Computer Vision – TP9
Active Contours

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Acknowledgement: Slides adapted from Kristen Grauman

Outline

- Introduction to Active Contours
- Energy functions
- Energy minimization

Topic: Introduction to Active Contours

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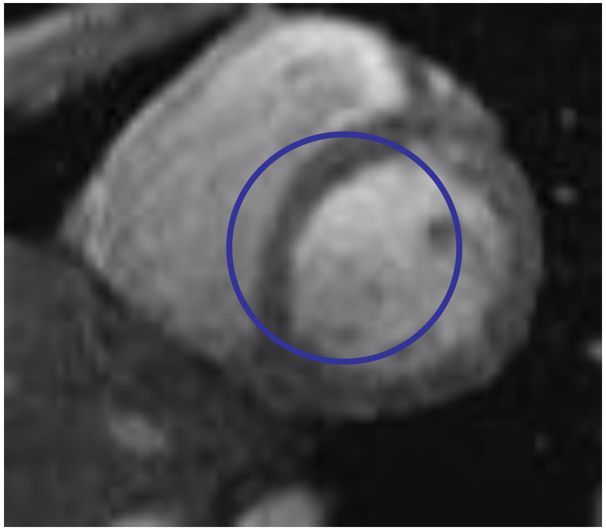
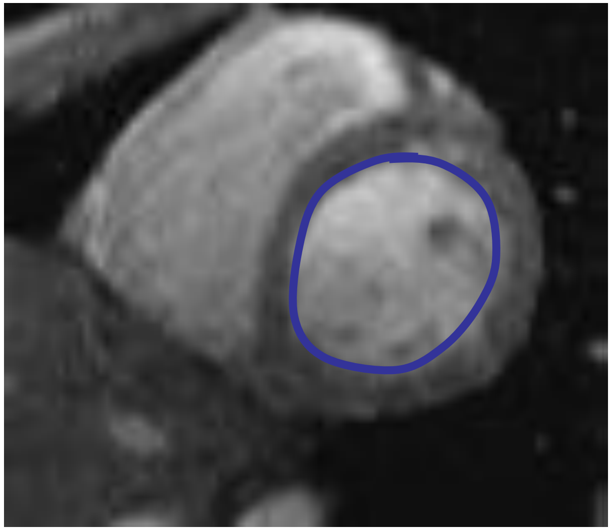
Active Contours

- Given: initial contour (model) near desired object



Active Contours

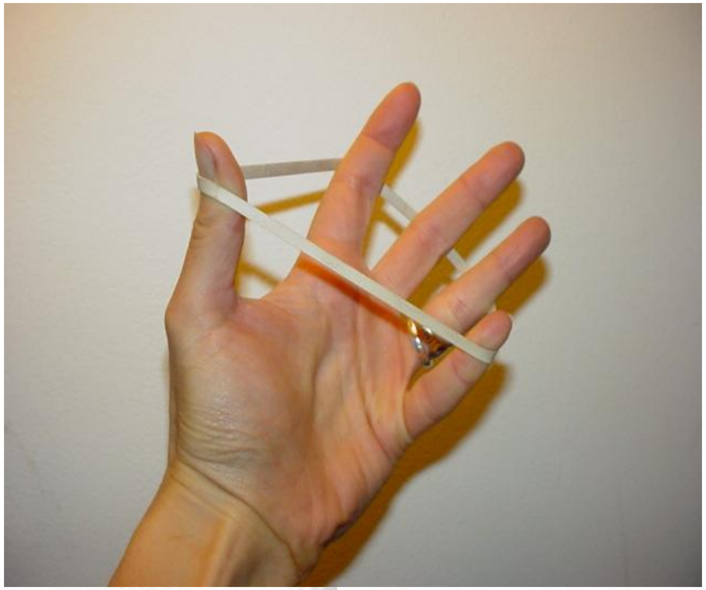
- Goal: evolve the contour to fit exact object boundary

