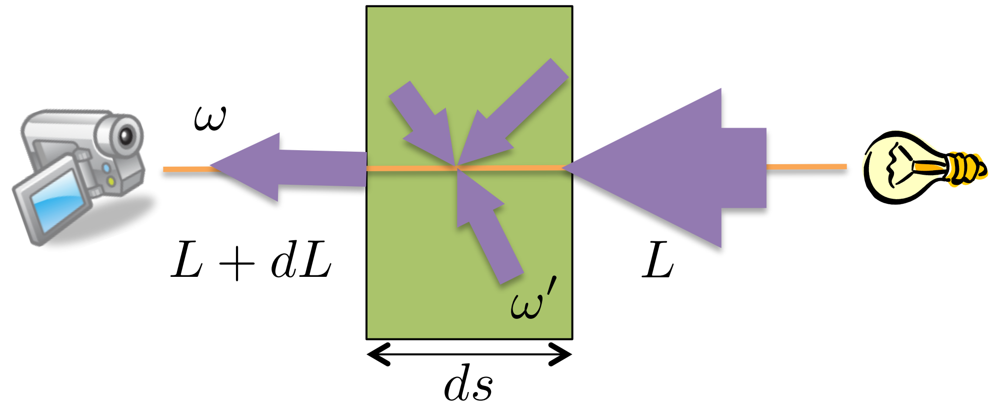
Integrating all incoming lights over the sphere

$$dL(x,\omega) = \sigma_s(x)\left(\int_\Omega \underbrace{p(x,\omega,\omega')}_{\substack{\text{Phase function}\\ \text{(from }\omega'\text{ to }\omega\text{ at x)}\\ \text{(usually common for all x)}}} \underbrace{L(x,\omega')}_{\substack{\text{Incident light}\\ \text{(from }\omega'\text{ at x)}}} d\omega'\right) ds$$

or

$$(\omega \cdot \nabla)L(x,\omega)$$

Rendering equation



Light transport equation
Volume rendering equation (differential form)

$$dL(x,\omega) = \underbrace{-\sigma_t(x)L(x,\omega)ds}_{\text{Attenuation term}} + \underbrace{\sigma_s(x)\left(\int_\Omega p(x,\omega,\omega')L(x,\omega')d\omega'\right)ds}_{\text{In-scattering term}}$$

$$(\omega \cdot \nabla)L(x,\omega) = \overbrace{-\sigma_t(x)L(x,\omega)}^{\text{Attenuation term}} + \overbrace{\sigma_s(x)\int_\Omega p(x,\omega,\omega')L(x,\omega')d\omega'}^{\text{In-scattering term}}$$

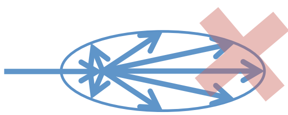
DOT with Diffuse Approximation

PDE solved by FEM

$$-\nabla \cdot D(x,t)\nabla L(x,t) + \sigma_a(x)L(x,t) + \frac{\partial L(x,t)}{\partial t} = Q(x,t)$$

Problem

No forward scattering

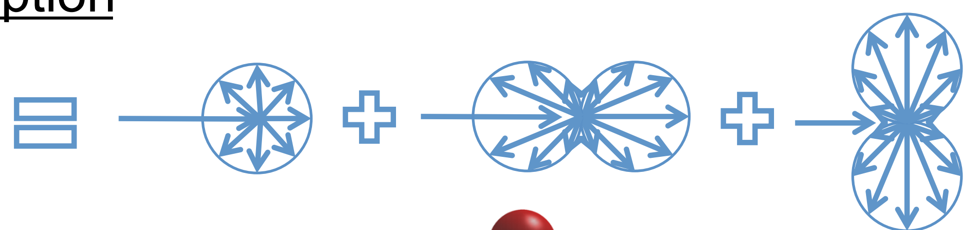


Blurred results

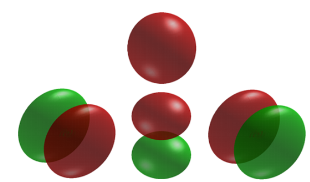
Main assumption

$p(x,\omega,\omega')$

$L(x,\omega')$



Diffuse Approximation
(<u>first</u> order spherical harmonics approximation)

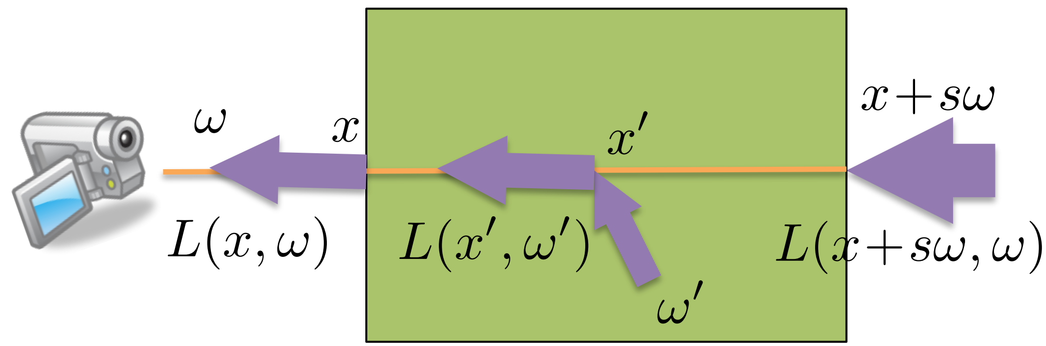


http://commons.wikimedia.org/wiki/File:Harmoniki.png

Time-resolved observation

$$\frac{1}{c}\frac{\partial L(x,\omega,t)}{\partial t} + (\omega\cdot\nabla)L(x,\omega,t) = \overbrace{-\sigma_t(x)L(x,\omega,t)}^{\text{Attenuation term}} + \overbrace{\sigma_s(x)\int_\Omega p(x,\omega,\omega')L(x,\omega',t)d\omega'}^{\text{In-scattering term}}$$

Rendering equation

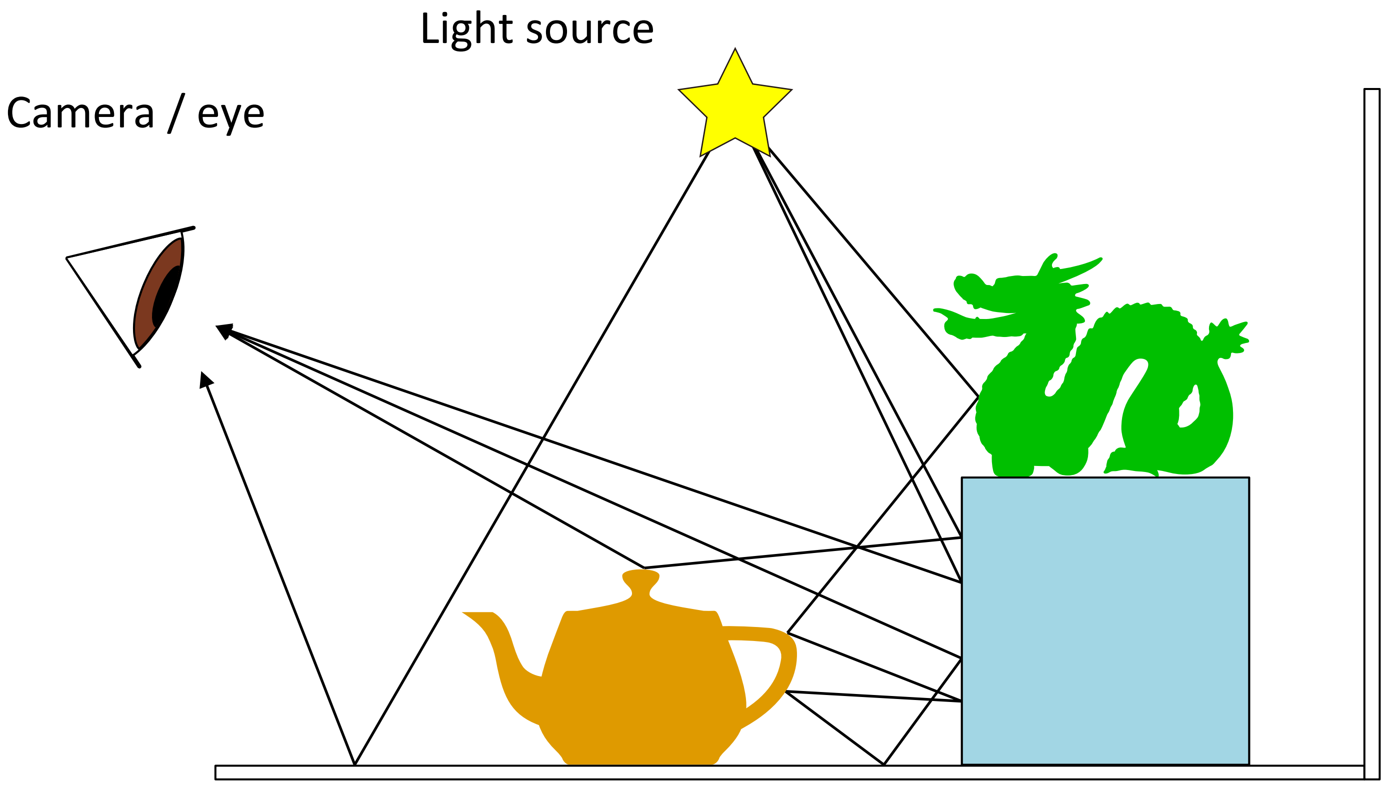


Light transport equation
Volume rendering equation (integral form)

