Semânticas de Linguagens de Programação - CC3033

Exercícios 8

Linguagem funcional FUN

1. Escreve um programa em **FUN** para calcular o máximo divisor comum entre dois inteiros x e y.

2. Considera o programa **FUN**

$$f_1\ x = \texttt{let}\ y = x \times x\ \texttt{in}\ y \times y$$
$$f_2\ x = \texttt{let}\ x = x \times x\ \texttt{in}\ x \times x$$

   com $\Gamma(f_1) = \Gamma(f_2) = \mathsf{int} \to \mathsf{int}$.

   (a) Mostra que admite tipo no ambiente Γ.

   (b) Verifica se o termo $\texttt{let}\ x = 2\ \texttt{in}\ f_1\ (f_2\ x)$ admite tipo em Γ

   (c) Avalia o termo

$$\texttt{let}\ x = 2\ \texttt{in}\ f_1\ (f_2\ x)$$

   na semântica de passagem de argumentos por valor $\Downarrow_P^v$.

3. Considera o programa **FUN**

$$g_1\ x = \texttt{let}\ x_2 = 1\ \texttt{in}\ \texttt{let}\ \texttt{fun}\ f\ x_1 : \mathsf{int} = (x_1 + x_2) : \mathsf{int}\ \texttt{in}\ (f\ x \times x)$$

   (a) Mostra que admite tipo no ambiente $\Gamma(g_1) = \mathsf{int} \to \mathsf{int}$.

   (b) Avalia o termo g_1 3 na semântica de passagem de argumentos por valor $\Downarrow_P^v$.

4. Seja o termo

$$(\texttt{let}\ \texttt{fun}\ sqr\ x : \mathsf{int} = (x \times x) : \mathsf{int}\ \texttt{in}\ (sqr\ 3) > (sqr\ 2))$$

   (a) Mostra que admite tipo.

   (b) Avalia-o na semântica de passagem de argumentos por valor $\Downarrow_P^v$.

5. Seja o termo

$$\texttt{let}\ x = 25\ \texttt{in}\ \texttt{let}\ \texttt{fun}\ f\ x : \mathsf{int} = (x + 3) : \mathsf{int}\ \texttt{in}\ (f\ 7) + x$$

   (a) Mostra que admite tipo no ambiente vazio.

   (b) Avalia-o na semântica de passagem de argumentos por valor $\Downarrow_P^v$.

6. Modifica a semântica de passagem de argumentos por valor $\Downarrow_P^v$ para que o âmbito dos identificadores seja estático. Sugestão: Na regra *letfun* incluí o ambiente na definição da função f.

   Indica qual o valor obtido para o termo seguinte usando o âmbito dinâmico e o usando o âmbito estático (para o ambiente vazio):

$$\texttt{let}\ x = 3\ \texttt{in}\ \texttt{let}\ \texttt{fun}\ f\ y : \mathsf{int} = (y + x) : \mathsf{int}\ \texttt{in}\ \texttt{let}\ x = 8\ \texttt{in}\ (f\ 2)$$

7. Define uma semântica de passagem de argumentos por nome para **FUN**.

8. Implementa em Haskell a linguagem **FUN** e a semântica de passagem de argumentos por valor $\Downarrow_P^v$ (supondo que os termos e programas admitem tipo).

Regras do sistema de tipos para FUN

$$\frac{}{\Gamma \vdash\ b : \mathsf{bool}}\ (\mathrm{b}) \qquad \frac{\Gamma \vdash\ t_1 : \mathsf{bool} \qquad \Gamma \vdash\ t_2 : \mathsf{bool}}{\Gamma \vdash\ t_1 \wedge t_2 : \mathsf{bool}}\ (\mathrm{and})$$

$$\frac{}{\Gamma \vdash\ n : \mathsf{int}}\ (\mathrm{n}) \qquad \frac{\Gamma \vdash\ t : \mathsf{bool}}{\Gamma \vdash\ \neg\, t : \mathsf{bool}}\ (\mathrm{not})$$

$$\frac{\Gamma(id) = \sigma}{\Gamma \vdash\ id : \sigma}\ (\mathrm{id}) \qquad \frac{\Gamma \vdash\ t_0 : \mathsf{bool} \qquad \Gamma \vdash\ t_1 : \sigma \qquad \Gamma \vdash\ t_2 : \sigma}{\Gamma \vdash\ \texttt{if}\ t_0\ \texttt{then}\ t_1\ \texttt{else}\ t_2 : \sigma}\ (\mathrm{if})$$

