For data structures definitions see for instance [1].

1. Show with Dafny the following propositions over integers using `lemmas`. Note you should consider {`:induction false`}. You may use a generic function for the power x^n, `function exp(x:int,n:int): int`.
   (a) $\forall n \geq 1, 3^n - 1$ is even.
   (b) $\forall n \geq 6, 4n < (n * n) - 7$
   (c) $\forall n \geq 1, 1 + 3 + \ldots + 2n - 1 = n * n$
   (d) $\forall n > 1, n! < n^n$

2. Consider the datatype for lists given in the lectures. Define a function that allows to Reverse a given list. Prove the following lemma:

```
lemma LengthReverse<T>(xs: List<T>)
  ensures Length(Reverse(xs)) == Length(xs)
{}
```

3. Implement and verify the correction of Binary search trees, AVL trees and Heaps using datatypes.

4. Implement and verify the correction of sorting algorithm *HeapSort*.

5. Implement and verify the correction of `QUEUE`, using datatypes.

References

[1] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein. *Introduction to Algorithms*. MIT, second edition, 2001.