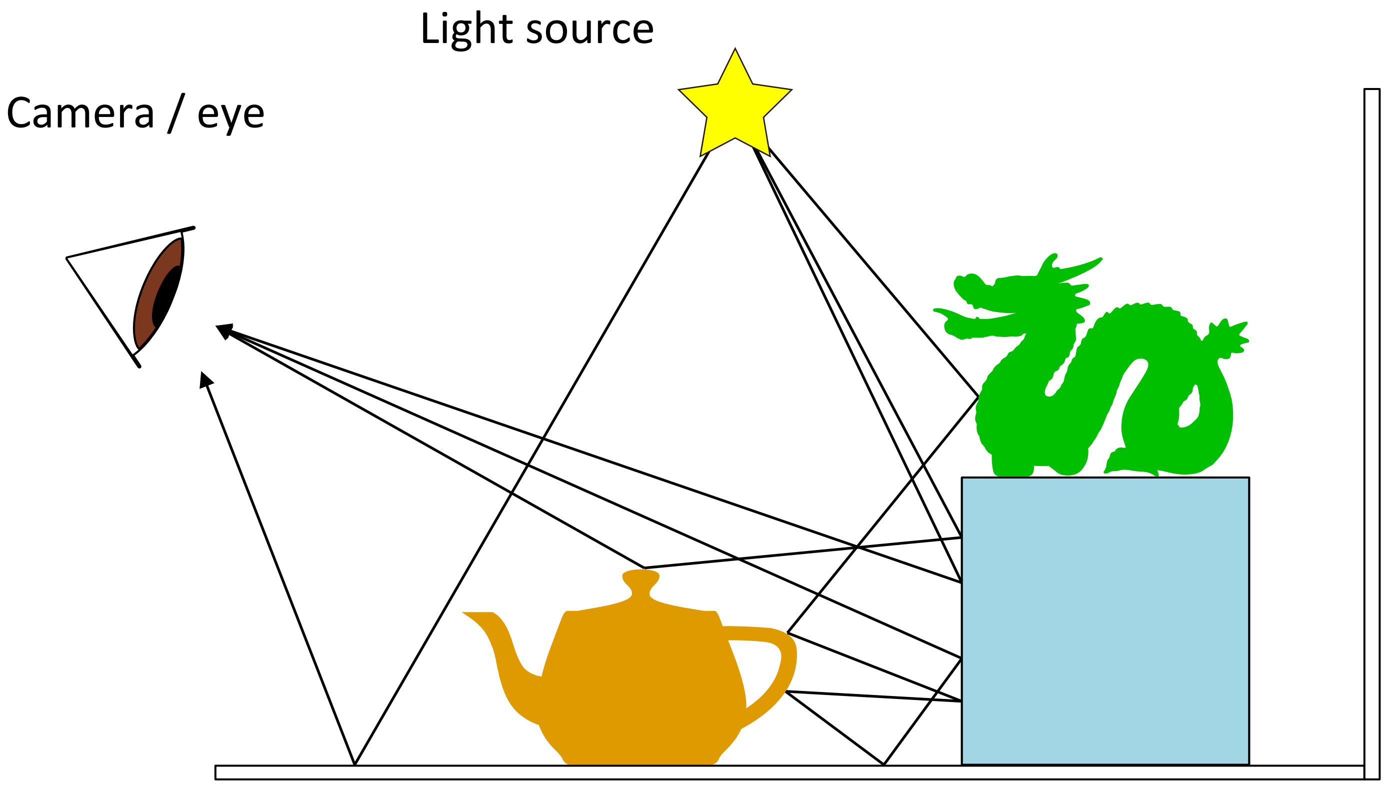
$$L(x,\omega) = \underbrace{e^{-\tau(x,x+s\omega)} L(x+s\omega,\omega)}_{\text{Attenuation term}}$$

$$+ \underbrace{\int_x^{x+s\omega} \underbrace{e^{-\tau(x,x')}\sigma_s(x')}_{\text{Attenuation of scattered light}} \left(\int_\Omega p(x',\omega,\omega') L(x',\omega') d\omega' \right) dx'}_{\text{In-scattering term}}$$

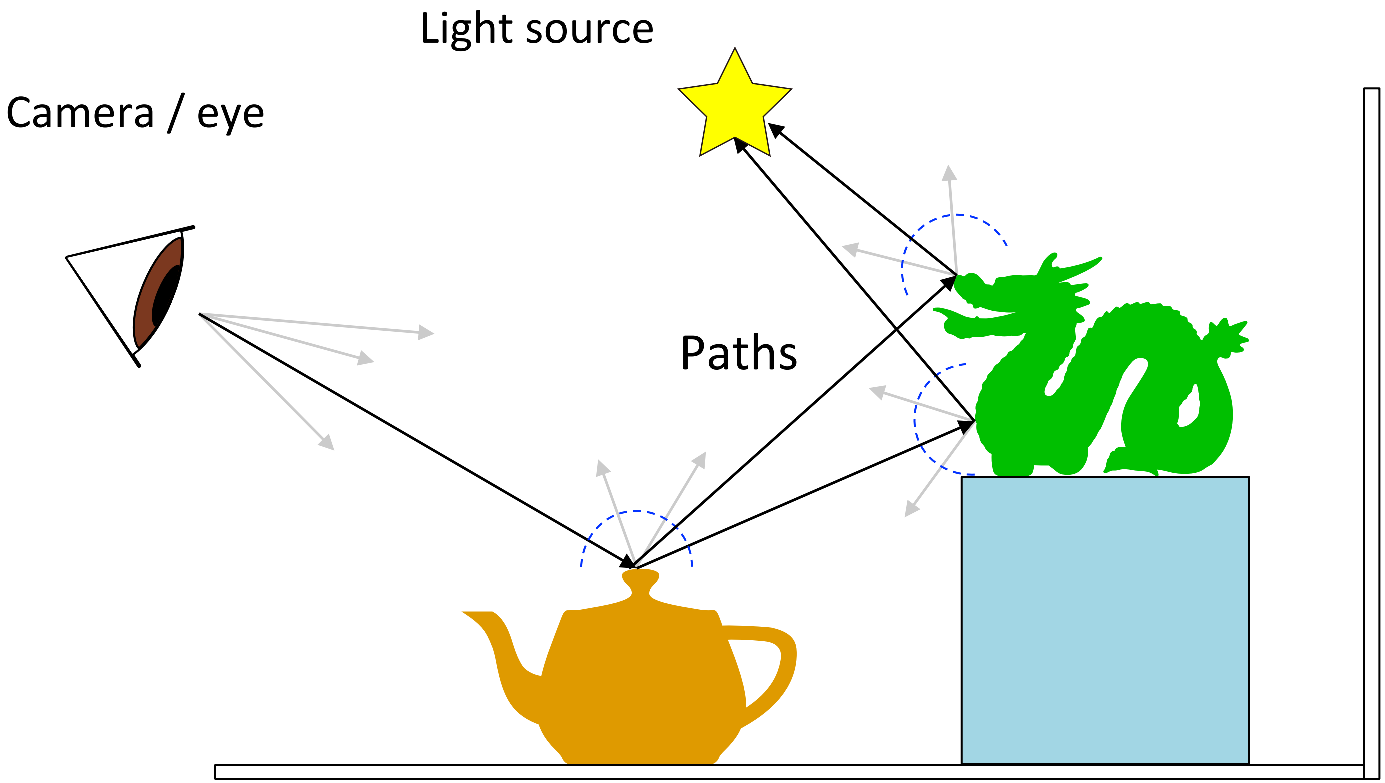
Light Paths:
from light sources to cameras



Light Paths:
from light sources to cameras



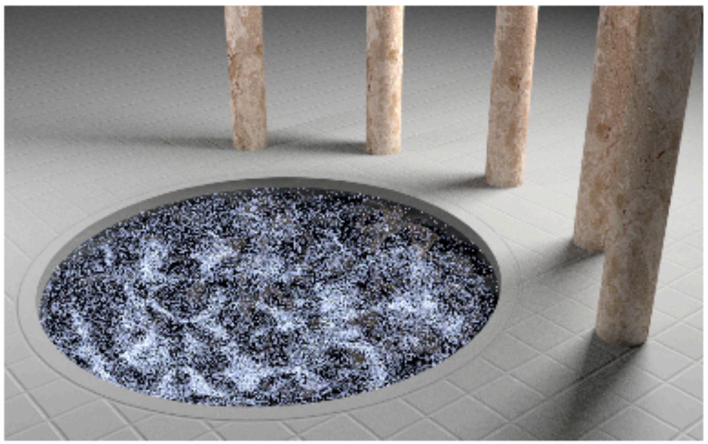
Path (Ray) tracing: from cameras to light source



Path integral approach

introduced by Eric Veach (1997 Ph.D thesis)

http://graphics.stanford.edu/papers/veach_thesis/



(a) Path tracing with 210 samples per pixel.

