Verificação de Programas

Nelma Moreira

Departamento de Ciência de Computadores da FCUP

Verificação de Programas/Program Verification
Aula/Lecture 1

Software and cathedrals are much the same.
First we build them, then we pray.

Reliability
Correctness
Safety
Robustness

Formal methods are methods based on mathematical techniques for the rigorous specification (modelling), development (synthesis) and verification (analysis) of software and hardware systems, with the aim of achieving higher levels of quality.

URL:`www.dcc.fc.up.pt/~nam/Teaching/vp2024/`

Assignments

1. 3 practical assignments (40%)
2. final exam (60%)

1. Brief introduction to formal methods and formal verification
2. Deductive verification
3. Automatic theorem provers: SAT and SMT solvers
4. Brief introduction to interactive theorem provers

1. Partial and total correctness calculus (Hoare logics).
2. Weak-preconditions and verification condition generators.
3. Tools for the specification, verification and certification programs: Dafny
4. Correction of imperative and object oriented programs with Dafny

Automatic and interactive theorem provers

1. Proposition satisfiability problem (SAT).
2. Decidable theories: integers, reals, *arrays*, pointers etc.
3. SMT, Satisfiability modulo theories: algorithms and combination.
4. SMT tools (Z3)
5. Introduction to interactive theorem provers (Coq)

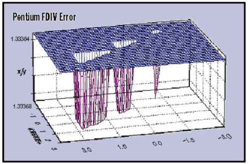
Livros recomendados/Books

- Logic in Computer Science, [HR04] (Cap. 3, 4, 6)
- Rigorous Software Development, [AFPMdS11]
- Semantics with Applications [NN07]
- Deductive procedures [KS16]

Formal methods in the design of Information and Computer Systems (ICS)

1. Formal specifications: languages *Z*, *VDM*, *B*, *JML*
2. Ensure that the specifications satisfy certain properties
3. Derive implementations from the specifications (synthesis)
4. Verify the implementations w.r.t the specifications

Ups!



Intel Pentium bug caused loss of reputation and money.

Ariane 5 crashed within a few minutes after launch

Software bug caused Toyota to recall 1.2M Prius cars

Software race condition caused northeast blackout of 2003

Critical systems and software errors

- Arianne-5, 1996
- Marte "Path Finder"
- Airbuses

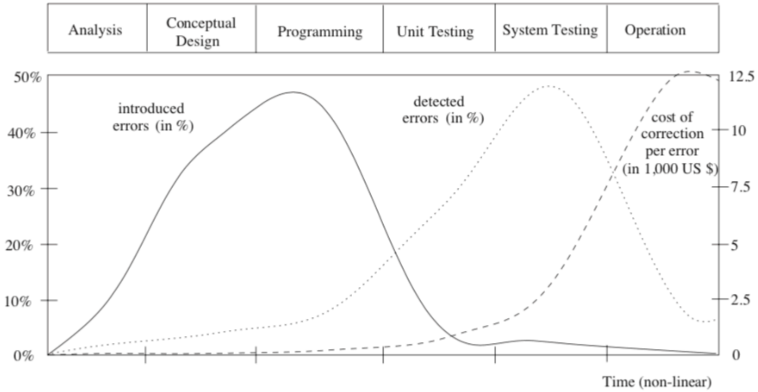
Control systems:

- Nuclear Plants
- Traffic Controllers
- Medical Tools
- etc.

In general, for each 1000 lines of code there is an error.

But, for instance Windows 95 had at least 5000 errors!

Software cycle of life, errors and costs



- How to ensure at the specification level the desired behaviour: model validation problem
- How to ensure that the implementation has the same behaviour as the specification: formal relation between the specification and the implementation problem
- Study of the specification: animation, transformation or proving of properties
- Implementations can be:
  - derived from specifications;
  - ensure their correctness: by construction (correct-by-construction), verification by formal proofs

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Formal Methods Tools

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Central notions and techniques/verification tools

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    - different mechanisms imply different interpretations of the notions of: states, transitions, state transformations
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- The behaviour of the system is described by:
  - operations, available mechanisms, or actions that can be performed
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- Describe how the system operations modify the state
- Formalization based on:
  - discrete mathematics;
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- Category and set theories: states described by mathematical structures (sets, relations or functions); transitions expressed as invariants, pre-conditions and post-conditions.
- Automata based models: to model systems with a concurrent behaviour; to define how the system reacts to events; adequated for reactive systems, concurrent or communication protocols.
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  - Functional languages based on λ-calculus: Scheme, SML, Haskell and OCaml; proof assistants such as ACL2, Coq, PVS, HOL, Isabelle e Agda, are based on typed variants and extensions of λ-calculus. By the Curry-Howard isomorphism the type is a formula and the λ-term a proof.
  - Rewritting systems such as ELAN or SPIKE: the behaviour of functional symbols is described by equational systems and the execution is based of the notion of reduction (as in the λ-calculus).

