Community Structure in Networks

CS224W: Analysis of Networks
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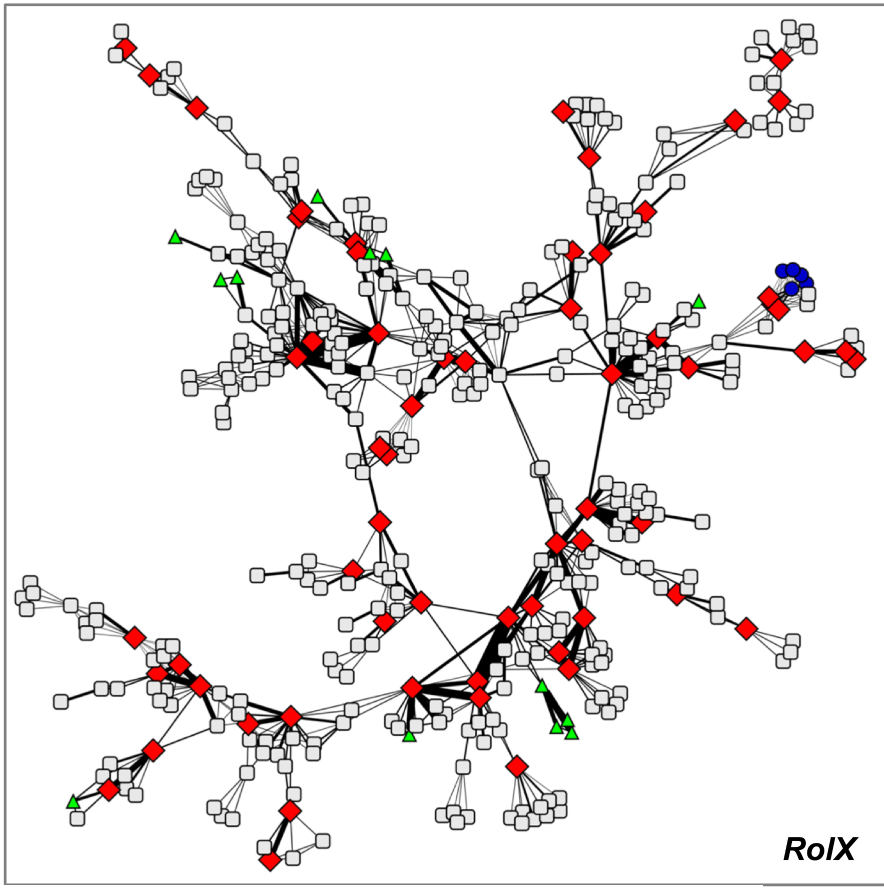
http://cs224w.stanford.edu

Roles and Communities: Example

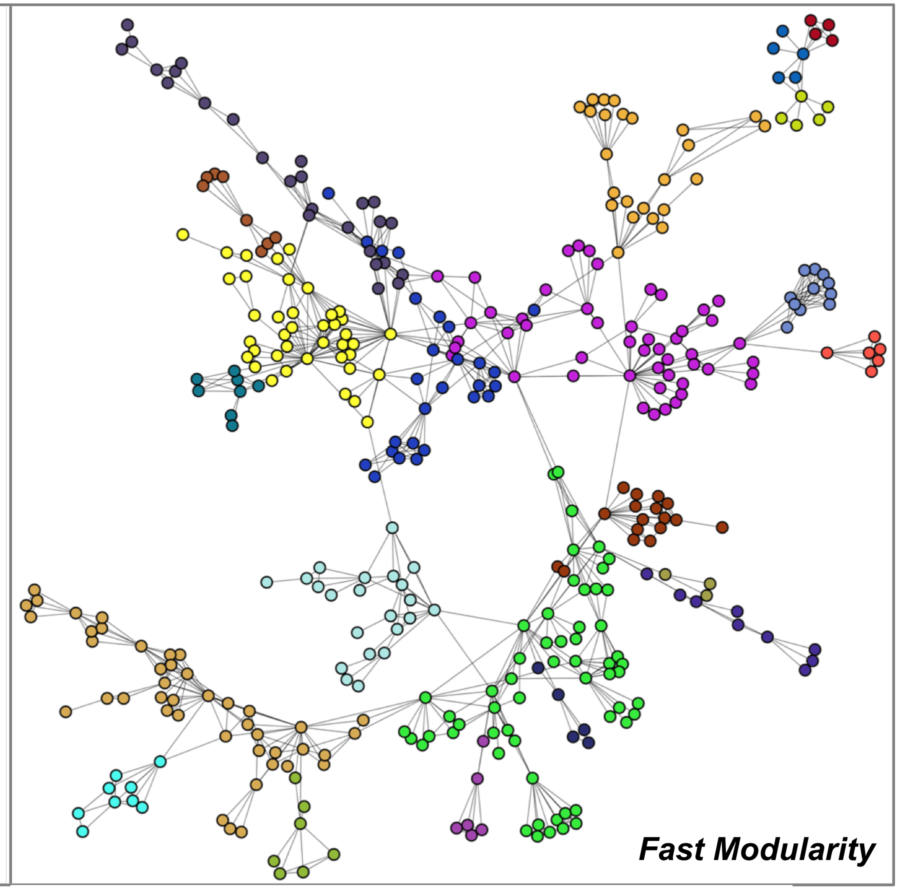
Roles



Henderson, *et al.*, KDD 2012

Nodes with different structural roles (connector node, bridge node, etc.)

Communities



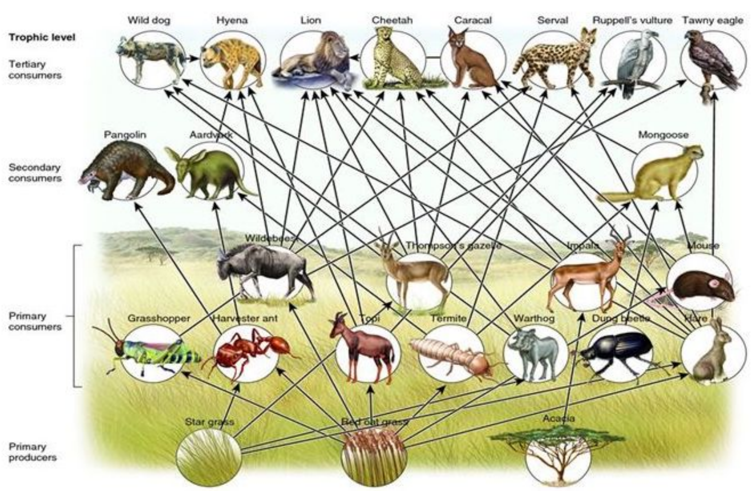
Clauset, *et al.*, Phys. Rev. E 2004

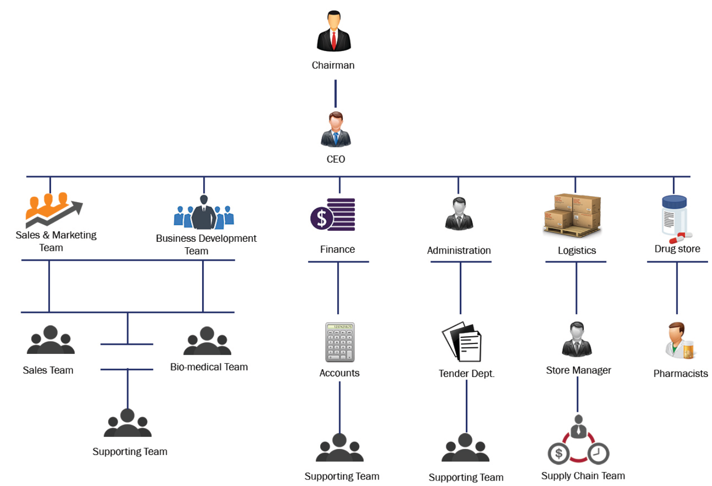
Nodes belonging to the same cluster/community

Structural Roles in Networks

What are Roles?

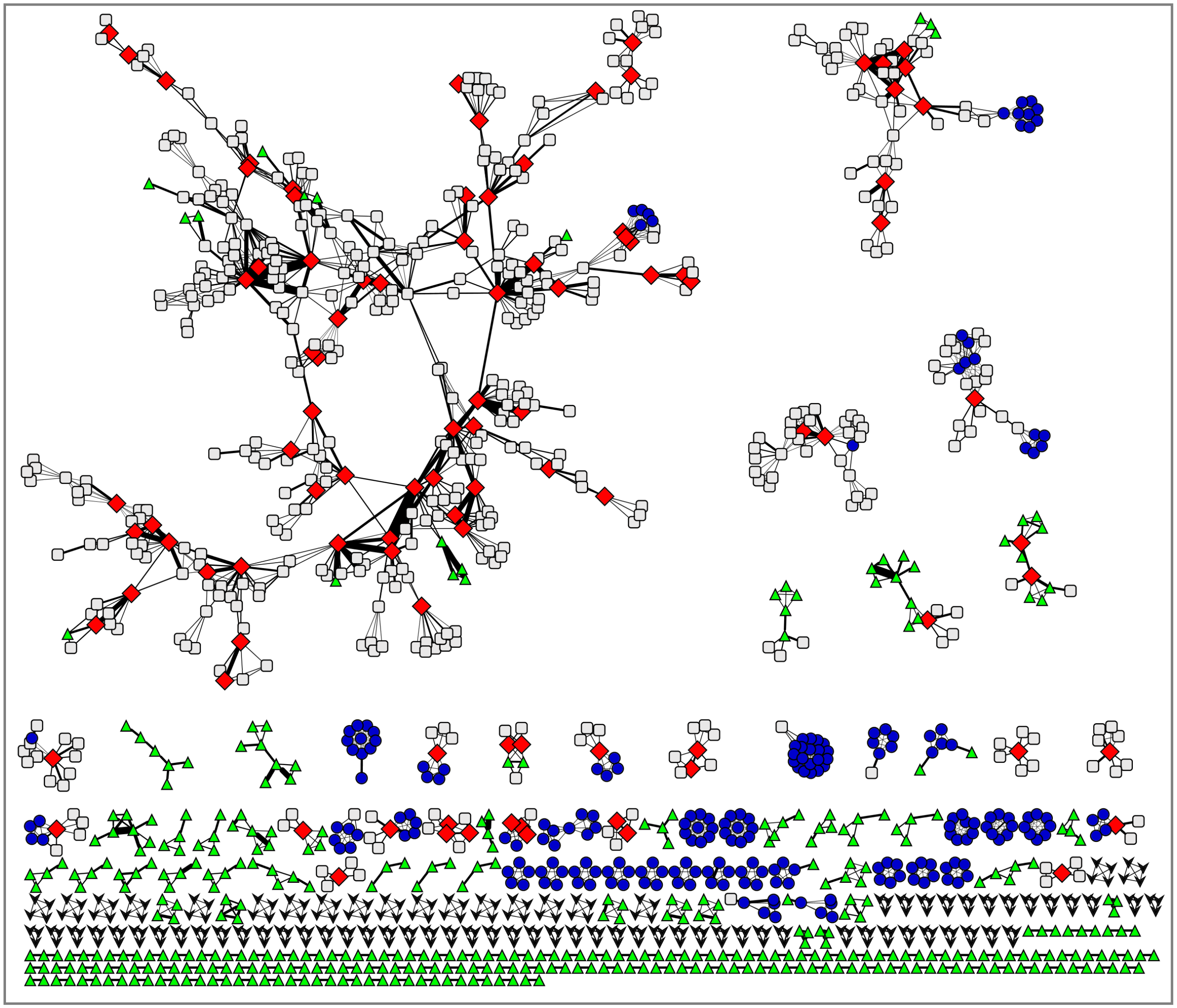
- **Roles are "functions" of nodes in a network:**
  - Roles of **species** in **ecosystems**
  - Roles of **individuals** in **companies**





- **Roles are measured by structural behaviors:**
  - Centers of stars
  - Members of cliques
  - Peripheral nodes, etc.