(b) Metropolis light transport with 100 mutations per pixel [the same computation time as (a)].

Pixel value of pixel j $I_j = \int_\Omega f_j(\bar{x})\, d\mu(\bar{x})\,.$

All possible paths of length k $\Omega_k \ni$ Paths of length k $\bar{x} = \mathbf{x}_0 \mathbf{x}_1 \ldots \mathbf{x}_k$

All possible paths of any length (path space) $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

Path measure $d\mu_k(\mathbf{x}_0 \ldots \mathbf{x}_k) = dA(\mathbf{x}_0) \cdots dA(\mathbf{x}_k)$

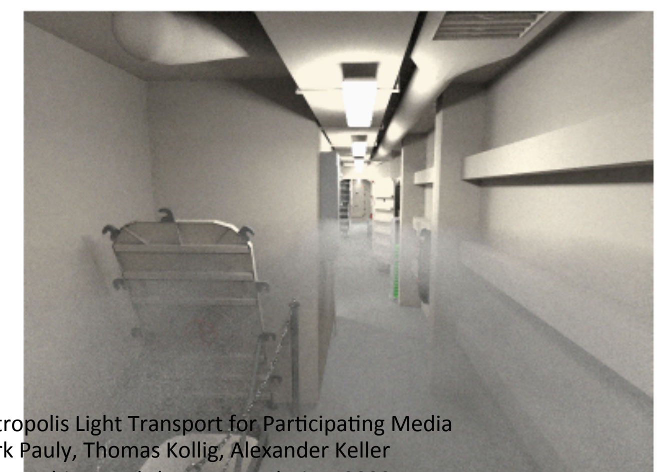
Product of measures at each vertex

Monte Carlo Integration $E\left[\frac{f_j(\bar{X})}{p(\bar{X})}\right] = \int_\Omega \frac{f_j(\bar{x})}{p(\bar{x})}\, p(\bar{x})\, d\mu(\bar{x})$

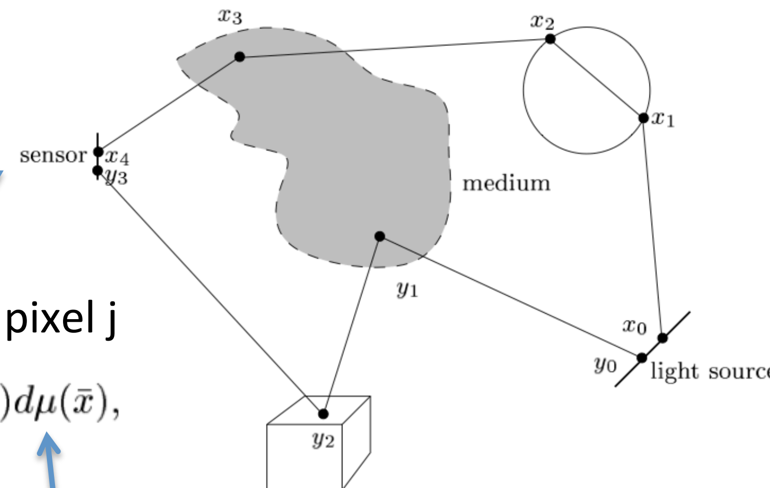
Path integral for participating media

introduced by Mark Pauly (1999 Master thesis)

http://graphics.stanford.edu/~mapauly/publications.html



Metropolis Light Transport for Participating Media
Mark Pauly, Thomas Kollig, Alexander Keller
Eurographics Workshop on Rendering, 2000



Pixel value of pixel j

$$I_j = \int_\Omega f_j(\bar{x}) d\mu(\bar{x}),$$

measure

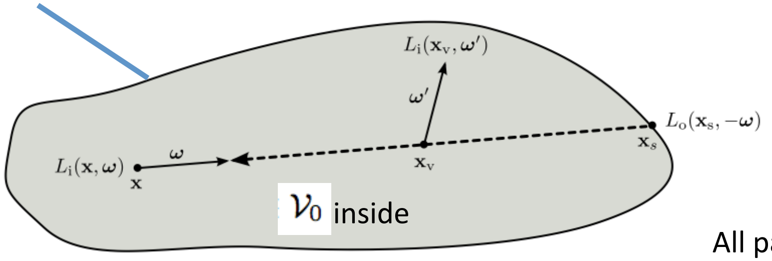
$$d\mu^l_{k,i}(\bar{x}) := \begin{cases} dA(x_i), & \text{Area measure if surface} \\ dV(x_i), & \text{Voxel measure if volume} \end{cases}$$

$$f_j(\bar{x}) = \boxed{\begin{array}{l} L_e(x_0 \to x_1) G(x_0 \leftrightarrow x_1) \tau(x_0 \leftrightarrow x_1) \cdot \\ \displaystyle\prod_{i=1}^{k-1} \hat{f}(x_{i-1} \to x_i \to x_{i+1}) G(x_i \leftrightarrow x_{i+1}) \tau(x_i \leftrightarrow x_{i+1}) \cdot \\ W_e^{(j)}(x_{k-1} \to x_k) \end{array}}$$

Anders Wang Kristensen, Efficient Unbiased Rendering using Enlightened Local Path Sampling, Ph.D thesis, 2010 URL

Details of paths in participating media

$\partial\mathcal{V}$ surface



$$I = \sum_{k=1}^{\infty} \int_{\Omega_k} L_e(\mathbf{x}_0 \to \mathbf{x}_1)\, G(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1) \left[\prod_{i=1}^{k-1} f_k(\mathbf{x}_{i-1} \to \mathbf{x}_i \to \mathbf{x}_{i+1})\, G(\mathbf{x}_i \leftrightarrow \mathbf{x}_{i+1}) \right] W_e(\mathbf{x}_{k-1} \to \mathbf{x}_k)\, d\mu_k(\mathbf{x}_0\, \mathbf{x}_1 \ldots \mathbf{x}_k).$$

All paths of length k

BRDF / scattering coeff, phase function

$$f_k(\mathbf{x}_i \to \mathbf{x} \to \mathbf{x}_o) = \begin{cases} f_s(\mathbf{x}_i \to \mathbf{x} \to \mathbf{x}_o) & \text{if } \mathbf{x} \in \partial\mathcal{V} \\ \sigma_s(\mathbf{x})\, f_p(\mathbf{x}_i \to \mathbf{x} \to \mathbf{x}_o) & \text{if } \mathbf{x} \in \mathcal{V}_0, \end{cases}$$

Geometric term

$$G(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2) = \mathcal{T}(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2)\, \frac{g(\mathbf{x}_1 \to \mathbf{x}_2)\, g(\mathbf{x}_2 \to \mathbf{x}_1)}{\|\mathbf{x}_1 - \mathbf{x}_2\|^2}.$$

Camera response function

Cos term

$$g(\mathbf{x} \to \mathbf{x}') = \begin{cases} |\mathbf{n}_g(\mathbf{x}) \cdot \boldsymbol{\omega}_{\mathbf{x} \to \mathbf{x}'}| & \text{if } \mathbf{x} \in \partial\mathcal{V} \\ 1 & \text{otherwise.} \end{cases}$$

Attenuation

$$\mathcal{T}(\mathbf{x} \leftrightarrow \mathbf{x}') = \begin{cases} \exp\left[-\tau(\mathbf{x} \leftrightarrow \mathbf{x}')\right] & \text{if } \|\mathbf{x} - \mathbf{x}'\| \le d_{\partial\mathcal{V}}(\mathbf{x}, \boldsymbol{\omega}_{\mathbf{x} \to \mathbf{x}'}), \\ 0 & \text{otherwise.} \end{cases}$$

$$\tau(\mathbf{x} \leftrightarrow \mathbf{x}') = \int_0^{\|\mathbf{x} - \mathbf{x}'\|} \sigma_t(\mathbf{x} + s\, \boldsymbol{\omega}_{\mathbf{x} \to \mathbf{x}'})\, ds.$$

To be estimated!

$$\sigma_t(\mathbf{x}_v) = \sigma_a(\mathbf{x}_v) + \sigma_s(\mathbf{x}_v).$$

$$\mu_k^l(A) = \int_A \prod_{i=0}^{k} d\mu_{k,i}^l(\bar{\mathbf{x}})$$ measure

$$d\mu_{k,i}^l(\mathbf{x}_0\, \mathbf{x}_1 \ldots \mathbf{x}_k) \equiv \begin{cases} dA(\mathbf{x}_i) \\ dV(\mathbf{x}_i) \end{cases}$$

PROPOSED METHOD
TO THE INVERSE PROBLEM

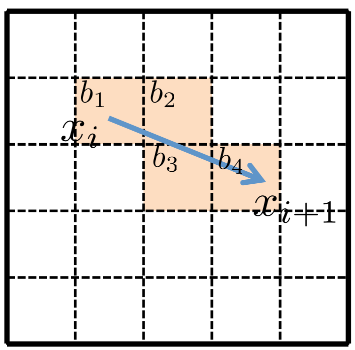
Discretization. How?