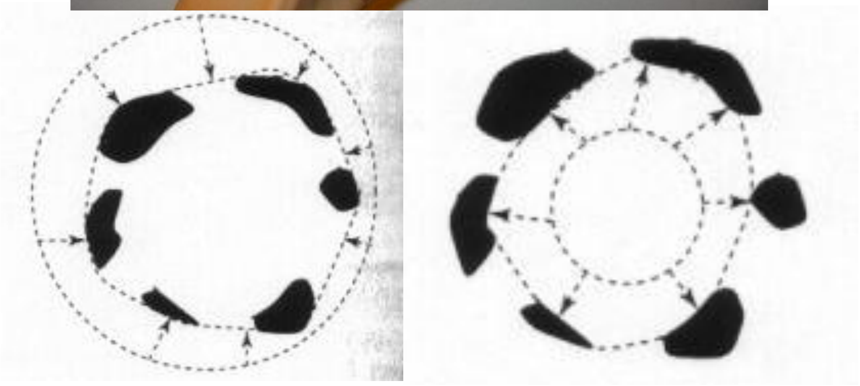


- How?
  - Reward solutions next to high image gradients
  - Punish solutions that deform shape too much
  - Iteratively find the ‘best’ solution to these requirements

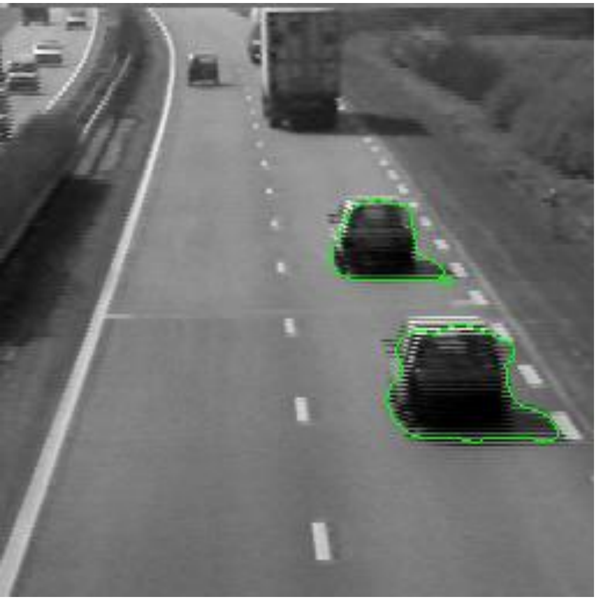
Intuition - Elastic Band

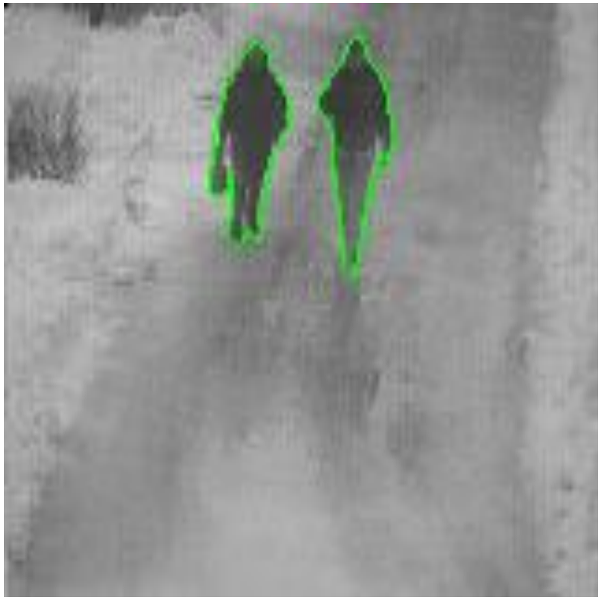
- Contour evolves to a low-energy solution, but is hindered by obstacles
- Better intuition: Gravity
  - Contour is ‘attracted’ to specific image features
  - Contour resists to any deformation of its shape