Example of Roles



◆ centers of stars
● members of cliques
▲ peripheral nodes

Network Science
Co-authorship network
[Newman 2006]

Roles versus Groups in Networks

- **Role**: A collection of nodes which have similar positions in a network:
  - Roles are based on the similarity of ties among subsets of nodes
  - Different from **community** (or cohesive subgroup)
    - Group is formed based on adjacency, proximity or reachability
    - This is typically adopted in current data mining

Nodes with the same role need not be in direct, or even indirect interaction with each other

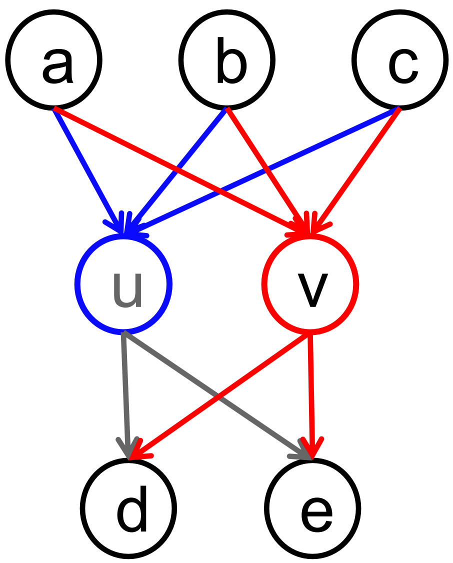
Roles and Communities

- **Roles**:
  - A group of nodes with similar structural properties
- **Communities:**
  - A group of nodes that are well-connected to each other
- Roles and communities **are complementary**

- Consider the social network of a CS Dept:
  - **Roles:** Faculty, Staff, Students
  - **Communities:** AI Lab, Info Lab, Theory Lab

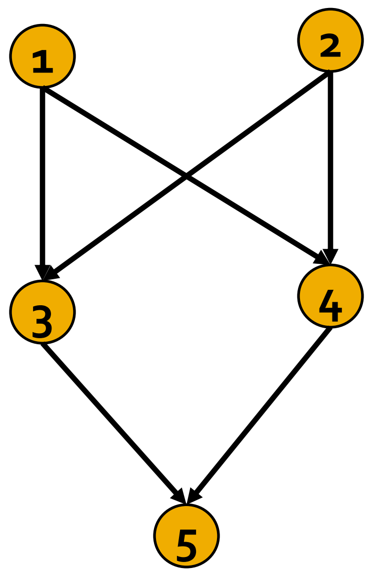
Roles: More Formally

- **Structural equivalence:** Nodes u and v are structurally equivalent if they have the same relationships **to all other nodes** [Lorrain & White 1971]
  - Structurally equivalent nodes are likely to be similar in other ways – *i.e.*, friendships in social networks



Structural Equivalence: Example

- Nodes u and v are **structurally equivalent:**
  - For all the other nodes k, node u has tie to k iff node v has tie to k
- **Example:**



Adjacency matrix

	1	2	3	4	5
1	-	0	1	1	0
2	0	-	1	1	0
3	0	0	-	0	1
4	0	0	0	-	1
5	0	0	0	0	-

- *E.g.*, nodes 3 and 4 are structurally equivalent

Discovering Structural Roles in Networks

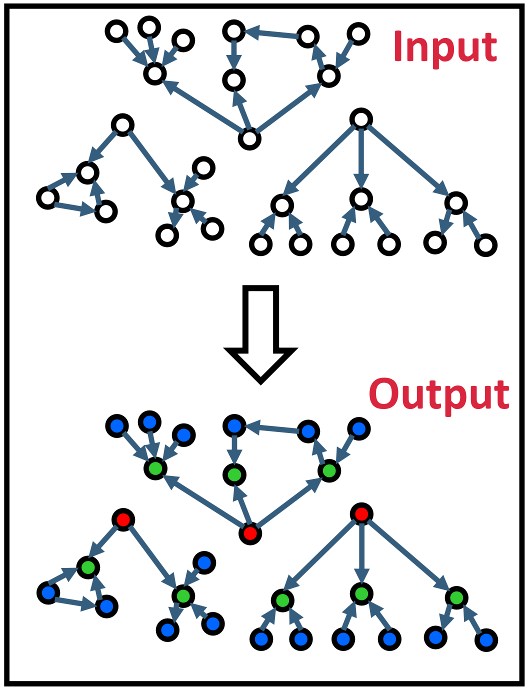
Why Are Roles Important?

Task	Example Application
Role query	Identify individuals with similar behavior to a known target
Role outliers	Identify individuals with unusual behavior
Role dynamics	Identify unusual changes in behavior
Identity resolution	Identify/de-anonymize, individuals in a new network
Role transfer	Use knowledge of one network to make predictions in another
Network comparison	Compute similarity of networks, determine compatibility for knowledge transfer

Structural Role Discovery Method

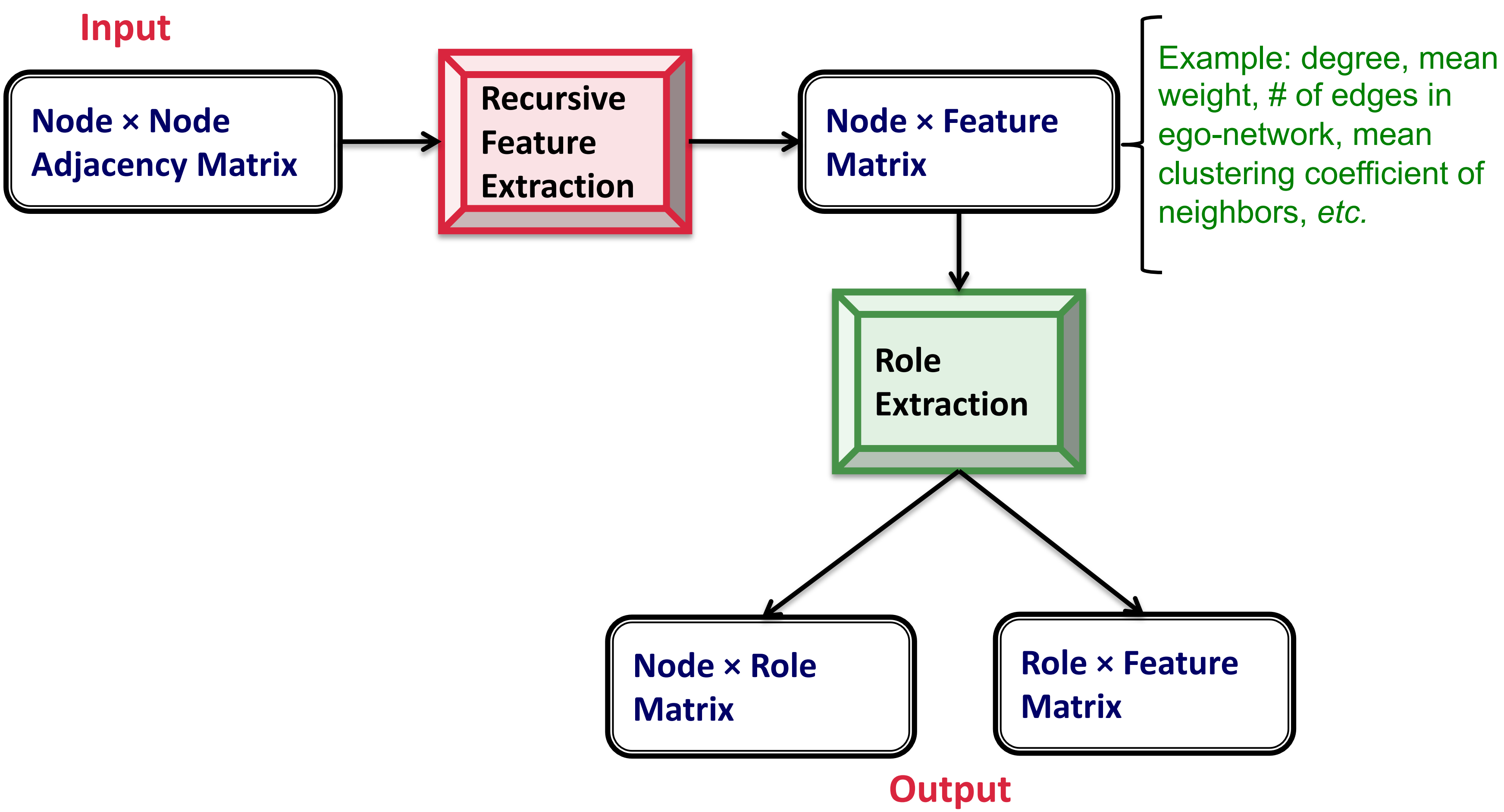
- **RolX:** Automatic discovery of nodes' structural roles in networks
  [Henderson, et al. 2011b]
  - Unsupervised learning approach
  - No prior knowledge required
  - Assigns a mixed-membership of roles to each node
  - Scales linearly in #(edges)

Role Discovery



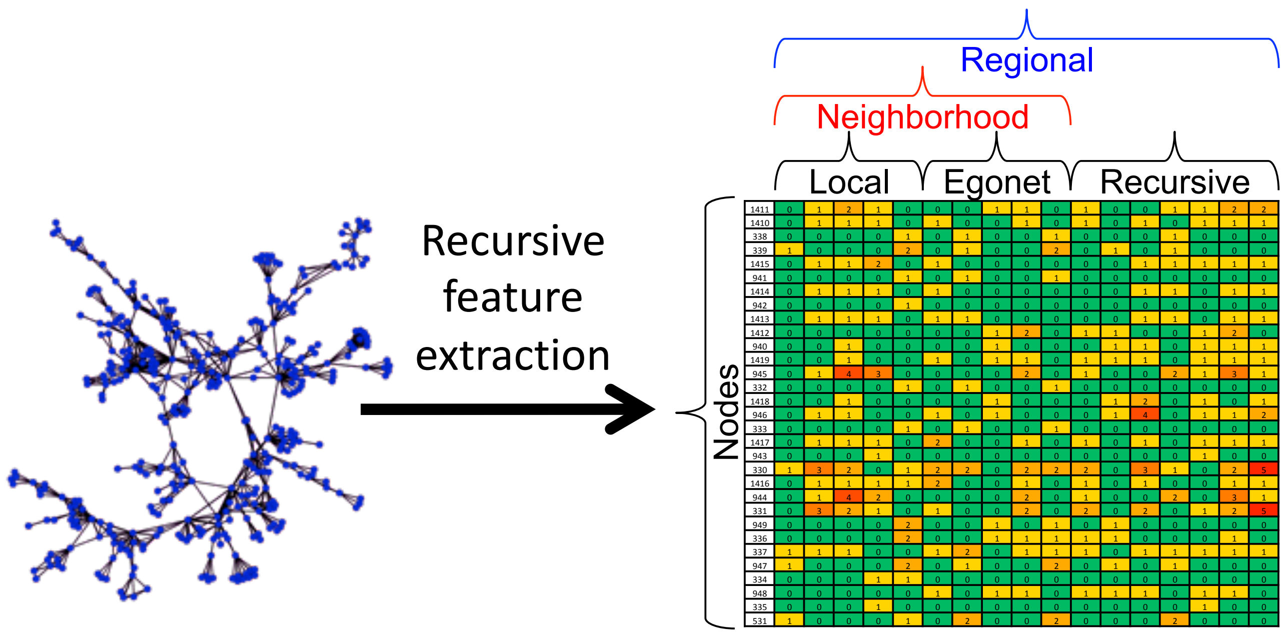
- ✓ Automated discovery
- ✓ Behavioral roles
- ✓ Roles generalize

RolX: Approach Overview



Recursive Feature Extraction

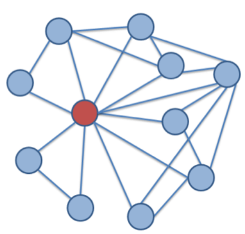
- **Recursive feature extraction** [Henderson, et al. 2011a] turns network connectivity into structural features



- **Neighborhood features:** What is a node's connectivity pattern?
- **Recursive features:** To what **kinds** of nodes is a node connected?