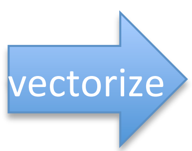
Extinction coefficient at voxel b

Length that the path x_i->x_{i+1} passes voxel b

$$\int_0^1 \sigma_t((1-s)x_i + sx_{i+1})ds \approx \sum_b \sigma_t[b]\; d_{x_i,x_{i+1}}[b] = \boldsymbol{\sigma}_t \cdot \boldsymbol{d}_{x_i,x_{i+1}}$$

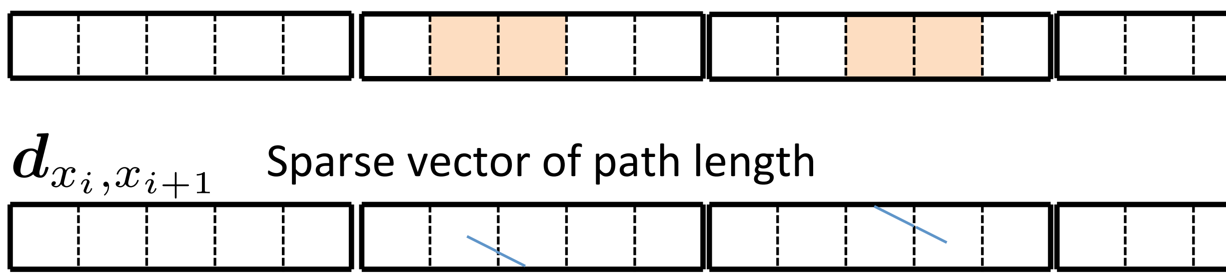
Inner product of sparse vectors

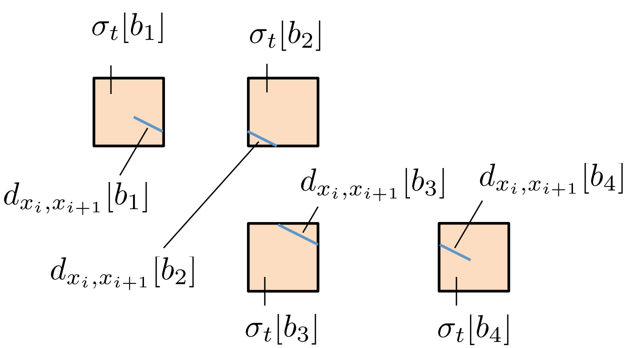




$\boldsymbol{\sigma}_t$ Sparse vector of coefficients

$\boldsymbol{d}_{x_i,x_{i+1}}$ Sparse vector of path length





Discretization. How?

Extinction coefficient
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$$\int_0^1 \sigma_t((1-s)x_i + sx_{i+1})ds \approx \sum_b \sigma_t[b]\; d_{x_i,x_{i+1}}[b] = \boldsymbol{\sigma}_t \cdot \boldsymbol{d}_{x_i,x_{i+1}}$$

before

$$I = \sum_{k=1}^{\infty} \sum_{\Omega_k} L_e(x_0,x_1) \frac{e^{-\int_0^1 \sigma_t((1-s)x_0+sx_1)ds}}{\|x_0 - x_1\|^2}$$
$$\left[\prod_{i=1}^{k-1} \sigma_s(x_i) f_p(x_{i-1},x_i,x_{i+1}) \frac{\boxed{e^{-\int_0^1 \sigma_t((1-s)x_i+sx_{i+1})ds}}}{\|x_i - x_{i+1}\|^2}\right]$$
$$W_e(x_{k-1},x_k)d\mu_k$$

after

$$I \approx \sum_{k=1}^{\infty} \sum_{\Omega_k} L_e(x_0,x_1) \frac{e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{d}_{x_0,x_1}}}{\|x_0 - x_1\|^2}$$
$$\left[\prod_{i=1}^{k-1} \sigma_s(x_i) f_p(x_{i-1},x_i,x_{i+1}) \frac{\boxed{e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{d}_{x_i,x_{i+1}}}}}{\|x_i - x_{i+1}\|^2}\right]$$
$$W_e(x_{k-1},x_k)d\mu_k$$

Addition of Vectorized paths

$$\left[\prod_{i=1}^{k-1} \sigma_s(x_i) f_p(x_{i-1}, x_i, x_{i+1}) \frac{e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{d}_{x_i, x_{i+1}}}}{\|x_i - x_{i+1}\|^2}\right]$$

$$= \underbrace{\left[\prod_{i=1}^{k-1} \sigma_s(x_i) f_p(x_{i-1}, x_i, x_{i+1})\right]}_{\text{Assumed to be known}} \left[\frac{e^{-\boldsymbol{\sigma}_t \cdot (\sum_{i=0}^{k-1} \boldsymbol{d}_{x_i, x_{i+1}})}}{\prod_{i=0}^{k-1} \|x_i - x_{i+1}\|^2}\right]$$

Everything else

$$\Rightarrow \quad I = \sum_{k=1}^{\infty} L_e(x_0, x_1) \sum_{\Omega_k} H_k \; e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_k}$$

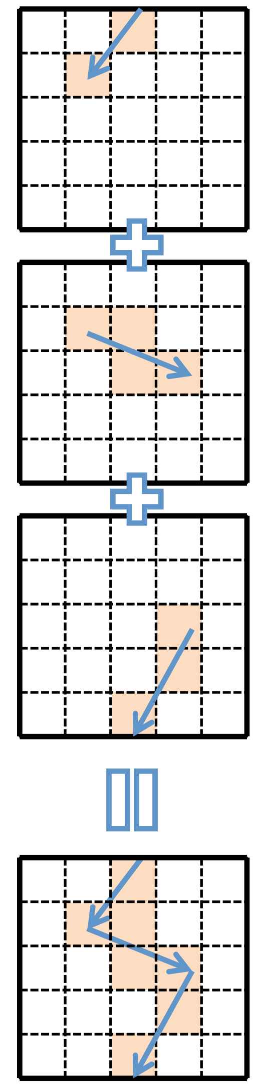
$$\Rightarrow \quad I = L_e(x_0, x_1) \sum_{\Omega} H_k \; e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_k}$$

$\boldsymbol{d}_{x_0, x_1}$

$\boldsymbol{d}_{x_1, x_2}$

$\boldsymbol{d}_{x_2, x_3}$

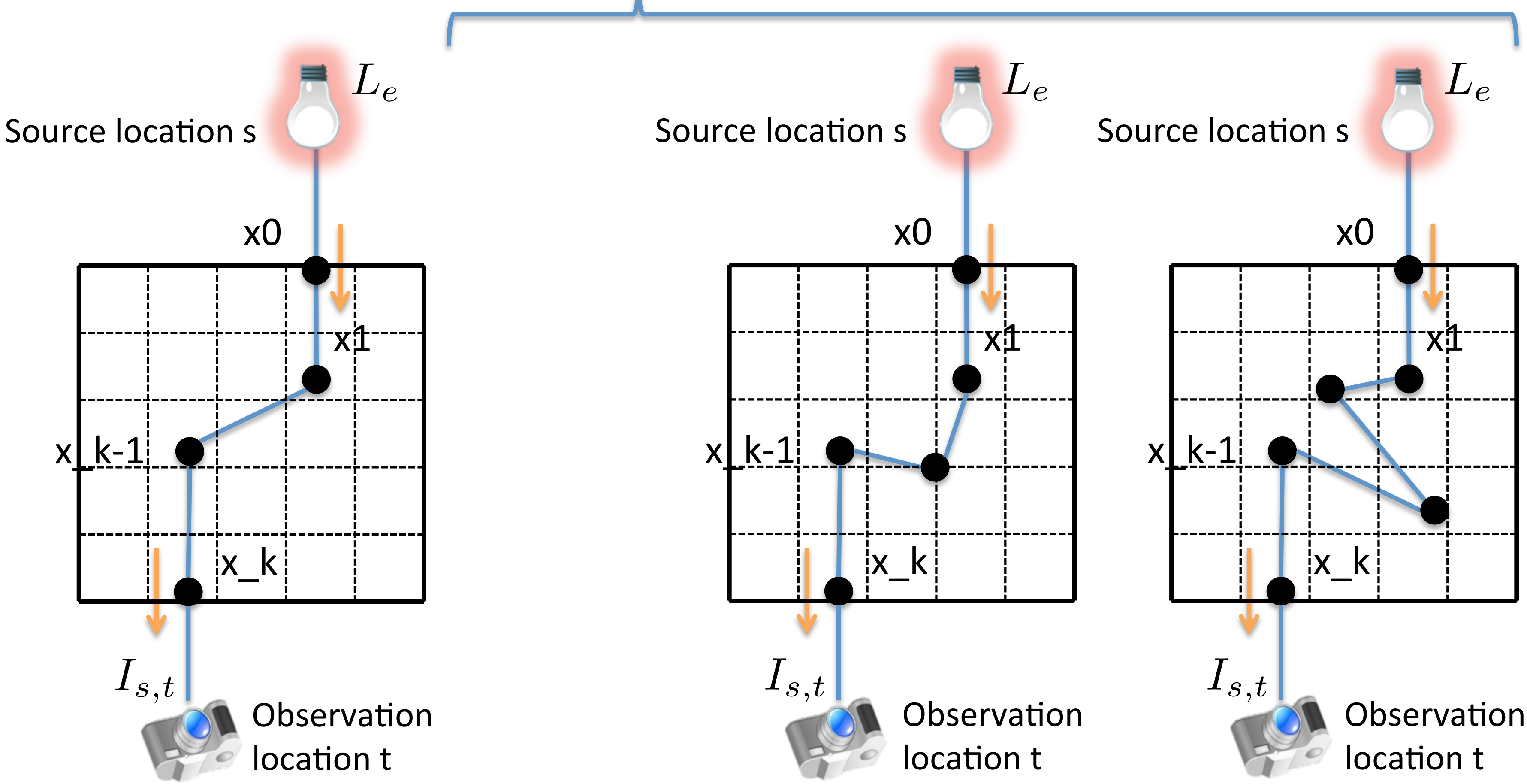
$\boldsymbol{D}_k = \sum \boldsymbol{d}_{x_i, x_{i+1}}$