$$\frac{\Gamma \vdash\ t_1 : \mathsf{int} \qquad \Gamma \vdash\ t_2 : \mathsf{int}}{\Gamma \vdash\ t_1\ op\ t_2 : \mathsf{int}}\ (\mathrm{op}) \qquad \frac{\Gamma \vdash\ t_1 : \sigma \to \tau \qquad \Gamma \vdash\ t_2 : \sigma}{\Gamma \vdash\ (t_1\ t_2) : \tau}\ (\mathrm{Ap})$$

$$\frac{\Gamma \vdash\ t_1 : \mathsf{int} \qquad \Gamma \vdash\ t_2 : \mathsf{int}}{\Gamma \vdash\ t_1\ bop\ t_2 : \mathsf{bool}}\ (\mathrm{bop}) \qquad \frac{\Gamma \vdash\ t_1 : \sigma \qquad \Gamma, x : \sigma \vdash\ t_2 : \tau}{\Gamma \vdash\ \texttt{let}\ x = t_1\ \texttt{in}\ t_2 : \tau}\ (\mathrm{let})$$

$$(\mathrm{letfun})\ \frac{\Gamma, x_1 : \sigma_1, \ldots x_n : \sigma_n, f : \rho \vdash\ t_1 : \sigma \qquad \Gamma, f : \rho \vdash\ t_2 : \tau}{\Gamma \vdash\ \texttt{let fun}\ f\ x_1 : \sigma_1\ x_2 : \sigma_2 \ldots\ x_n : \sigma_n = t_1 : \sigma\ \texttt{in}\ t_2 : \tau}$$

onde $\rho = \sigma_1 \to \sigma_2 \to \cdots \to \sigma_n \to \sigma$.

Semântica com passagem de argumentos por valor para FUN (âmbito dinâmico), dado um programa P, $f_i\ x_1 \cdots x_{a_i}\ = d_i$, $1 \leq i \leq k$.

$$(\mathrm{n})\ \frac{}{\Gamma \vdash n\ \Downarrow_P^v\ n} \qquad \frac{\Gamma \vdash t\ \Downarrow_P^v\ b}{\Gamma \vdash \neg\, t\ \Downarrow_P^v\ \neg\, b}\ (\mathrm{not})$$

$$(\mathrm{b})\ \frac{}{\Gamma \vdash b\ \Downarrow_P^v\ b}$$

$$(\mathrm{f})\ \frac{ar(f) > 0}{\Gamma \vdash f\ \Downarrow_P^v\ f} \qquad \frac{\Gamma \vdash t_0\ \Downarrow_P^v\ \mathsf{true} \qquad \Gamma \vdash t_1\ \Downarrow_P^v\ v_1}{\Gamma \vdash \texttt{if}\ t_0\ \texttt{then}\ t_1\ \texttt{else}\ t_2\ \Downarrow_P^v\ v_1}\ (\mathrm{ifT})$$

$$(\mathrm{id})\ \frac{\Gamma(x) = v}{\Gamma \vdash x\ \Downarrow_P^v\ v} \qquad \frac{\Gamma \vdash t_0\ \Downarrow_P^v\ \mathsf{false} \qquad \Gamma \vdash t_2\ \Downarrow_P^v\ v_2}{\Gamma \vdash \texttt{if}\ t_0\ \texttt{then}\ t_1\ \texttt{else}\ t_2\ \Downarrow_P^v\ v_2}\ (\mathrm{ifF})$$

$$(\mathrm{op})\ \frac{\Gamma \vdash t_1\ \Downarrow_P^v\ n_1 \qquad \Gamma \vdash t_2\ \Downarrow_P^v\ n_2}{\Gamma \vdash t_1\ op\ t_2\ \Downarrow_P^v\ n_1\ op\ n_2} \qquad \frac{\Gamma \vdash t_1\ \Downarrow_P^v\ v_1 \qquad \Gamma, x = v_1 \vdash t_2\ \Downarrow_P^v\ v}{\Gamma \vdash \texttt{let}\ x = t_1\ \texttt{in}\ t_2\ \Downarrow_P^v\ v}\ (\mathrm{let})$$