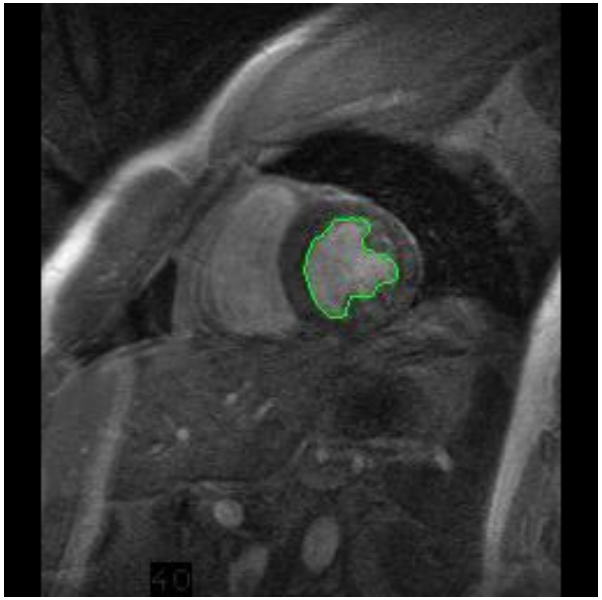




Strong motivation – Moving deformable objects





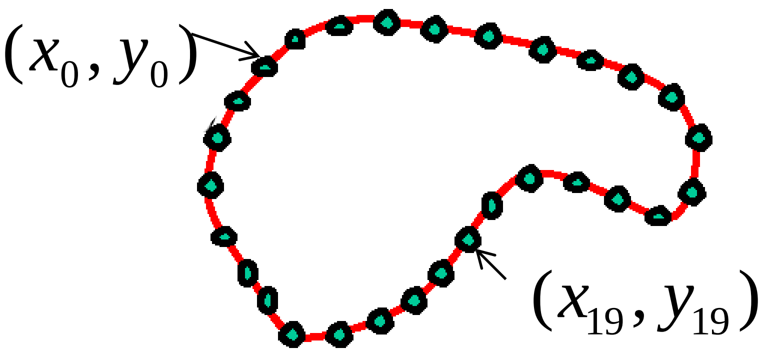


Things we need to consider

- Representation of the contours
- Defining the energy functions
  - External
  - Internal
- Minimizing the energy function

Representation

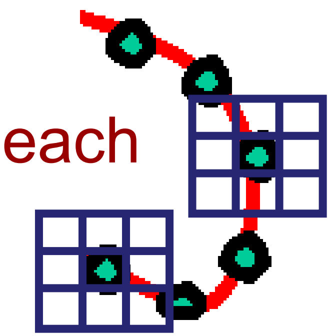
- We'll consider a discrete representation of the contour, consisting of a list of 2d point positions ("vertices")



$$v_i = (x_i, y_i),$$

for $i = 0, 1, \ldots, n-1$

- At each iteration, we'll have the option to move each vertex to another nearby location ("state")



Topic: Energy functions

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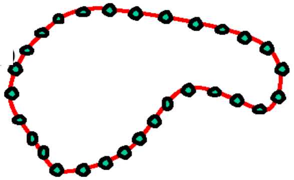
Energy function

The total energy (cost) of the current snake is defined as:

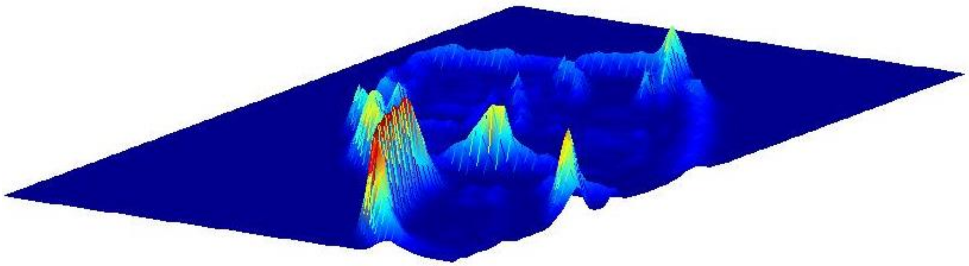
$$E_{total} = E_{external} + E_{internal}$$