The forward model

Observation at s,t

$$I_{s,t} = L_e \sum_{\Omega} H_{k(s,t)} \; e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_{k(s,t)}}$$



The inverse problem

Cost function with constraints

$$\min_{\boldsymbol{\sigma}_t} \sum_{s,t} |I_{s,t} - L_e \sum_{\Omega} H_{k(s,t)} \; e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_{k(s,t)}}|^2 \qquad \text{subject to} \qquad \boldsymbol{\sigma}_t \succ 0$$

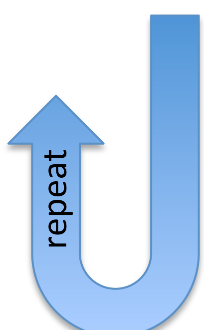
observation

model

Coefficient must be positive

Solved by the log-barrier interior point method

$$\min_{\boldsymbol{\sigma}_t} f_0(\boldsymbol{\sigma}_t) \quad \text{subject to} \quad f_i(x) \le 0, \quad i = 1, \cdots, m,$$



Initial value $\boldsymbol{\sigma}_t$

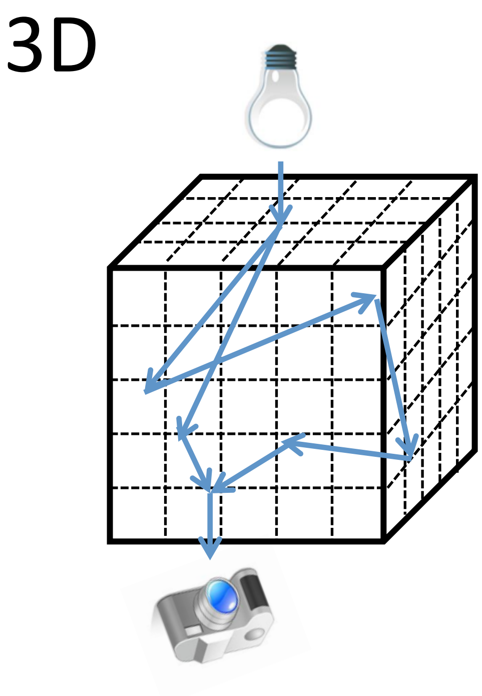
Log-barrier functions

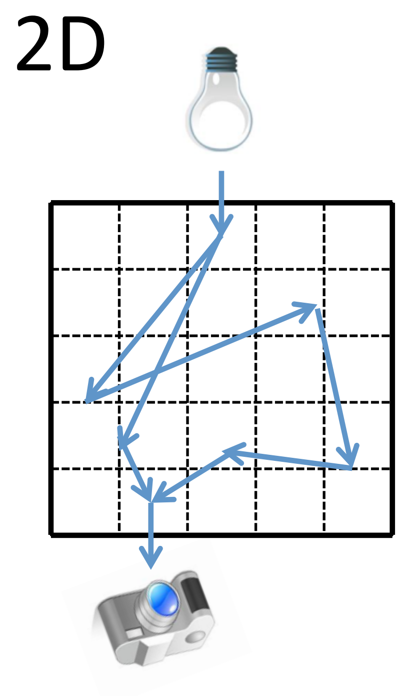
Solve $\boldsymbol{\sigma}_t \leftarrow \underset{\boldsymbol{\sigma}_t}{\operatorname{argmin}} f_0(\boldsymbol{\sigma}_t) + \sum_{i=1}^{m} -(1/t)\log(-f_i(\boldsymbol{\sigma}_t))$

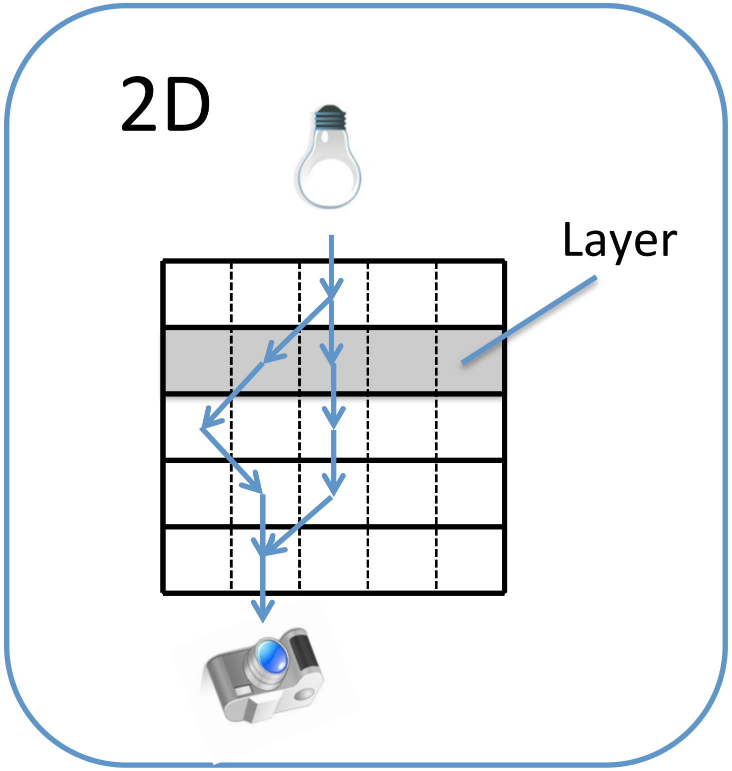
by Quasi-Newton

Update t (larger)