Recursive Feature Extraction

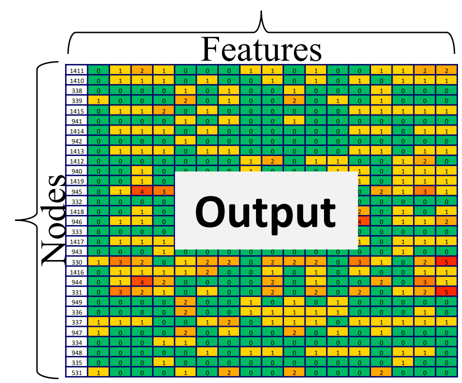
- **Idea:** Aggregate features of a node and use them to **generate new recursive features**
- **Base set of a node's neighborhood features:**
  - **Local features:** All measures of the node degree:
    - If network is directed, include in- and out-degree, total degree
    - If network is weighted, include weighted feature versions
  - **Egonetwork features:** Computed on the node's egonet:
    - **Egonet** includes the node, its neighbors, and any edges in the induced subgraph on these nodes
    - #(within-egonet edges), #(edges entering/leaving egonet)



Egonet for **red node**

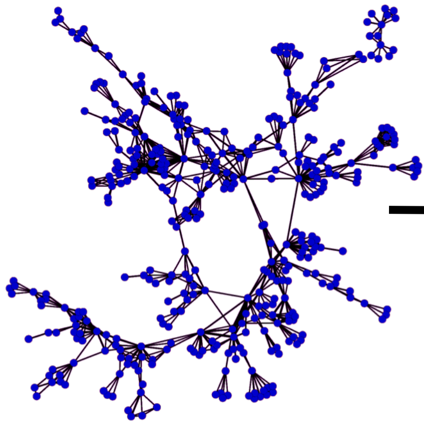
Recursive Feature Extraction

- Start with the base set of node features
- Use the **set of current node features** to generate **additional features**:
  - Two types of **aggregate functions:** means and sums
    - *E.g.,* mean value of "unweighted degree" feature among all neighbors of a node
    - Compute means and sums over all current features, including other recursive features
  - Repeat
- The number of possible recursive features **grows exponentially** with each recursive iteration:
  - Reduce the number of features using a **pruning technique:**
    - Look for pairs of features that are highly correlated
    - Eliminate one of the features whenever two features are correlated above a user-defined threshold

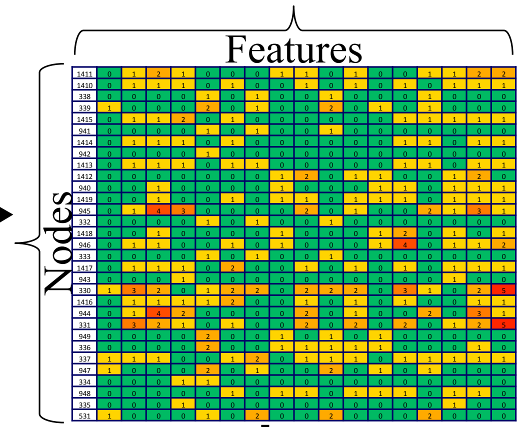


Role Extraction

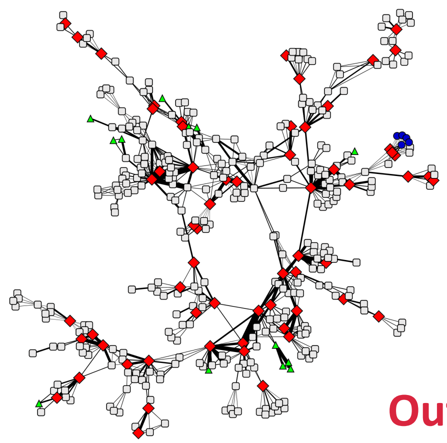
Input



Recursively extract features



1) Can compare nodes based on their structural similarity
2) Can cluster nodes to identify different structural roles



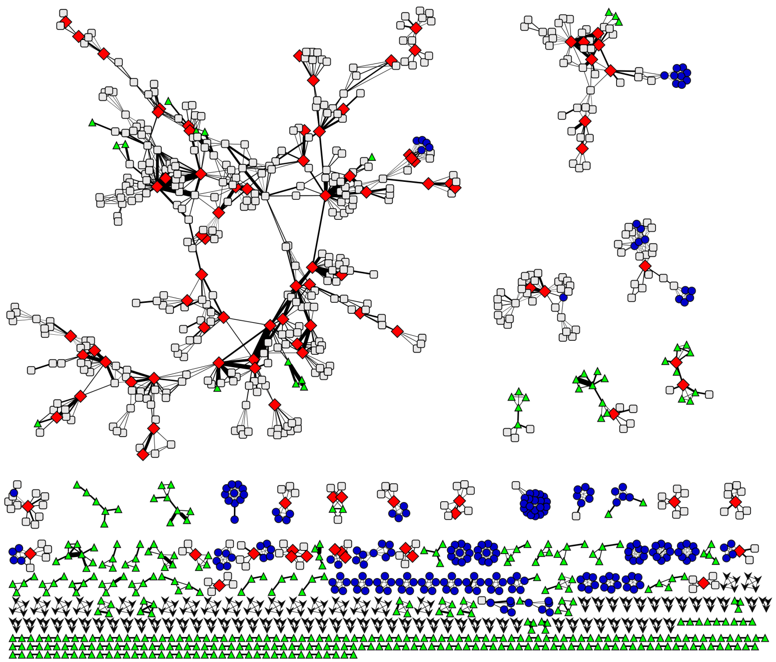
Output

e.g, RolX uses a clustering technique called non-negative matrix factorization

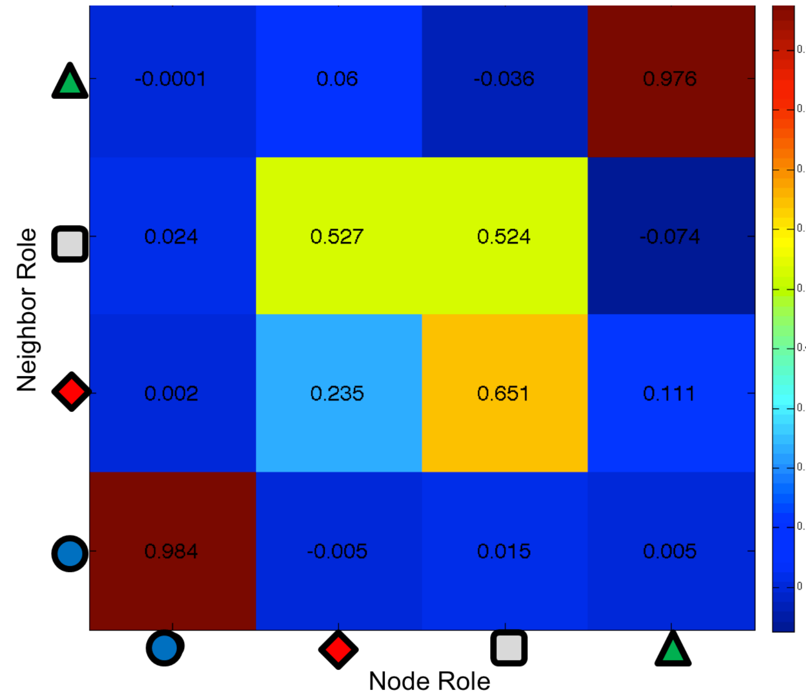
Application: Structural Similarity

- **Task:** Cluster nodes based on their structural similarity
- **Two networks:**
  - Network science co-authorship network:
    - Nodes: Network scientists; Edges: The number of co-authored papers
  - Political books co-purchasing network:
    - Nodes: Political books on Amazon; Edges: Frequent co-purchasing of books by the same buyers
- **Setup:** For each network:
  - Use RolX to assign each node a distribution over the set of discovered, structural roles
  - Determine similarity between nodes by comparing their role distributions

Structural Sim: Co-authorship Net



Role-colored graph: each node is colored by the primary role that RolX finds



Role affinity heat-map

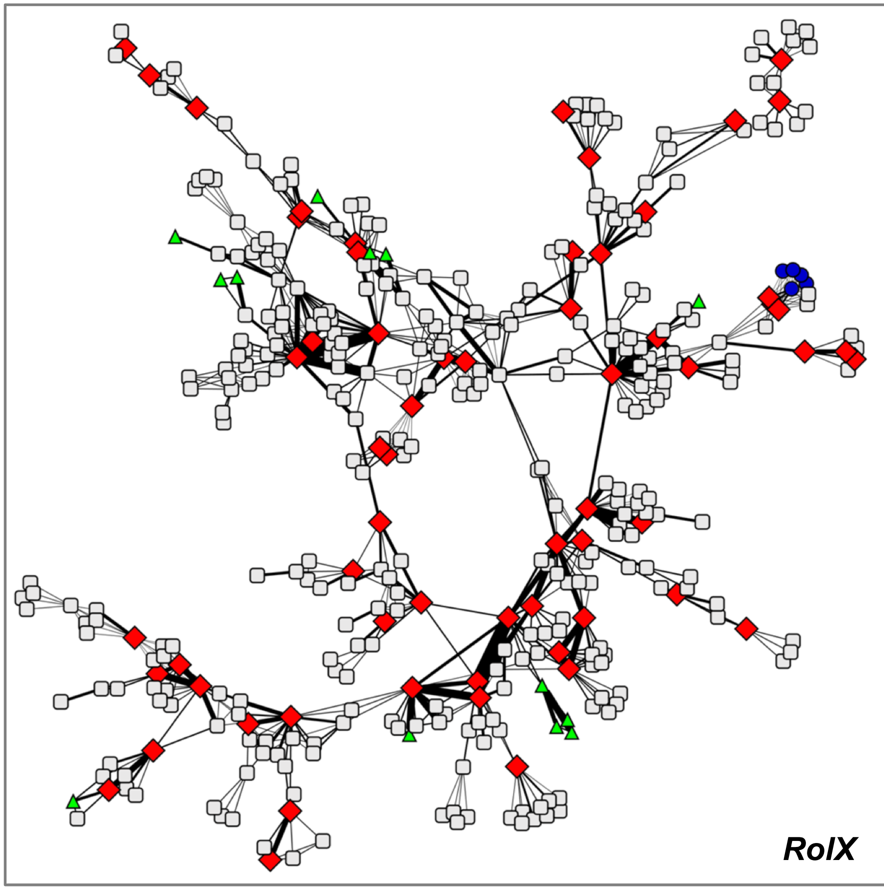
Making sense of roles:

- **Blue circle: Tightly knit**, nodes that participate in tightly-coupled groups
- **Red diamond: Bridge nodes**, that connect groups of nodes
- **Gray rectangle: Main-stream**, most of nodes, neither a clique, nor a chain
- **Green triangle: Pathy**, nodes that belong to elongated clusters