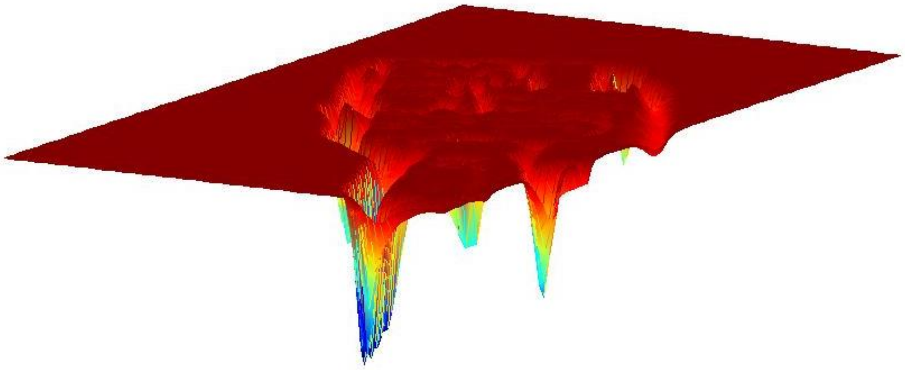
- External energy: encourage contour to fit on places where specific image structures exist
- Internal energy: encourage prior shape preferences

A good fit between the current deformable contour and the target shape in the image will yield a low value for this cost function

External image energy



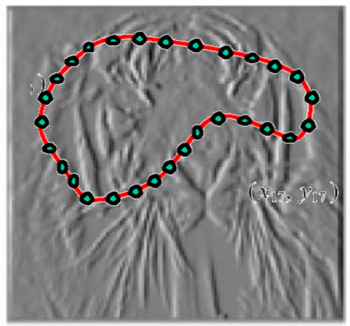
Magnitude of gradient $G_x(I)^2 + G_y(I)^2$

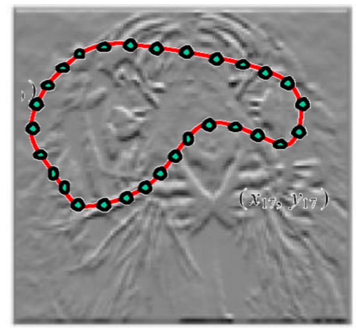


- (Magnitude of gradient) $-\left(G_x(I)^2 + G_y(I)^2\right)$

External image energy

- Gradient images $G_x(x, y)$ and $G_y(x, y)$





- External energy at a point on the curve is:

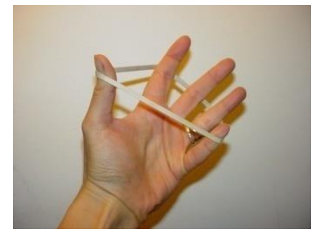
$$E_{external}(v) = -(\,|G_x(v)|^2 + |G_y(v)|^2\,)$$

- External energy for the whole curve:

$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

Internal energy

For a *continuous* curve, a common internal energy term is the "bending energy"



At some point *v(s)* on the curve, this is:

$$E_{internal}(v(s)) = \alpha \left| \frac{dv}{ds} \right|^2 + \beta \left| \frac{d^2 v}{d^2 s} \right|^2$$

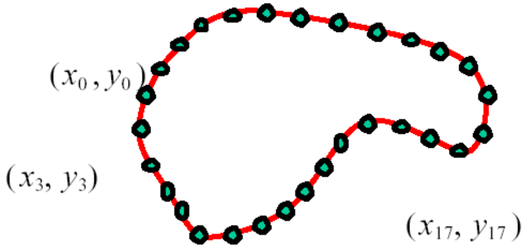
Tension, Elasticity

Stiffness, Curvature

Internal energy

- For our discrete representation:

$$v_i = (x_i, y_i) \qquad i = 0 \ldots n-1$$



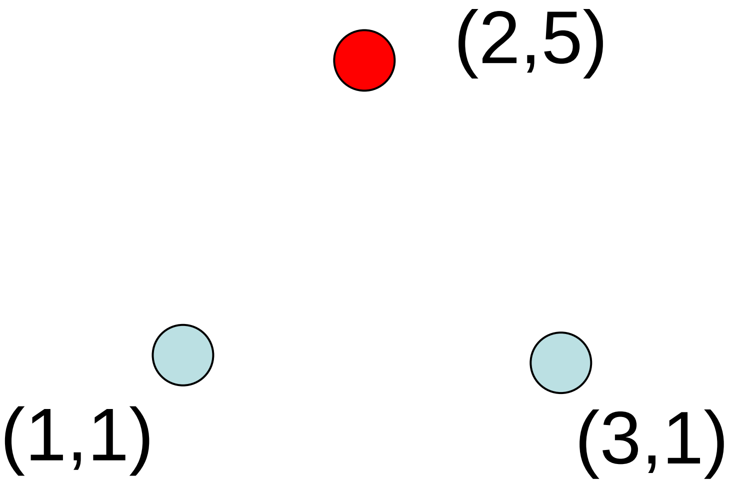
$$\frac{dv}{ds} \approx v_{i+1} - v_i \qquad \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