Target model







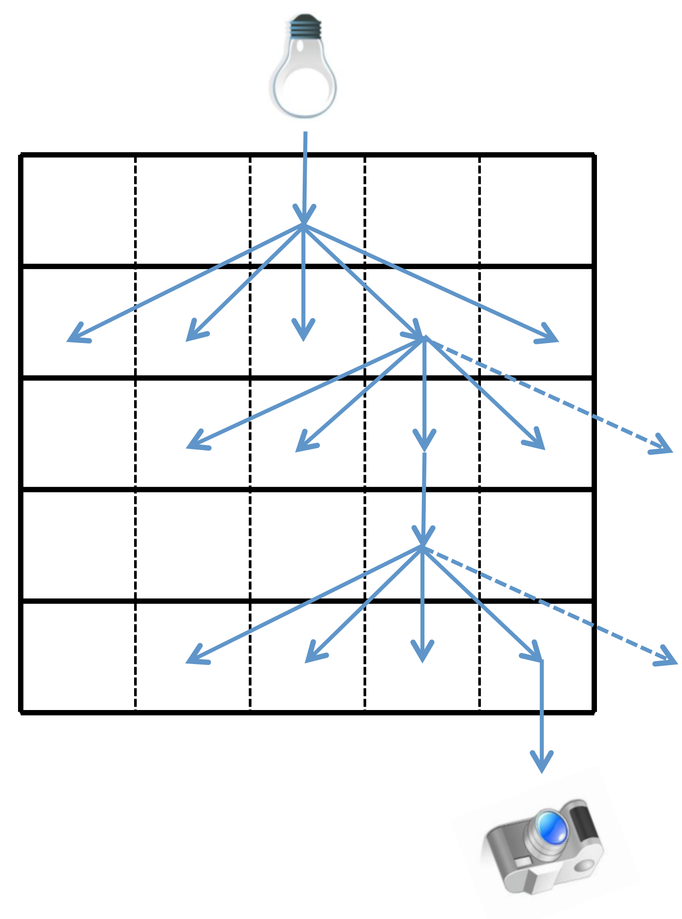
Ideal
Costly in space and time

Second option
Less expensive
but still large cost

Smallest cost
Rough approximation

Proposed 2D layered model

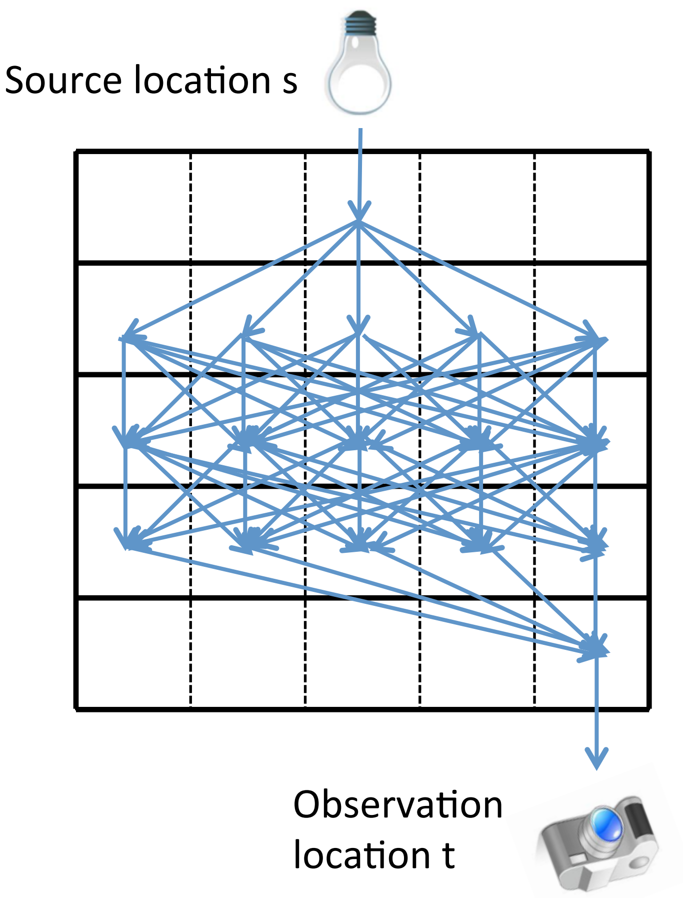
- Light scatters:
  - Layer by layer
  - At voxel centers
  - Forward only
    - Subject to a simplified phase function
  - With uniform/known scattering coefficients



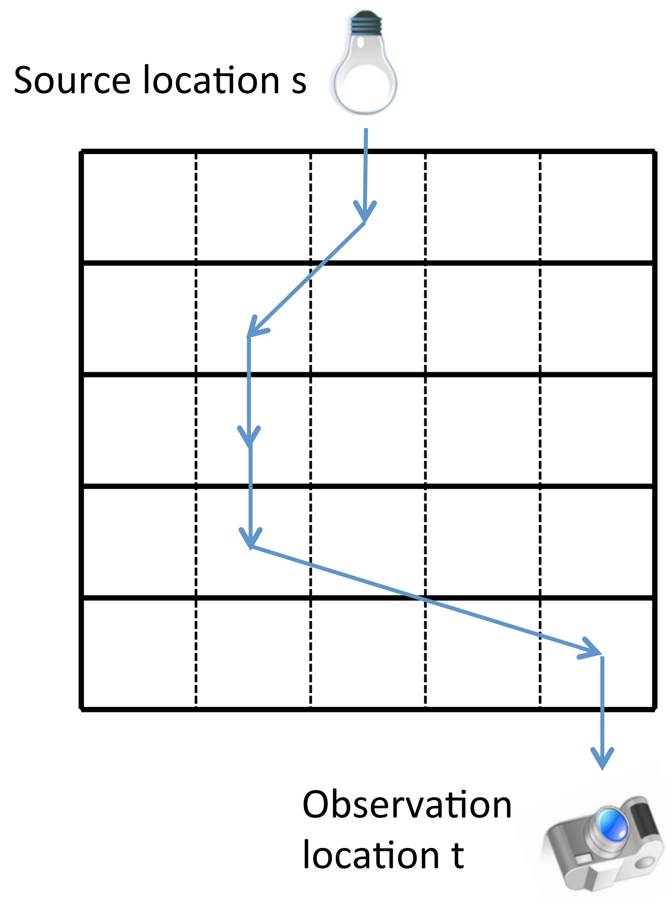
Enumerating paths

$$\min_{\boldsymbol{\sigma}_t} \sum_{s,t} |I_{s,t} - L_e \sum_{\Omega} H_{k(s,t)}\ e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_{k(s,t)}}|^2$$

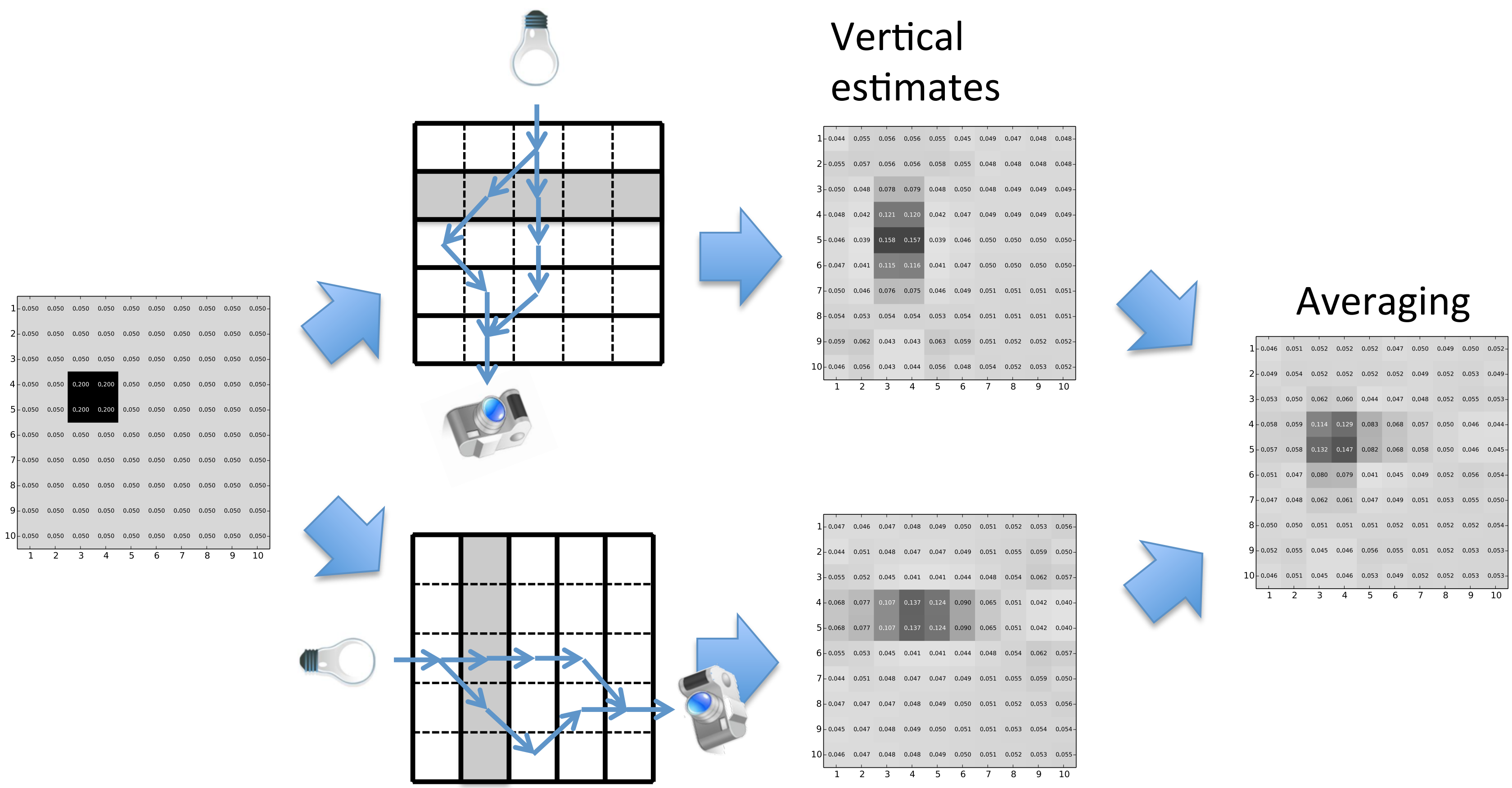
Exhaustive (with thresholding)