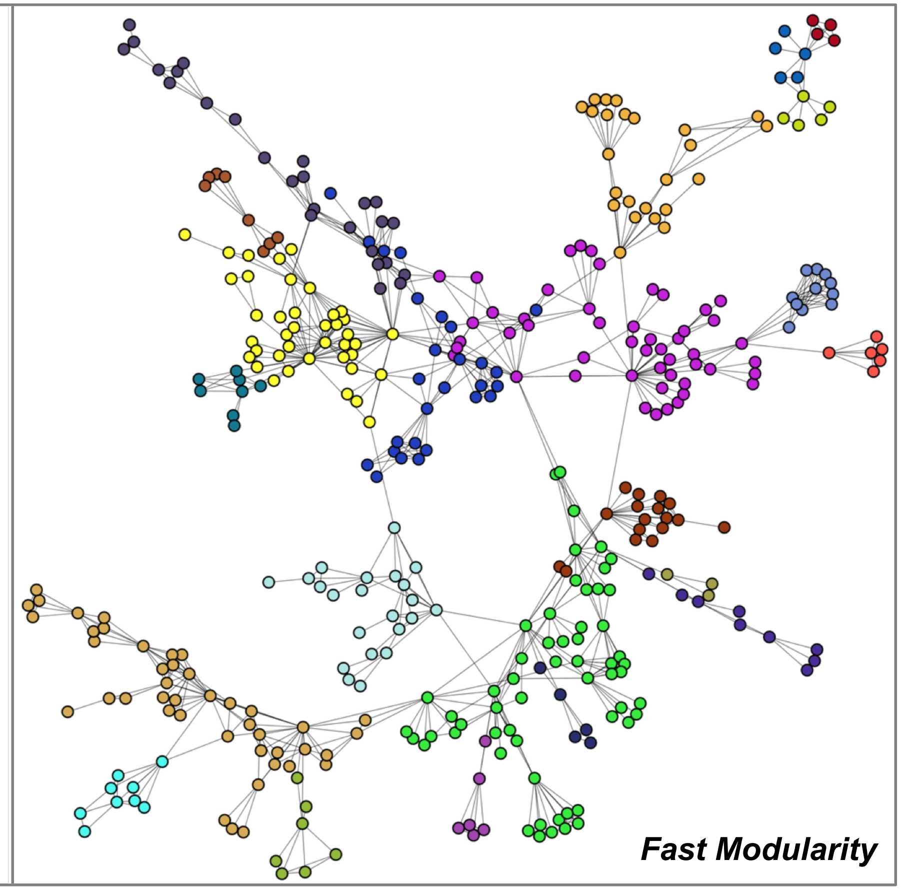
Community Structure in Networks

Roles and Communities: Example

Roles



Communities

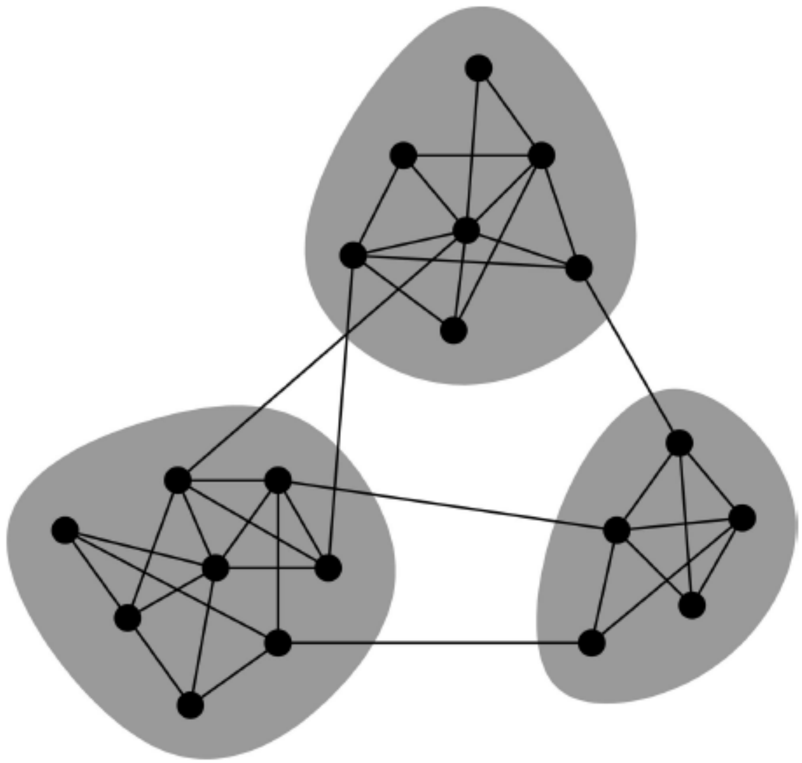


Henderson, *et al.,* KDD 2012

Clauset, *et al.,* Phys. Rev. E 2004

Networks & Communities

- We often think of networks "looking" like this:



- What led to such a conceptual picture?

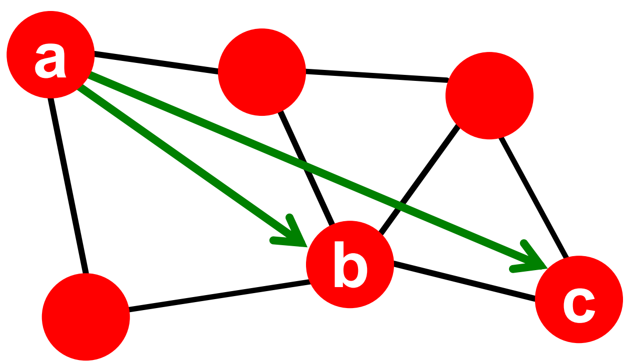
Networks: Flow of Information

- **How does information flow through the network?**
  - **What structurally distinct roles do nodes play?**
  - **What roles do different links ("short" vs. "long") play?**
- **How do people find out about new jobs?**
  - Mark Granovetter, part of his PhD in 1960s
  - People find the information through personal contacts
- **But:** Contacts were often **acquaintances** rather than close friends
  - **This is surprising:** One would expect your friends to help you out more than casual acquaintances
- **Why is it that acquaintances are most helpful?**

Granovetter's Answer

[Granovetter '73]

- **Two perspectives on friendships:**
  - **Structural:** Friendships span different parts of the network
  - **Interpersonal:** Friendship between two people is either **strong** or **weak**
- **Structural role: Triadic Closure**

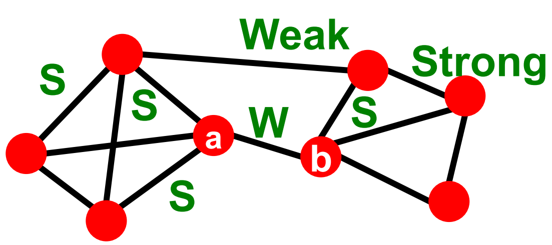


Which edge is more likely, a-b or a-c?

If two people in a network have a friend in common, then there is an increased likelihood they will become friends themselves.

Granovetter's Explanation

- **Granovetter makes a connection between social and structural role of an edge**
- **First point: Structure**
  - Structurally embedded edges are also socially strong
  - Long-range edges spanning different parts of the network are socially weak
- **Second point: Information**
  - Long-range edges allow you to gather information from different parts of the network and get a job
  - Structurally embedded edges are heavily redundant in terms of information access

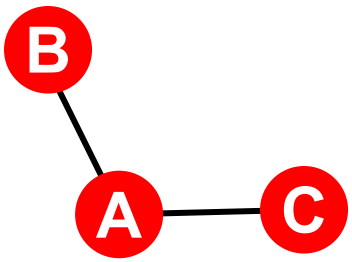


Triadic Closure

- **Triadic closure == High clustering coefficient**

Reasons for triadic closure:

- If B and C have a friend A in common, then:
  - B is **more likely to meet** C
    - (since they both spend time with A)
  - B and C **trust** each other
    - (since they have a friend in common)
  - A has **incentive** to bring B and C together
    - (since it is hard for A to maintain two disjoint relationships)



- **Empirical study by Bearman and Moody:**
  - Teenage girls with low clustering coefficient are more likely to contemplate suicide

Tie strength in real data

- **For many years Granovetter's theory was not tested**
- But, today we have large who-talks-to-whom graphs:
  - Email, Messenger, Cell phones, Facebook
- **Onnela et al. 2007:**
  - Cell-phone network of 20% of country's population
  - **Edge strength:** # phone calls