$$(\mathrm{bop})\ \frac{\Gamma \vdash t_1\ \Downarrow_P^v\ n_1 \qquad \Gamma \vdash t_2\ \Downarrow_P^v\ n_2}{\Gamma \vdash t_1\ bop\ t_2\ \Downarrow_P^v\ n_1\ bop\ n_2} \qquad \frac{\Gamma, f = \lambda x_1\ \ldots\ x_n.t_1 \vdash t_2\ \Downarrow_P^v\ v}{\Gamma \vdash \texttt{let fun}\ f\ x_1\ \ldots\ x_n = t_1\ \texttt{in}\ t_2\ \Downarrow_P^v\ v}\ (\mathrm{letfun})$$

$$(\mathrm{and})\ \frac{\Gamma \vdash t_1\ \Downarrow_P^v\ b_1 \qquad \Gamma \vdash t_2\ \Downarrow_P^v\ b_2}{\Gamma \vdash t_1\ \wedge\ t_2\ \Downarrow_P^v\ b_1\ \wedge\ b_2} \qquad \frac{\Gamma \vdash d_i\ \Downarrow_P^v\ v}{\Gamma \vdash f_i\ \Downarrow_P^v\ v \quad \text{se } def(f_i) = d_i}\ (\mathrm{fn0})$$

$$(\mathrm{Ap})\ \frac{\Gamma \vdash s\ \Downarrow_P^v\ f\, t_1\ \ldots\ t_i \qquad \Gamma \vdash t\ \Downarrow_P^v\ v \qquad i < ar(f) - 1}{\Gamma \vdash (s\ t)\ \Downarrow_P^v\ (f\, t_1\ \ldots\ t_i\ v)}$$

$$(\mathrm{fn})\ \frac{\Gamma \vdash s\ \Downarrow_P^v\ f_i\, v_1 \ldots v_{a_i - 1} \qquad \Gamma \vdash t\ \Downarrow_P^v\ v_{a_i} \qquad \Gamma \vdash d_i[\overrightarrow{x} \mapsto \overrightarrow{v}]\ \Downarrow_P^v\ v}{\Gamma \vdash (s\ t)\ \Downarrow_P^v\ v \quad \text{se } def(f_i) = \lambda x_1 \ldots x_{a_i}.d_i}$$

$$def(f_i) \quad = \quad \begin{cases} \Gamma(f_i) & \text{se } f_i \in dom(\Gamma) \\ \lambda x_1 \ldots x_{a_i}.d_i & \text{se } f_i\ x_1 \cdots x_{a_i}\ = d_i \in P \\ \lambda x_1\ x_2.x_1\ f_i\ x_2 & \text{se } f_i \text{ é um operador.} \end{cases}$$

Semântica com passagem de argumentos por valor para FUN (âmbito estático).

$$\frac{\Gamma, f = (\lambda x_1\ \ldots\ x_n.t_1, \Gamma) \vdash t_2\ \Downarrow_P^v\ v}{\Gamma \vdash \texttt{let fun}\ f\ x_1\ \ldots\ x_n = t_1\ \texttt{in}\ t_2\ \Downarrow_P^v\ v}\,(letfun_e)$$

$$\frac{\Gamma \vdash s\ \Downarrow_P^v\ f_i\ v_1 \ldots v_{a_i-1} \qquad \Gamma \vdash t\ \Downarrow_P^v\ v_{a_i} \qquad \Gamma' \vdash d_i[\overrightarrow{x} \mapsto \overrightarrow{v}]\ \Downarrow_P^v\ v}{\Gamma \vdash (s\ t)\ \Downarrow_P^v\ v \quad \text{se } def(f_i) = (\lambda x_1 \ldots x_{a_i}.d_i, \Gamma')}\,(fn_e)$$

$$\frac{\Gamma' \vdash d_i\ \Downarrow_P^v\ v}{\Gamma \vdash f_i\ \Downarrow_P^v\ v \qquad \text{se } def(f_i) = (d_i, \Gamma')}\,(fn0_e)$$

$$def(f_i) \quad = \quad \begin{cases} \Gamma(f_i) & \text{se } f_i \in dom(\Gamma) \\ (\lambda x_1 \ldots x_n.d_i, \emptyset) & \text{se } f_i\, x_1 \cdots x_n\ = d_i \in P \\ (\lambda x_1\ x_2.x_1\ f_i\ x_2, \emptyset) & \text{se } f_i \text{ é um operador} \end{cases}$$