(Bi-directional) Monte Carlo-like



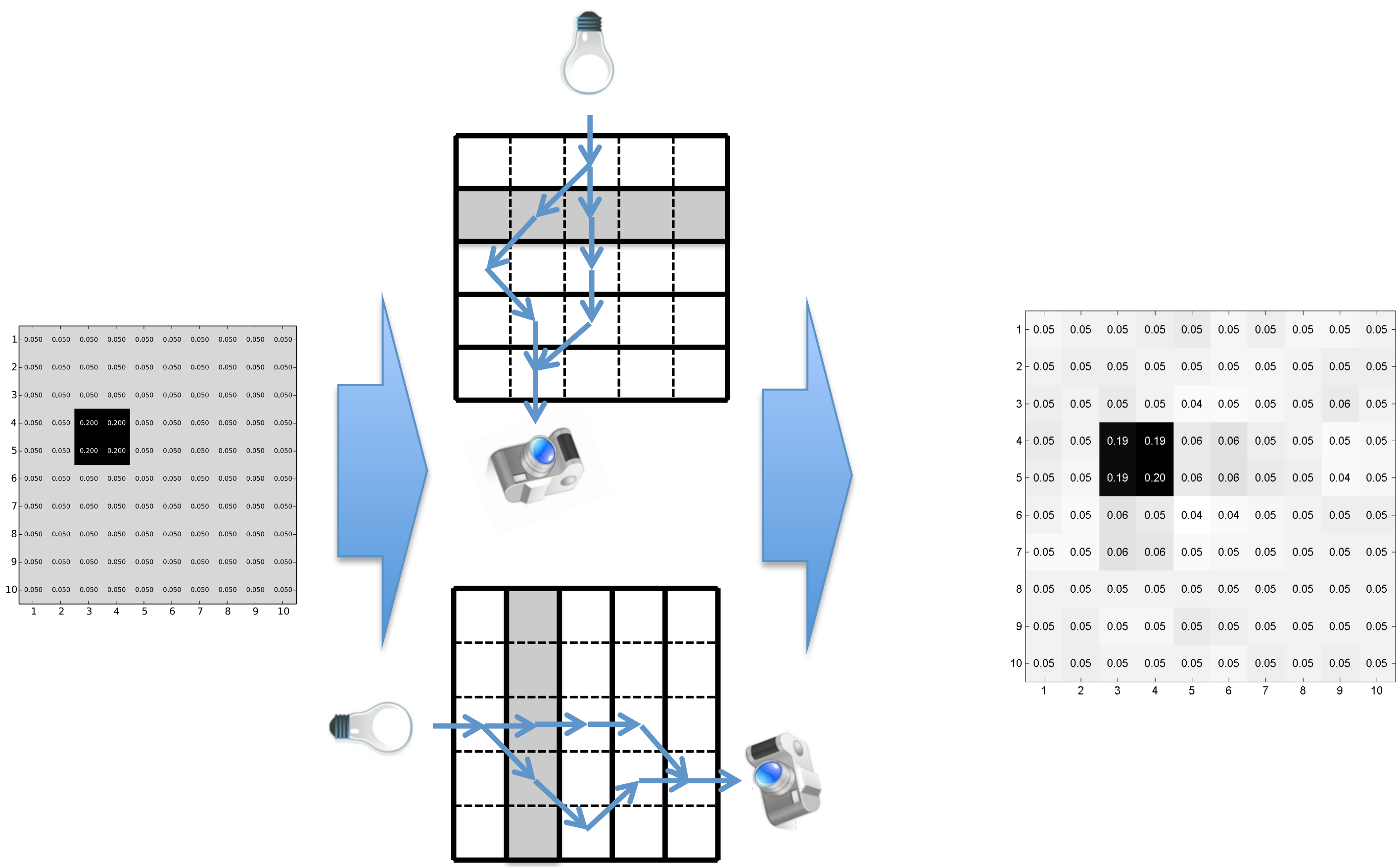
Different directions



Horizontal
Estimates

Toru Tamaki, Bingzhi Yuan, Bisser Raytchev, Kazufumi Kaneda, Yasuhiro Mukaigawa, "Multiple-scattering Optical Tomography with Layered Material," The 9th International Conference on Signal-Image Technology and Internet-Based Systems (SITIS 2013), December 2-5, 2013, Kyoto, Japan. (2013 12)

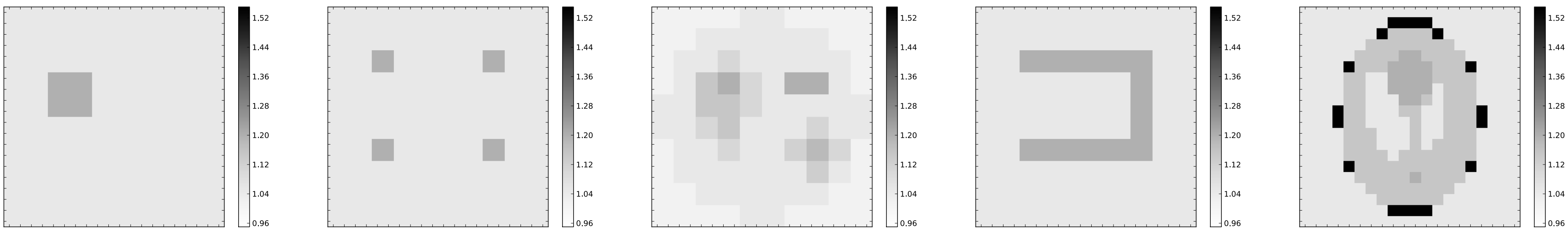
Joint optimization



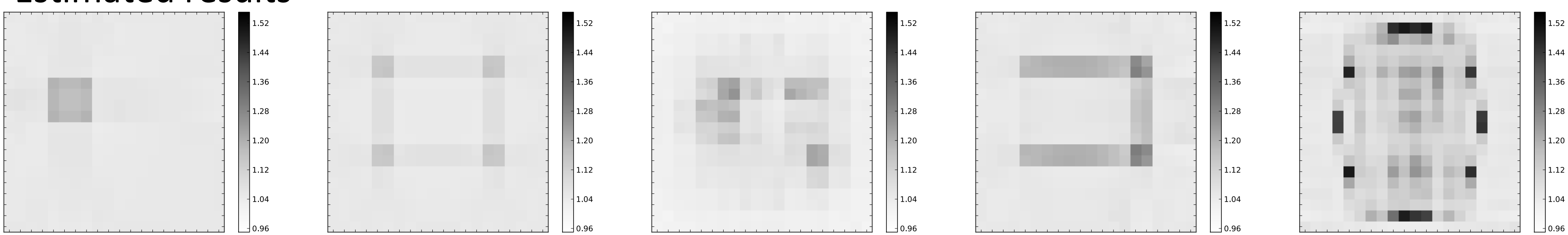
Bingzhi Yuan, Toru Tamaki, Takahiro Kushida, Bisser Raytchev, Kazufumi Kaneda, Yasuhiro Mukaigawa and Hiroyuki Kubo, "Layered optical tomography of multiple scattering media with combined constraint optimization", in Proc. of FCV 2015 : 21st Korea-Japan joint Workshop on Frontiers of Computer Vision, January 28--30, 2015, Shinan Beach Hotel, Mokpo, KOREA.

results of numerical simulation

Ground truth



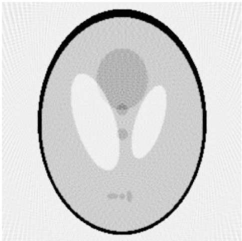
Estimated results



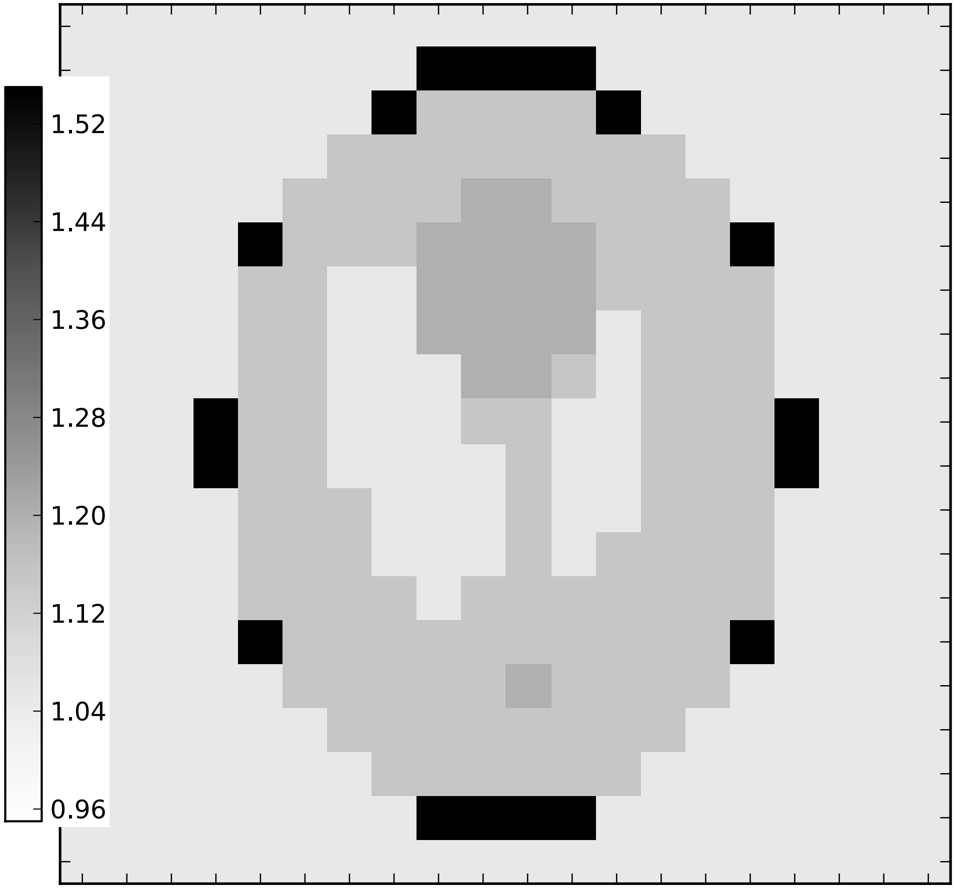
- 20x20 grid of 20[mm]x20[mm]
- Extinction coefficients: between 1.05 and 1.55 [mm^{-1}]
- Scattering coefficients (known): 1 [mm^{-1}]
- Phase function (known): a simplified approximation