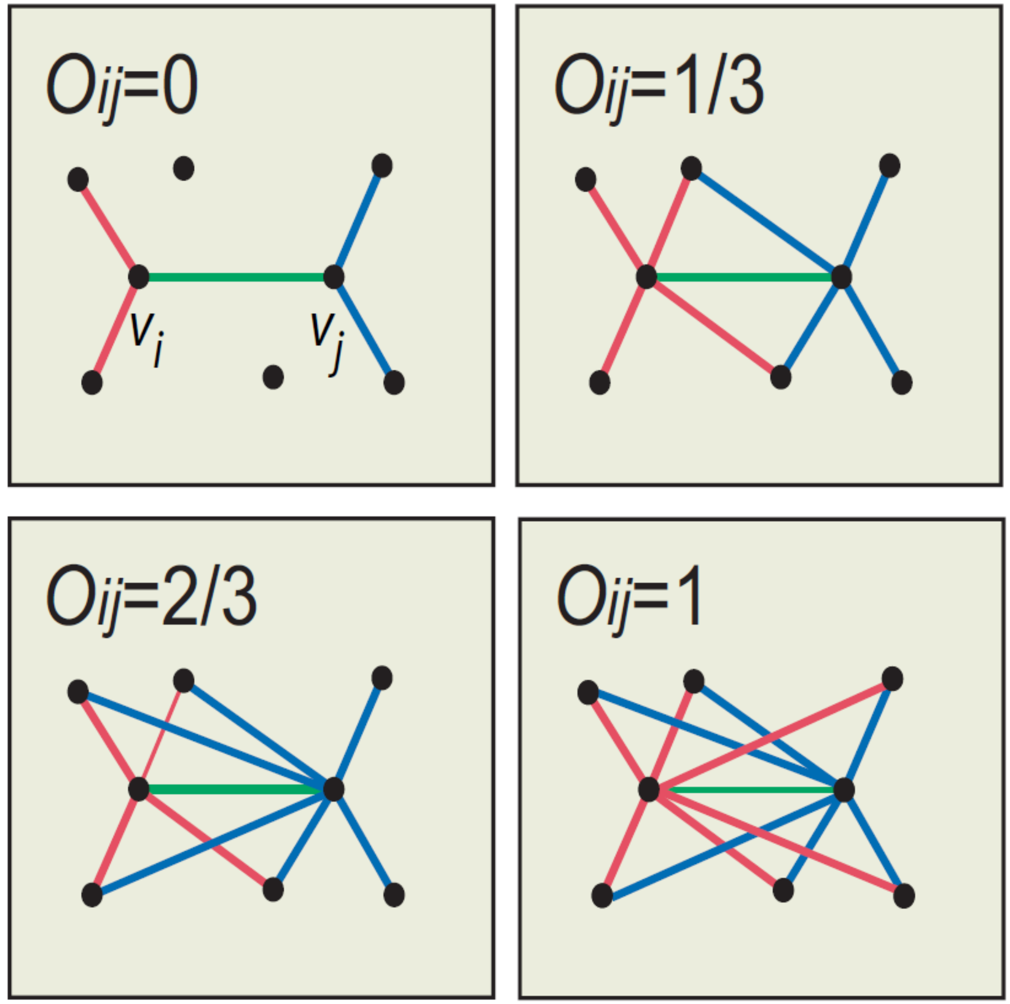
Neighborhood Overlap

- **Edge overlap:**

$$O_{ij} = \frac{|N(i) \cap N(j)|}{|N(i) \cup N(j)|}$$

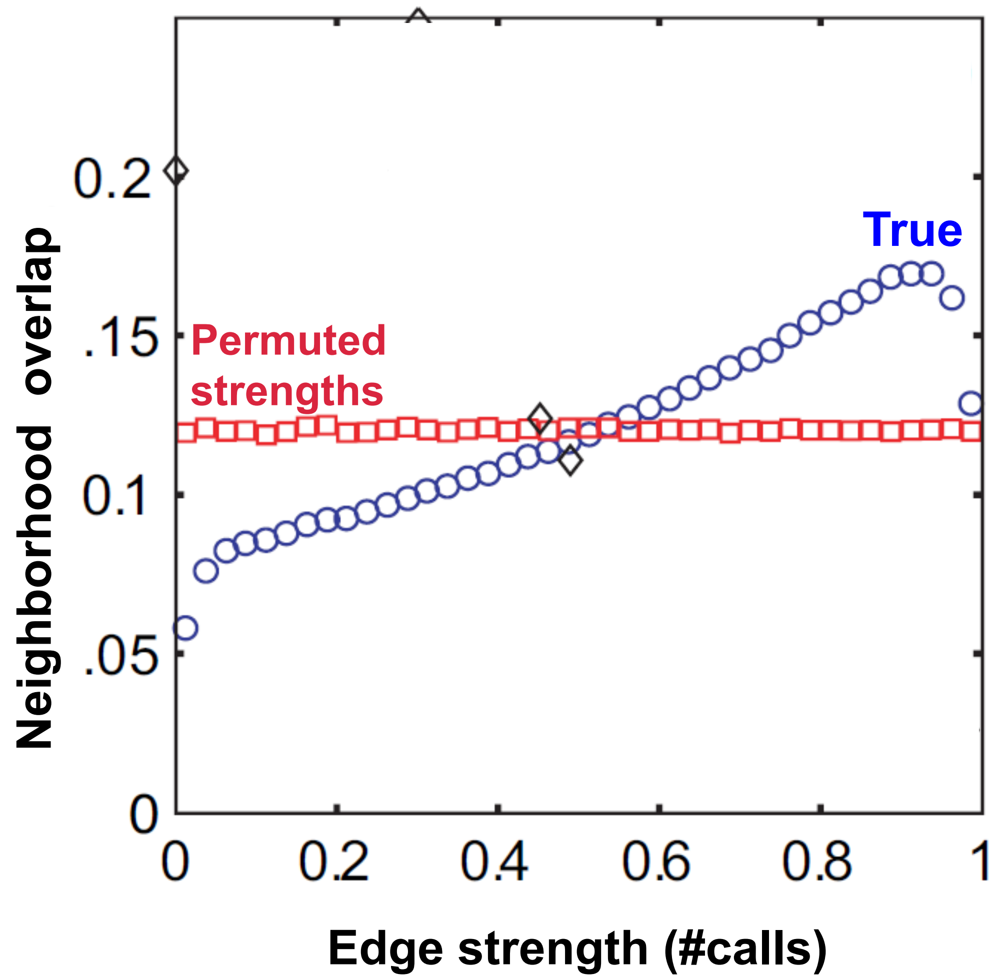
  - $N(i)$ … a set of neighbors of node i

- **Overlap $=$ 0** when an edge is a **local bridge**



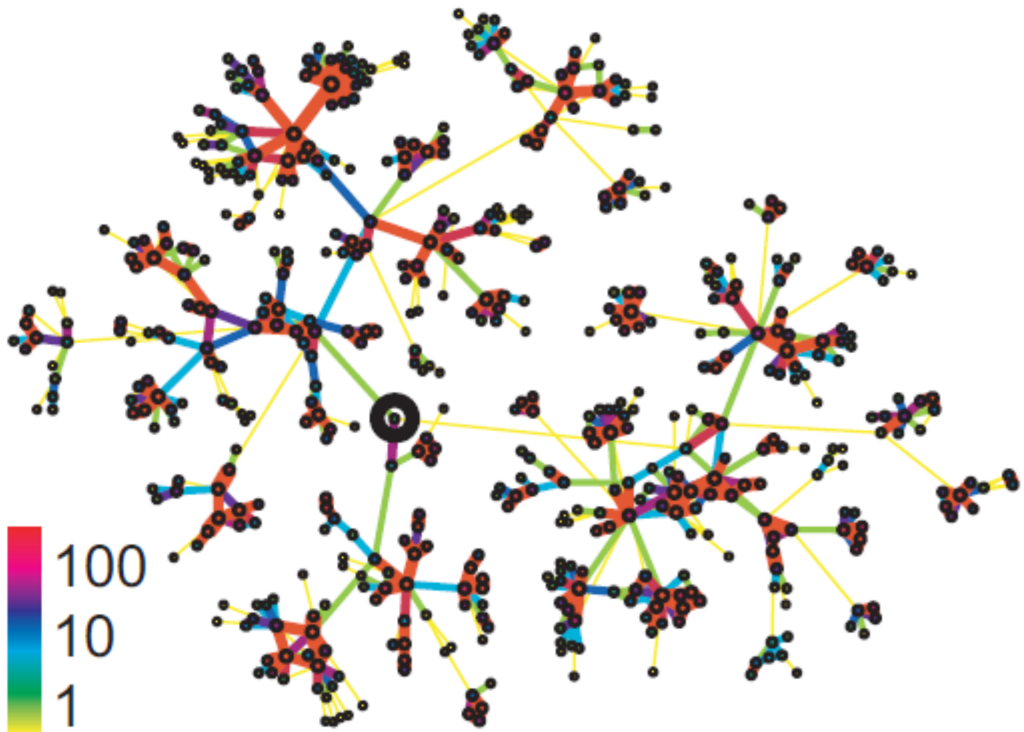
Phones: Edge Overlap vs. Strength

- Cell-phone network
- **Observation:**
  - Highly used links have high overlap!



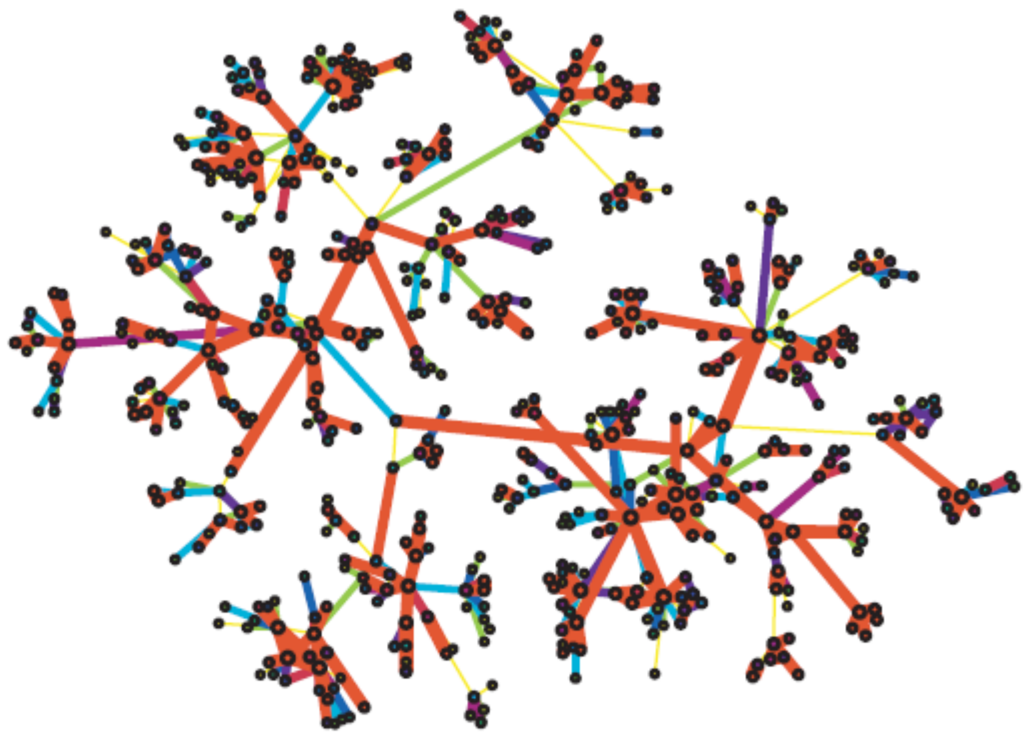
- **Legend:**
  - **True:** The data
  - **Permuted strengths:** Keep the network structure but randomly reassign edge strengths

Real Network, Real Tie Strengths



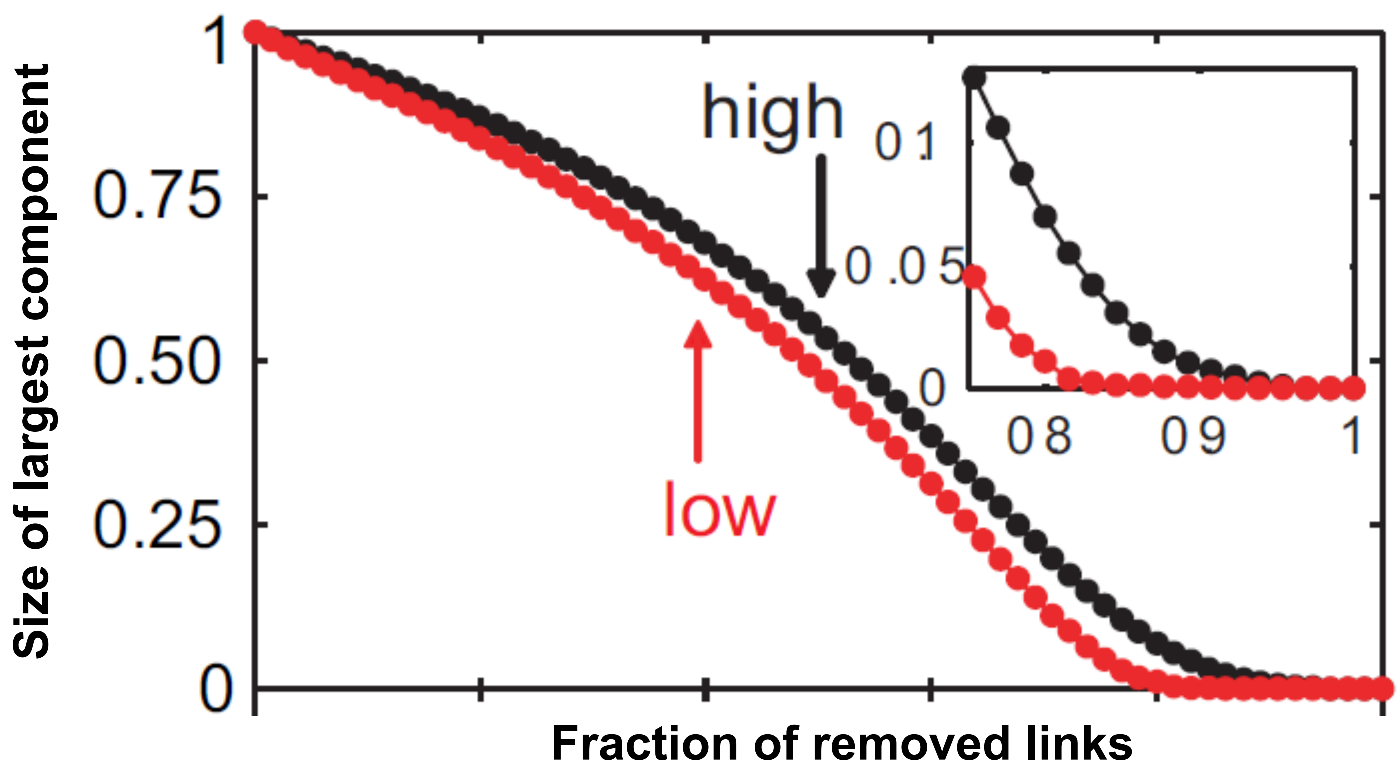
- **Real edge strengths in mobile call graph**
  - Strong ties are more embedded (have higher overlap)

Real Net, Permuted Tie Strengths



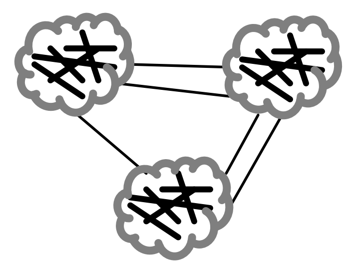
- **Same network, same set of edge strengths** but now **strengths are randomly shuffled**

