Verificação Formal de Software, 2019/20
FCUP

TP MC-5– Büchi Automata and Algorithm for Model Checking for LTL

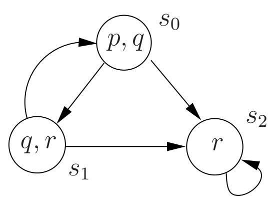
1. Consider the Büchi automaton

$$\mathcal{A}_\omega = (\Sigma = \{a, b\}, \{s_0, s_1, s_2\}, \{s_0\}, \delta, \{s_1, s_2\}),$$

where $\delta(s_0, a) = \{s_1, s_2\}$, $\delta(s_0, b) = \{s_1\}$, $\delta(s_1, a) = \{s_2\}$, $\delta(s_1, b) = \{s_1\}$, $\delta(s_2, a) = \delta(s_2, b) = \emptyset$. Determine $L(\mathcal{A}_\omega) \subseteq \Sigma^\omega$.

2. For each language, build a Büchi automaton that accepts it for $\Sigma = \{a, b, c\}$.
   (a) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains at least one infix } ab\}$;
   (b) $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains the infix } ab \text{ infinitely often}\}$.

3. Consider the system $\mathcal{M} = (S = \{s_0, s_1, s_2\}, \{s_0 \longrightarrow s_1, s_0 \longrightarrow s_2, s_1 \longrightarrow s_2, s_1 \longrightarrow s_0, s_2 \longrightarrow s_2\}, L(s_0) = \{p, q\}, L(s_1) = \{q, r\}, L(s_2) = \{r\})$, with $AP = \{p, q, r\}$



   (a) Obtain a Büchi automaton for $\mathcal{M}$ with initial state s_0, A_M.
   (b) For each formula φ below
      i. obtain for $\neg\varphi$ an equivalent formula ψ in negation normal form (i.e. with negations only for the atomic propositions)
      ii. obtain a Büchi automaton for the ψ
      iii. using the model checking algorithm determine if $\mathcal{M}, s_0 \models \varphi$, i.e. arguing that $L(A_M) \cap L(A_\varphi) = \emptyset$ or $L(A_M) \cap L(A_\varphi) \neq \emptyset$ giving a counterexample (a word that belongs to the intersection)

      1. $p \ \wedge \ q$
      2. $\mathrm{X}r$
      3. $\mathrm{X}(q \ \wedge \ r)$
      4. $\mathrm{G}\neg(p \wedge r)$
      5. $\mathrm{GF}p$

4. Let $AP = \{p, q\}$ and let $\Sigma = 2^{\{p,q\}}$.
   (a) Build a Büchi automaton B_1 for the formula $\neg\mathrm{G}(p \rightarrow \mathrm{XF}q)$.
   (b) Consider the transition system over AP, $T = (S = \{s_0, s_1, s_2, s_3\}, \{s_0 \longrightarrow s_1, s_1 \longrightarrow s_2, s_2 \longrightarrow s_0, s_2 \longrightarrow s_3, s_3 \longrightarrow s_0\}, L(s_0) = L(s_1) = L(s_2) = \{\}, L(s_3) = \{p, q\}$. Build a Büchi automaton for T, A_T.
   (c) Determine if $L(A_T) \cap L(B_1)$ is empty or not. Can you conclude that $T, s_0 \models \mathrm{G}(p \rightarrow \mathrm{XF}q)$ holds or not?

5. Consider the model $T = (S, \rightarrow, L)$ with

- $S = \{q_1, q_2, q_3, q_4\}$,
- $\rightarrow = \{q_1 \rightarrow q_2, q_2 \rightarrow q_2, q_3 \rightarrow q_1, q_3 \rightarrow q_2, q_3 \rightarrow q_4, q_4 \rightarrow q_3\}$,
- and $L(q_1) = \{\}, L(q_2) = \{b\}, L(q_3) = \{a\}, L(q_4) = \{a, b\}$).

(a) Represent (T, q_1) by a Büchi automaton

(b) For $\varphi = a\ U\ b$ determine a Büchi automaton $A_{\neg\varphi}$.

(c) Indicates the language accepted by $A_{\neg\varphi}$.

(d) Determine using the model checking algorithm if $T, q_1 \models \varphi$.

(e) Repeat the questions above for the formula $a \wedge Fb$ (recall that$F\varphi \equiv \mathsf{true} U\varphi$).)