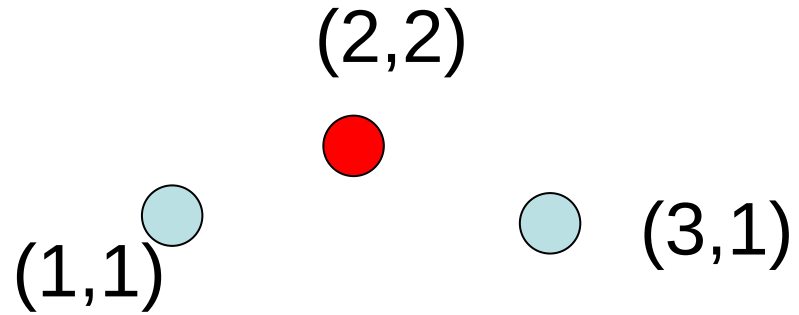
- Internal energy for the whole curve:

$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2 + \beta \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$

Example: compare curvature

$$E_{curvature}(v_i) = \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$
$$= (x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2$$





$$(3 - 2(2) + 1)^2 + (1 - 2(5) + 1)^2$$
$$= (-8)^2 = 64$$

$$(3 - 2(2) + 1)^2 + (1 - 2(2) + 1)^2$$
$$= (-2)^2 = 4$$

Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

$$E_{elastic} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2$$

- This rewards very small shapes!
- Instead -> Reward an 'average distance ***d*** between pairs of points'

$$= \alpha \cdot \sum_{i=0}^{n-1} \left((x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - \bar{d} \right)^2$$

Total energy

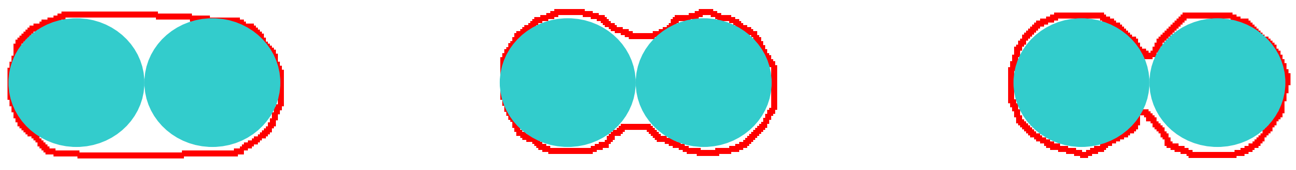
$$E_{total} = E_{internal} + \gamma E_{external}$$

$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left(\bar{d} - \|v_{i+1} - v_i\|\right)^2 + \beta \|v_{i+1} - 2v_i + v_{i-1}\|^2$$

Energy weights

- e.g., α weight controls the penalty for internal elasticity



large α medium α small α

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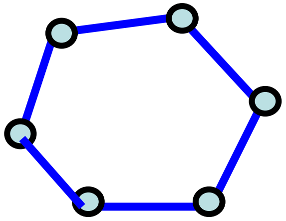
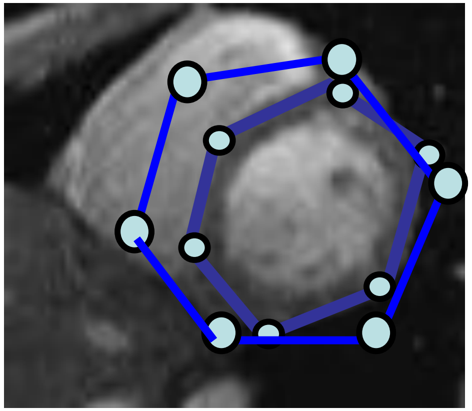
Recap: deformable contour



- A simple elastic snake is defined by:
  - A set of *n* points,
  - An internal energy term (tension, bending, plus optional shape prior)
  - An external energy term (gradient-based)