Link Removal by Strength



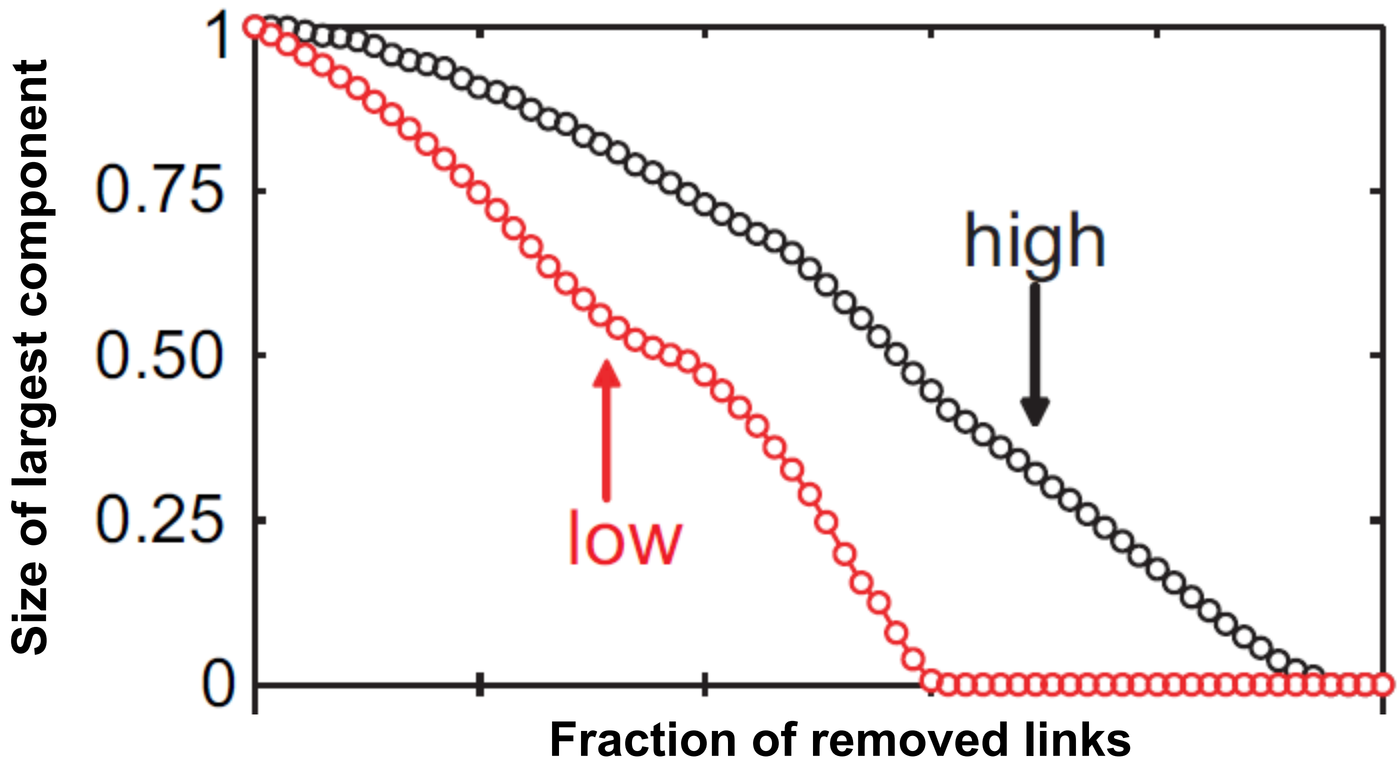
Low disconnects the network sooner

- Removing links by **strength (#calls)**
  - Low to high
  - High to low



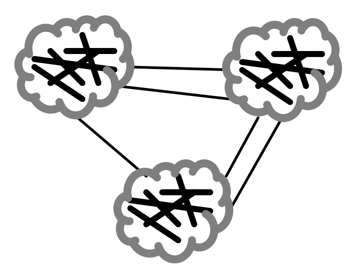
Conceptual picture of network structure

Link Removal by Overlap



Low disconnects the network sooner

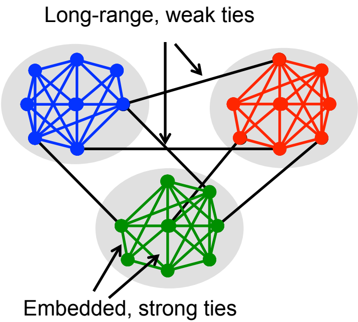
- Removing links based on **overlap**
  - Low to high
  - High to low

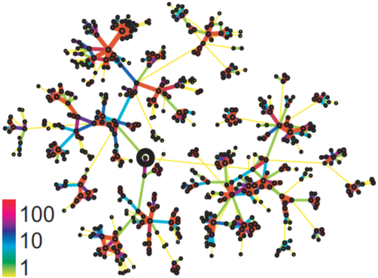


Conceptual picture of network structure

Closing the loop

- We often think of (social) networks as having the following structure



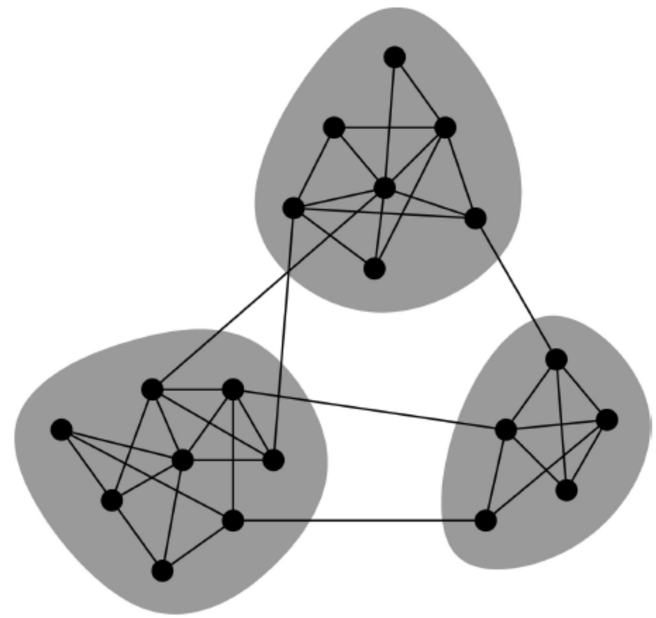


- Conceptual picture supported by Granovetter's strength of weak ties

Network Communities

Network Communities

- Granovetter's theory suggest that networks are composed of **tightly connected sets of nodes**

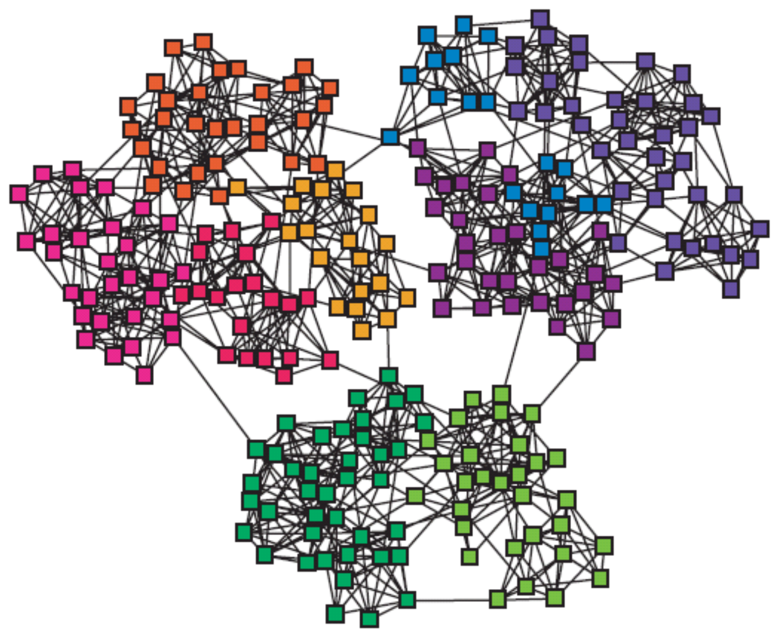


Communities, clusters, groups, modules

- **Network communities:**
  - Sets of nodes with **lots** of **internal** connections and **few external** ones (to the rest of the network).

Finding Network Communities

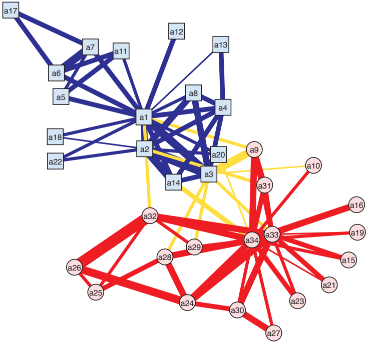
- **How to automatically find such densely connected groups of nodes?**
- Ideally such automatically detected clusters would then correspond to real groups
- **For example:**



Communities, clusters, groups, modules

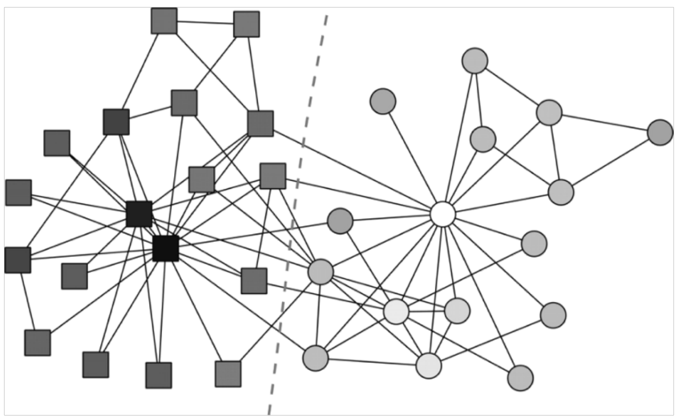
Zachary's karate club

- Social interactions among members of a karate club in the 70s

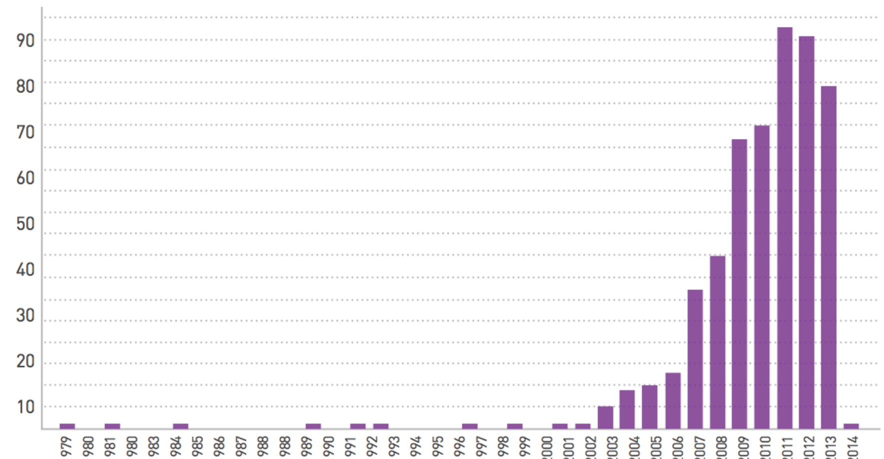


- Zachary witnessed the club split in two during his study
  - $\Rightarrow$ Toy network, yet canonical for community detection algorithms
  - $\Rightarrow$ Offers "ground truth" community membership (a rare luxury)

Zachary’s Karate Club



Citation history
of the Zachary’s Karate club paper



W.W. Zachary, *J. Anthropol. Res.* 33:452-473 (1977).
A.-L. Barabási, *Network Science: Communities.*

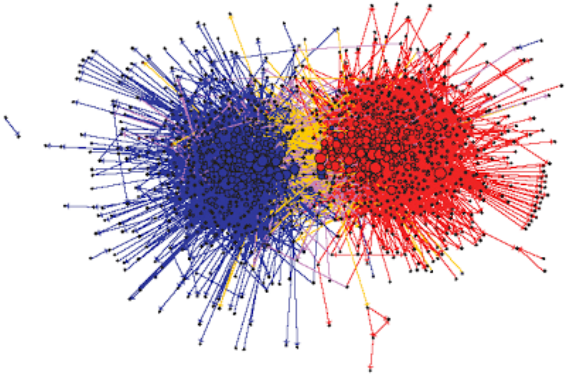
The first scientist at any conference on networks who uses Zachary's karate club as an example is inducted into the Zachary Karate Club Club, and awarded a prize.