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Verification of Information and Computer Systems (ICS)

1. asynchronous/synchronous
2. analogic/digital hardware
3. mono/multi processors
4. languages: imperative, functional, logic, object oriented
5. sequential or multi-*threaded*
6. Convencional operating systems or real-time
7. Embedded systems
8. Distributed systems

1. Transformational: reads input data and produces an output; should terminate. Ex: compiler
2. Interactive: interact with the user through events; do not terminate. Ex: operating system
3. Reactive: the interaction is determined by the environment. Ex: flights database access; train controllers

Types of ICS

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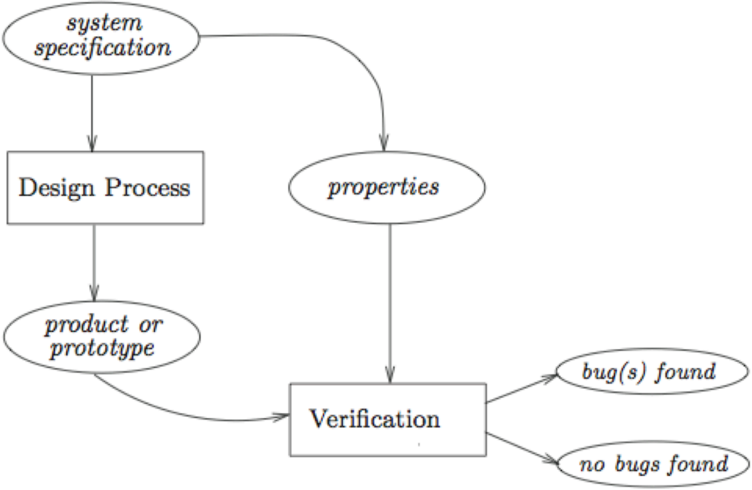
Modelling systems platform

Specification Language to describe the properties one wants to verify

Verification Method proofs can be made

- "by hand, in paper"
- with some automation
- using automatic or interactive computational tools

Formal Verification



Deduction systems vs. Models DS: the system is described by a set of formulas Γ, the specification is a formula φ, and we want

$$\Gamma \vdash \varphi$$

(in general semi-automatic)

M: The system is described by a model $\mathcal{M}$, the specification is a formula φ, and we want

$$\mathcal{M} \models \varphi$$

. (in general automatic for finite models)

Automation Degree automatic/interactive

Complete vs. Properties A specification describes one property or the behaviour of the whole system.

Domain Hardware/Software; sequential/functional or concurrent/reactive

Static vs Dynamic The verification is performed during runtime or before execution.

Program verification

Interactive, Deduction systems, Property verification; Terminating

Model checking

Automatic, Model based, Verification of properties, concurrent and reactive systems, dynamic

But the approaches are not strict and techniques may be mixed. For example for embedded system or *proof carrying code* systems.

There are 3 categories:

- Proofs made by hand and can be informally described
- Tools that allow the formal definition of the proofs.
- Computer assisted proofs

Logics:

- propositional logic, first-order logic, high-order logic
- classic logic versus intuicionistic logic
- modal and temporal logics

Proof Tools

- Automatic proof tools: use a decidable logic fragment
  - ELAN: first-order rewrite
  - ACS2: first-order logic
  - SMT Solvers (Satisfability Module Theory): Yices, CVC3, Z3, Alt-Ergo, Simplify: integers, reais, "arrays", etc.
  - Allow reason about infinite sets
- Interactive proof tools: allow more expressive logics, potentially nondecidable. Coq, Matita, HOL, Lean, etc
  - Combine two capacities: proof check and assisted proof construction
  - Proofs are build interactively using tactics: case, elim, change, rewrite, simpl, discriminate, injection, induction.

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