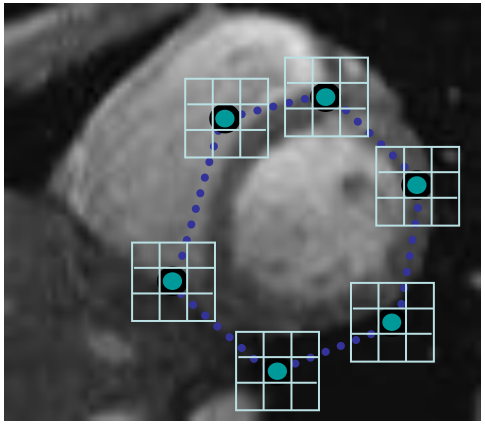
- To use this to segment an object:
  - Initialize in the vicinity of the object
  - Modify the points to minimize the total energy

Energy minimization: greedy

- For each point, search window around it and move to where energy function is minimal
  - Typical window size, e.g., 5 x 5 pixels

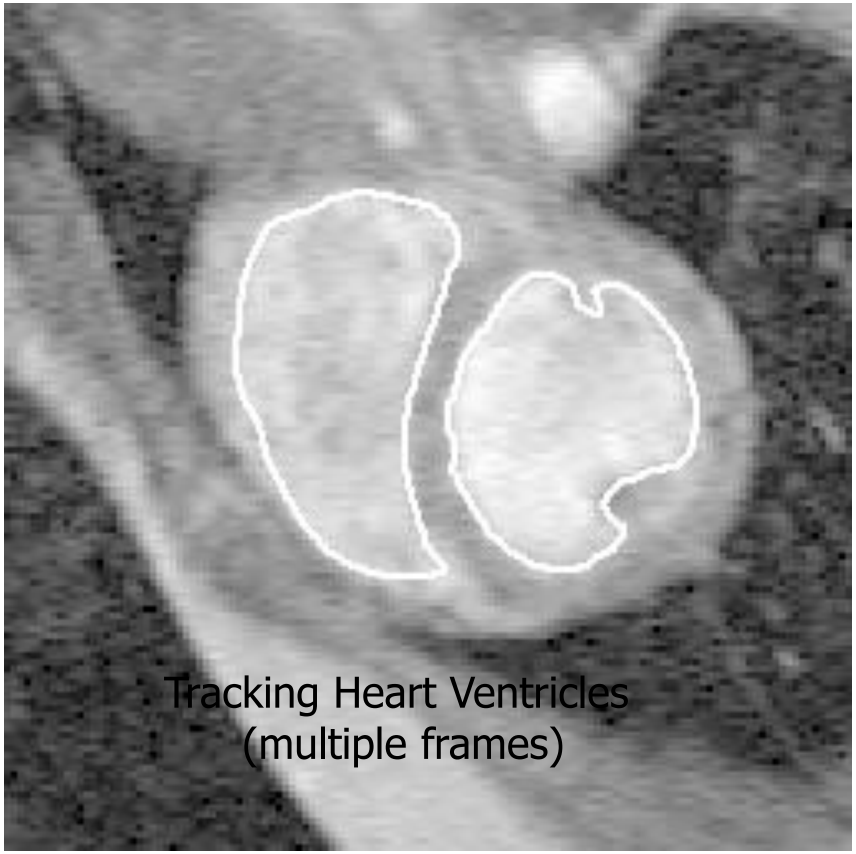
- Stop when predefined number of points have not changed in last iteration, or after max number of iterations

- Note:
  - Convergence not guaranteed
  - Need decent initialization



Tracking via deformable contours

1. Use final contour/model extracted at frame t as an initial solution for frame t+1
2. Evolve initial contour to fit exact object boundary at frame t+1
3. Repeat, initializing with most recent frame



Deformable contours: pros and cons

Pros:

- Useful to track and fit non-rigid shapes
- Contour remains connected
- Possible to fill in “subjective” contours
- Flexibility in how energy function is defined, weighted.

Cons:

- Must have decent initialization near true boundary, may get stuck in local minimum
- Parameters of energy function must be set well based on prior information

Resources

- Szeliski, “Computer Vision: Algorithms and Applications”, Springer, 2011
  - Chapter 5 – “Segmentation”