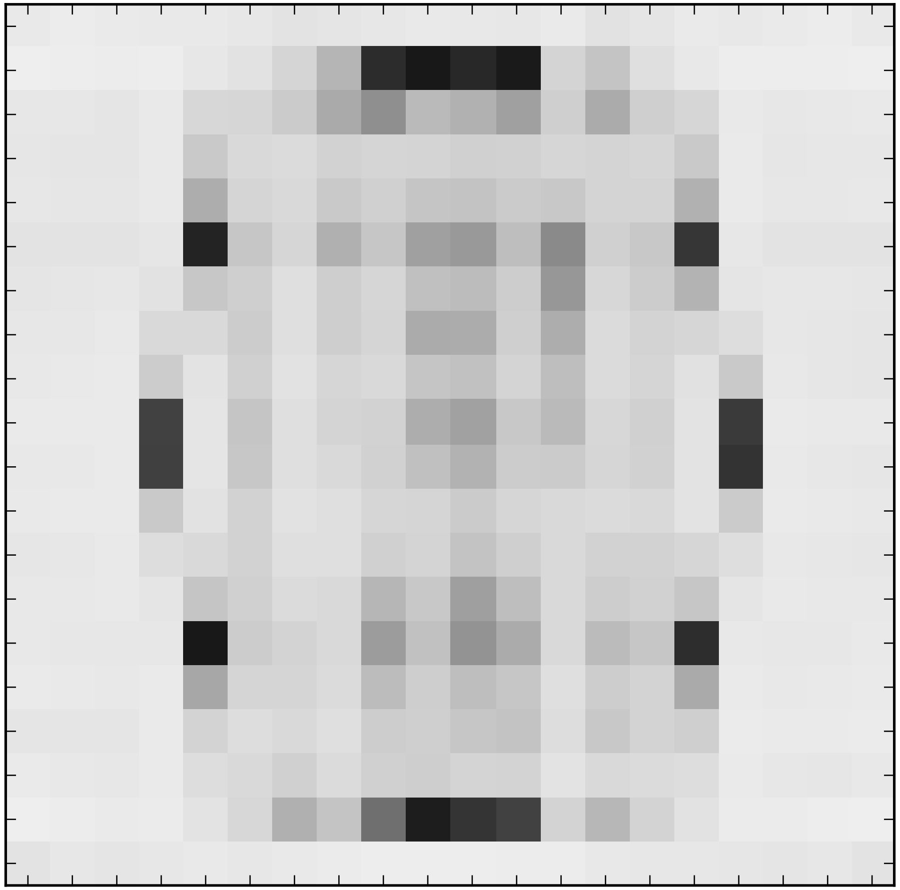
- Fixed 742 paths for each s,t (exhaustive with th.)
- Joint optimization with interior point method

Shepp-Logan phantom

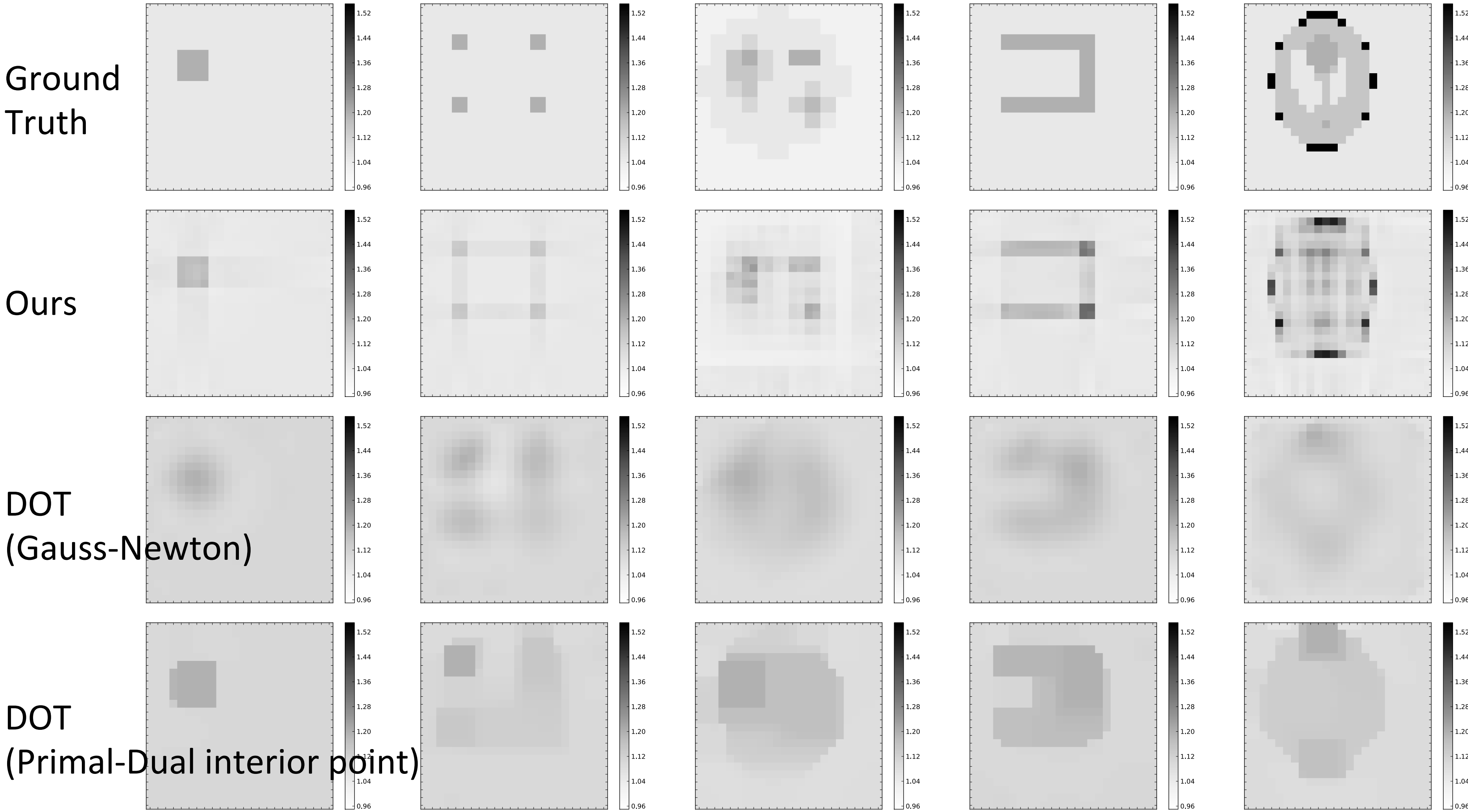


20 x 20





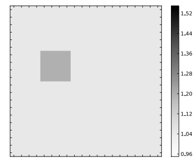
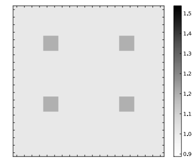
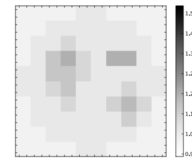
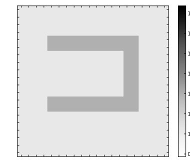
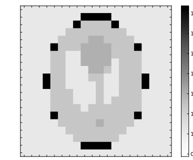
Comparison with DOT



- 24x24 grid of 24[mm]x24[mm]
- 24x24x2 = 1152 triangle meshes for FEM
- 16 light sources and 16 detectors around the square

- With EIDORS (Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software) eidors3d.sourceforge.net

Accuracy and cost

							
RMSE	Ours $\sigma^2 = 0.4$	0.007662 (0.730%)	0.01244 (1.18%)	0.026602 (2.53%)	0.021442 (2.04%)	0.051152 (4.87%)	✓
	DOT (GN)	0.053037 (5.05%)	0.060597 (5.77%)	0.7605 (7.53%)	0.059534 (5.67%)	0.0855 (8.14%)	
	DOT (PD)	0.052466 (5.25%)	0.0626 (5.97%)	0.081081 (8.11%)	0.066042 (6.60%)	0.080798 (8.08%)	
Computation time [s]	Ours $\sigma^2 = 0.4$	257	217	382	306	504	✗
	DOT (GN)	0.397	0.390	0.407	0.404	0.453	
	DOT (PD)	1.11	1.09	1.14	1.08	1.15	

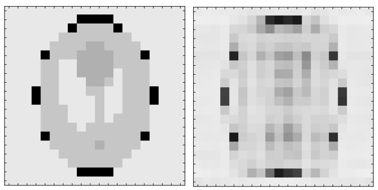
Our implementation: MATLAB

Summary

- Optical tomography for scattering media
- Observation model: path integral
- Least-square approach with constraints

$$\min_{\boldsymbol{\sigma}_t} \sum_{s,t} |I_{s,t} - L_e \sum_{\Omega} H_{k(s,t)} \; e^{-\boldsymbol{\sigma}_t \cdot \boldsymbol{D}_{k(s,t)}}|^2$$

- Solver: interior point method
- Experiments: numerical simulation



- Future work: a lot