Chris Moore (9 May 2013).
Mason Porter (NetSci, June 2013).
Yong-Year Ahn (Oxford University, July 2013)
Marián Boguñá (ECCS, September 2013).
Mark Newman (Netsci, June 2014)

http://networkkarate.tumblr.com/)

Political blogs

- The political blogosphere for the US 2004 presidential election

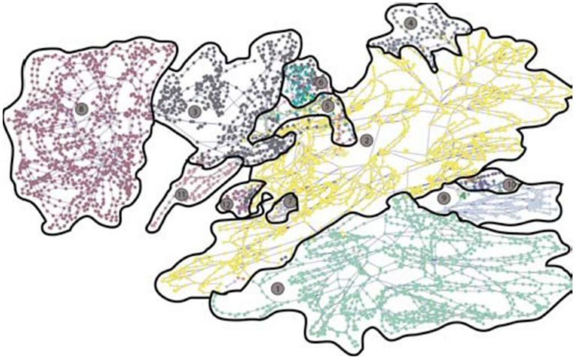


- Community structure of liberal and conservative blogs is apparent

  ⇒ People have a stronger tendency to interact with "equals"

Electrical power grid

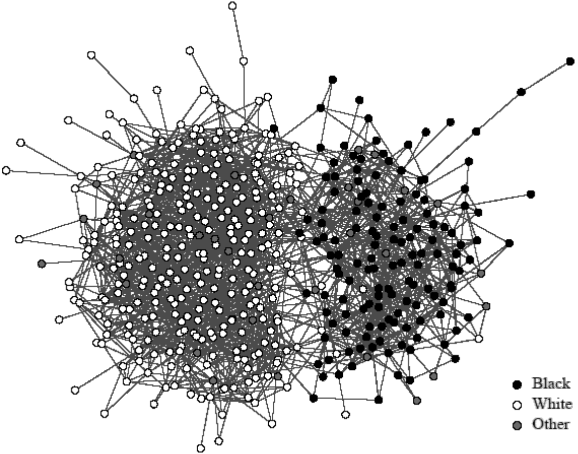
- Split power network into areas with minimum inter-area interactions



- **Applications:**
  - Decide control areas for distributed power system state estimation
  - Parallel computation of power flow
  - Controlled islanding to prevent spreading of blackouts

High-school students

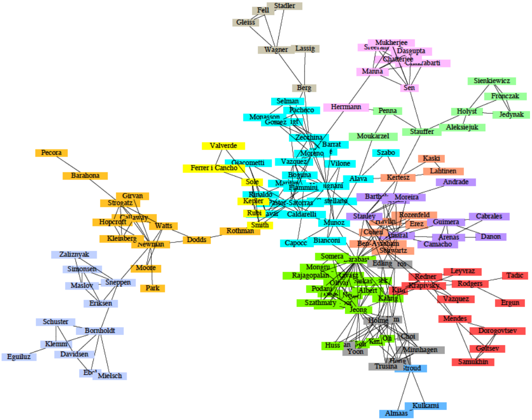
- Network of social interactions among high-school students



- Strong assortative mixing, with race as latent characteristic

Physicists working on Network Science

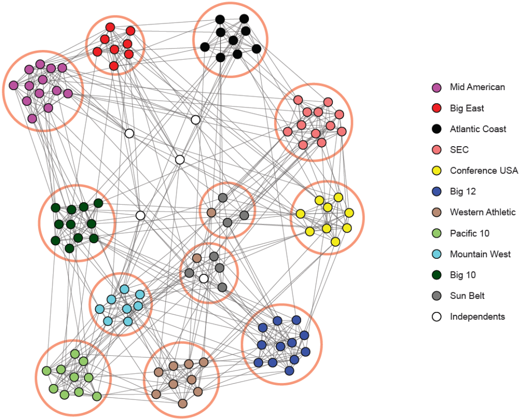
- Coauthorship network of physicists publishing networks' research



- Tightly-knit subgroups are evident from the network structure

College football

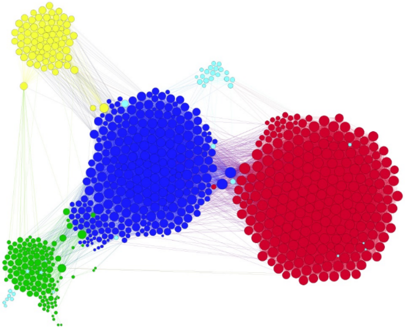
- Vertices are NCAA football teams, edges are games during Fall'00



- Communities are the NCAA conferences and independent teams

Facebook friendships

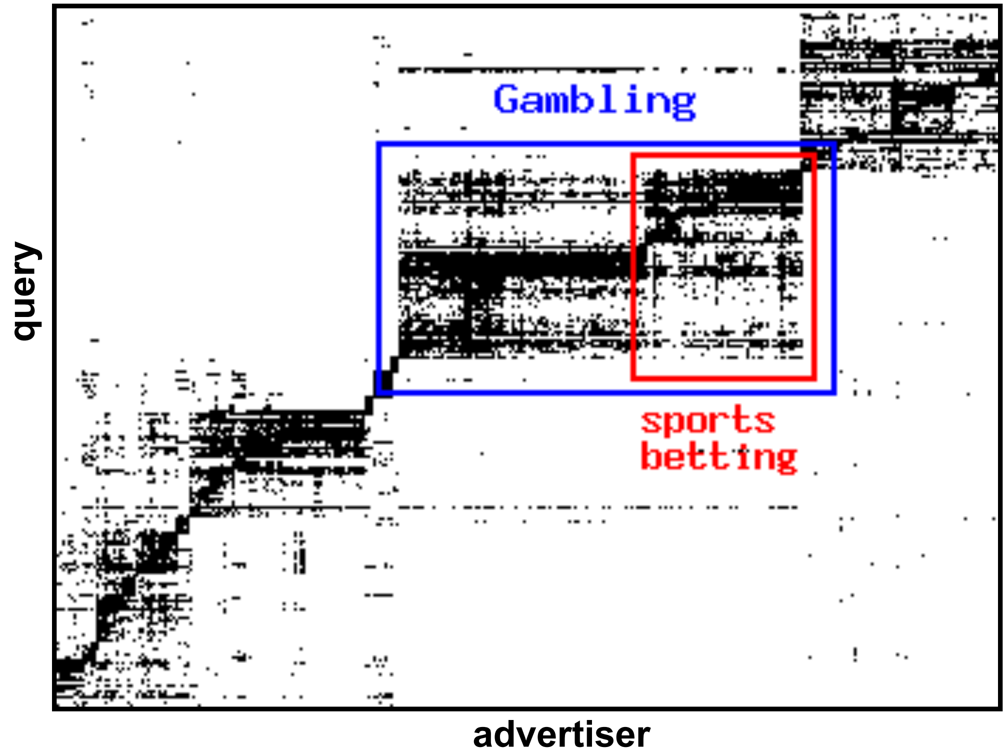
- Facebook egonet with 744 vertices and 30K edges



- Asked "ego" to identify social circles to which friends belong
  - $\Rightarrow$ Company, high-school, basketball club, squash club, family

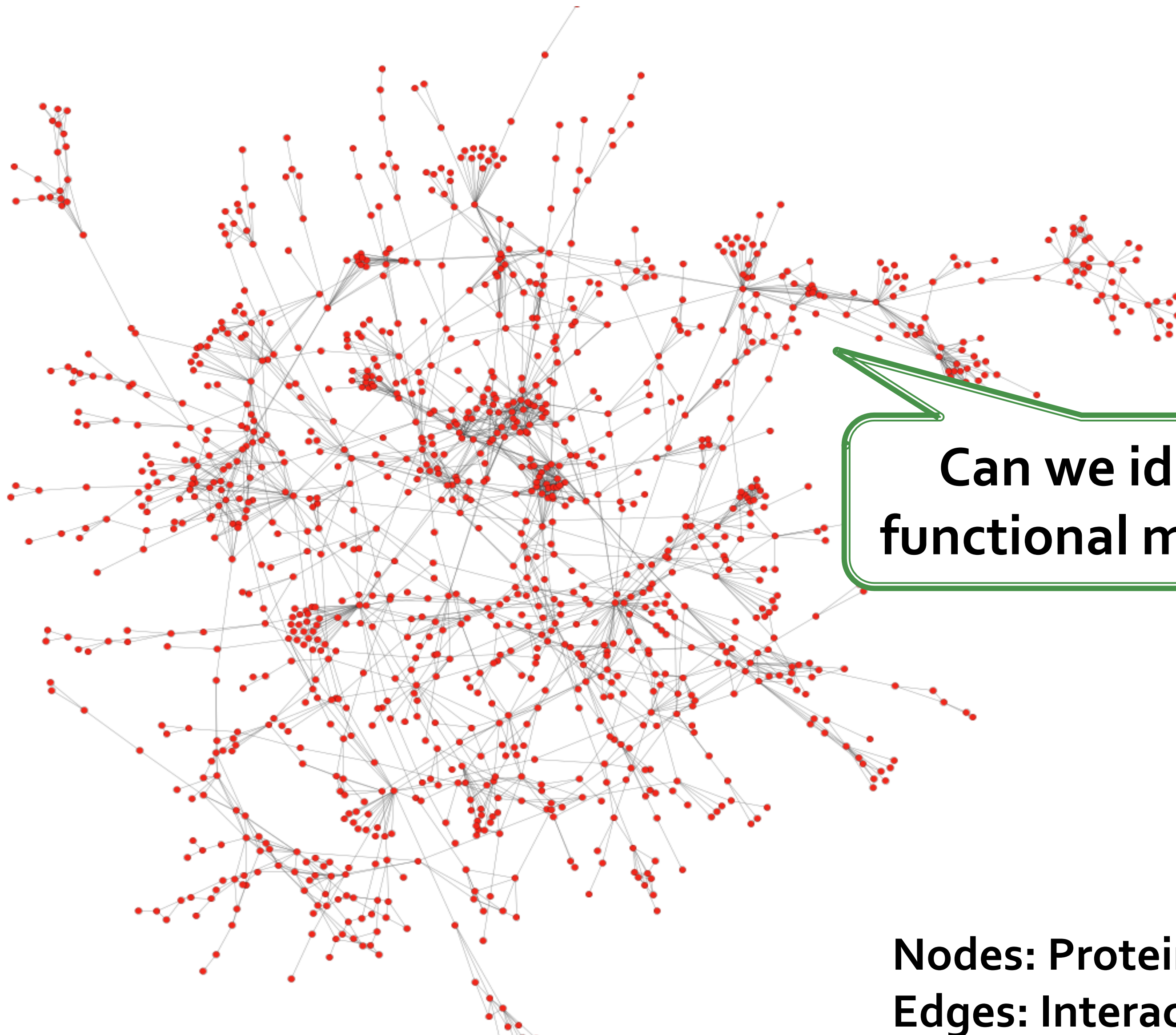
Micro-Markets in Sponsored Search

Find micro-markets by partitioning the "query-to-advertiser" graph in web search:



Nodes: advertisers and queries/keywords; Edges: Advertiser advertising on a keyword.

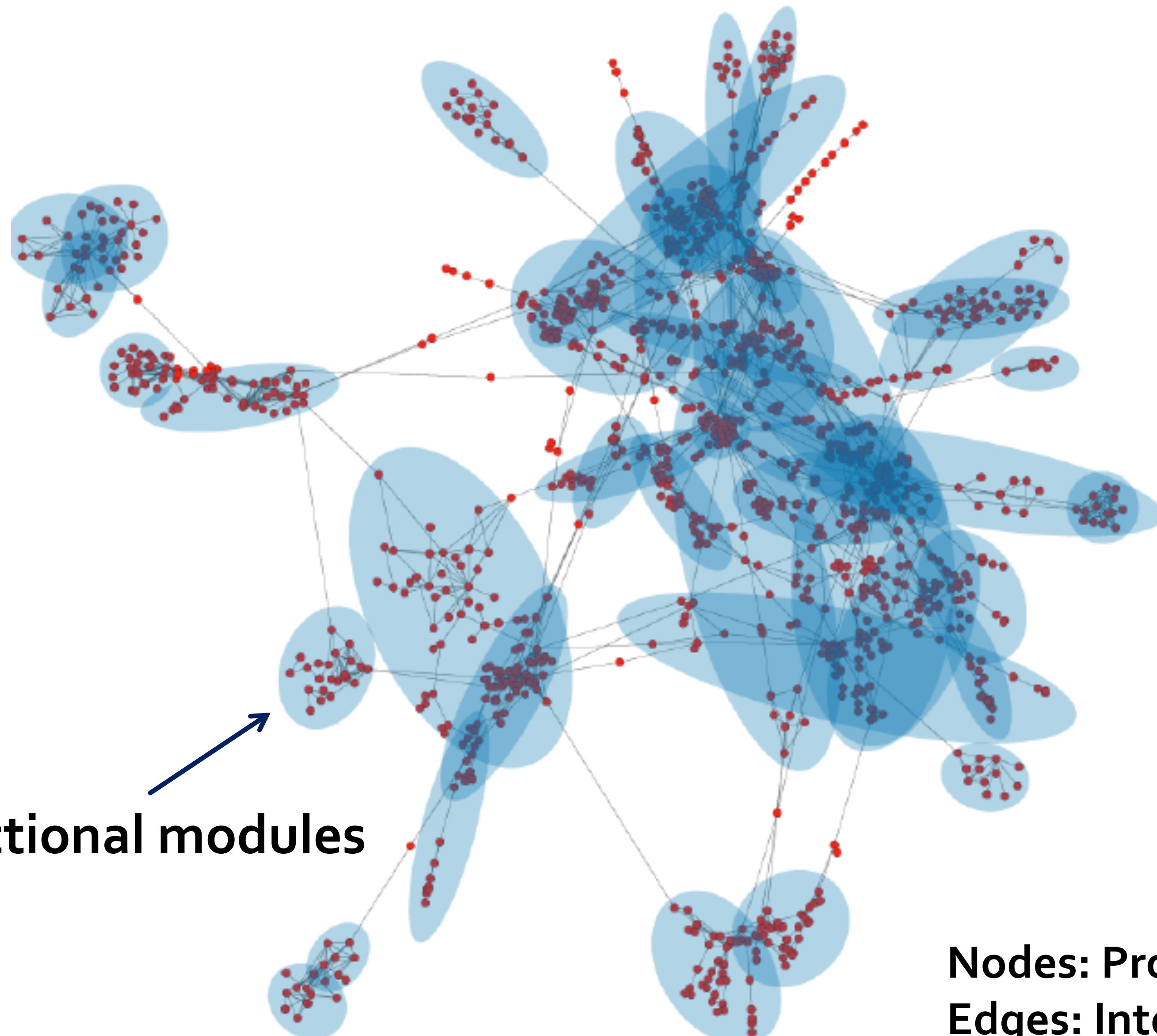
Protein-Protein Interactions



Can we identify functional modules?

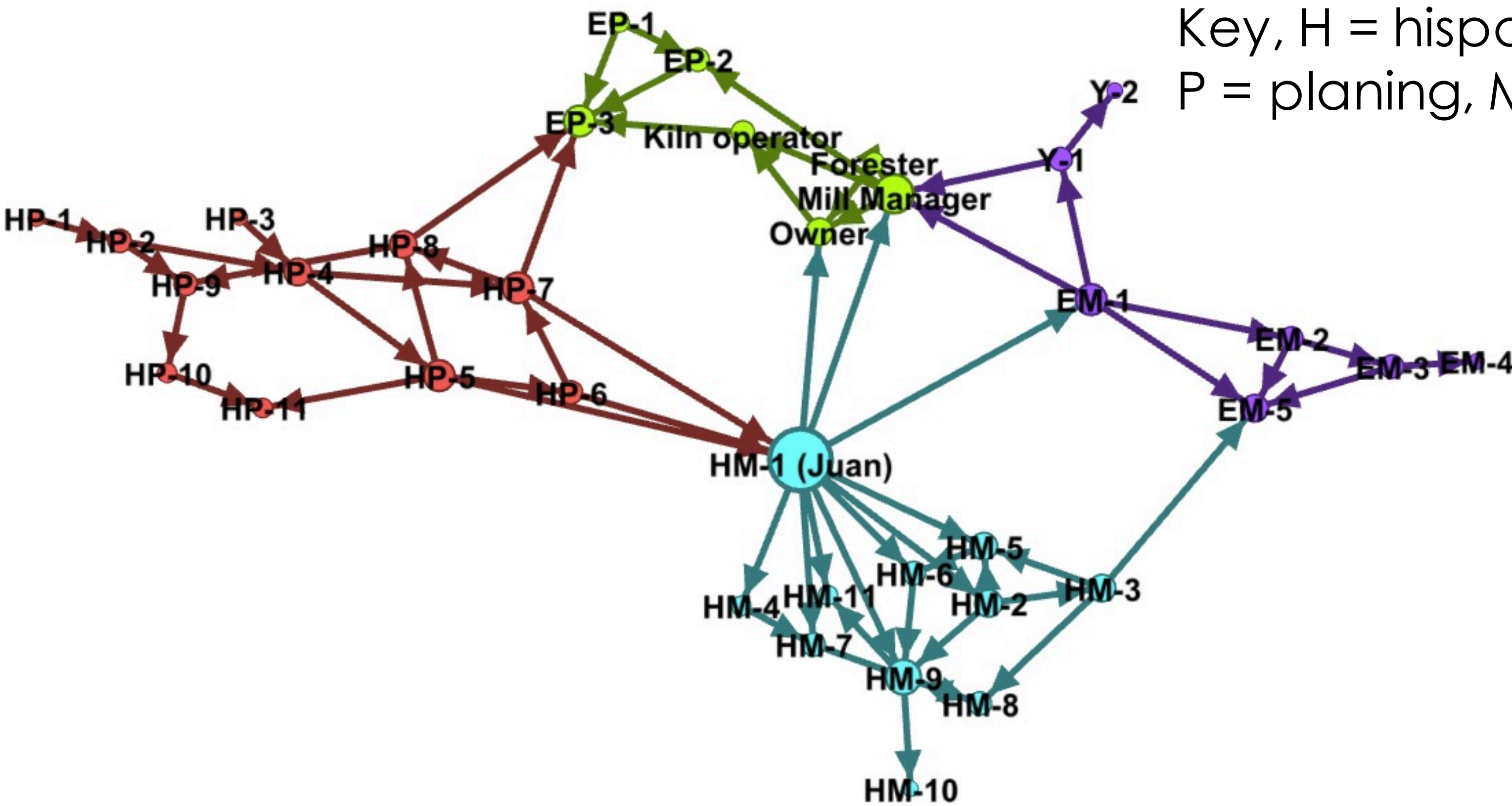
Nodes: Proteins
Edges: Interactions

Protein-Protein Interactions



Nodes: Proteins
Edges: Interactions

Why look for community structure?



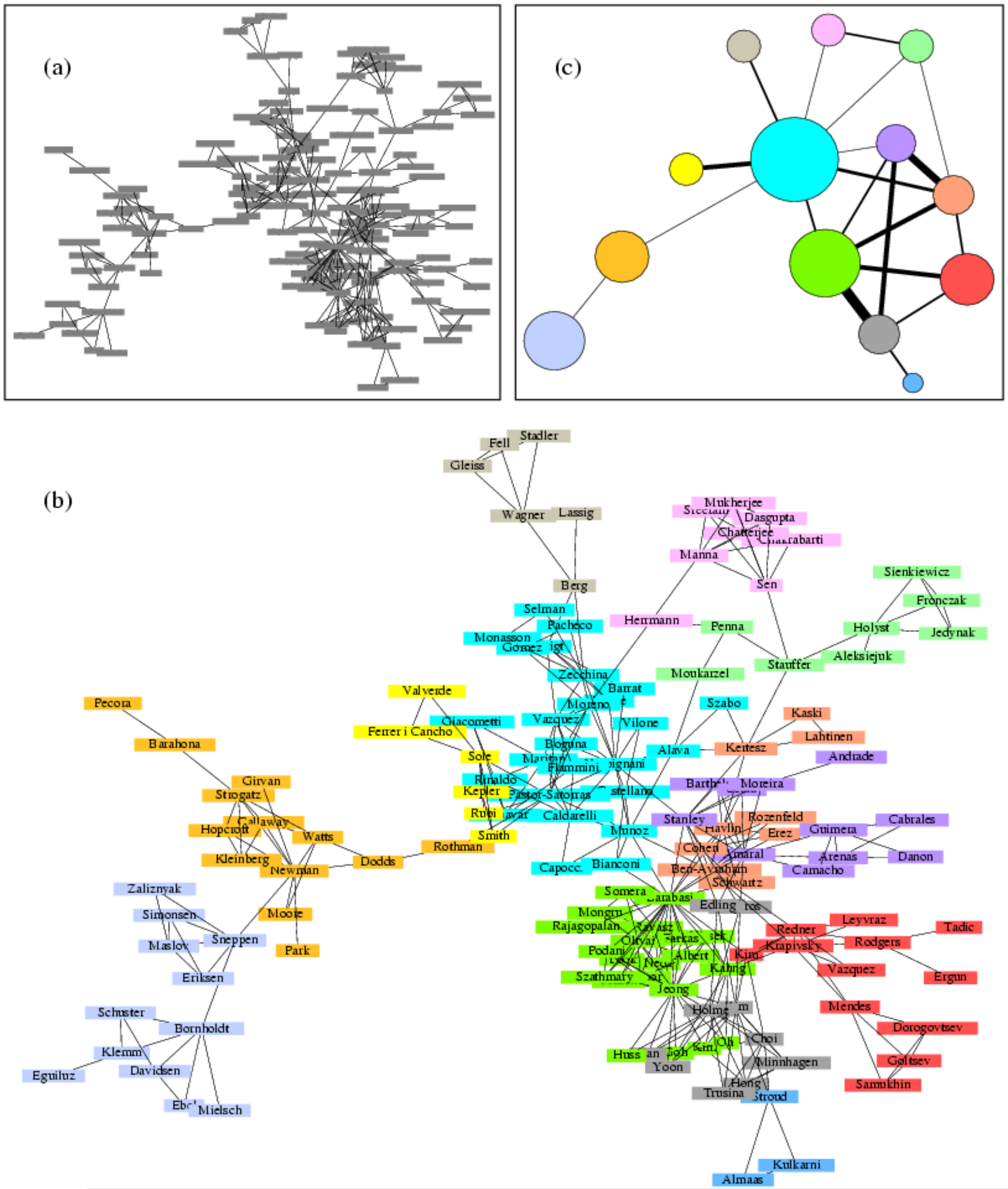
Key, H = hispanic, E = english
P = planing, M = milling, Y = yard

- The management at the sawmill was having difficulty persuading the workers to adopt a new plan, even though everyone would benefit. In particular the Hispanic workers (H) were reluctant to agree. The management called in a sociologist who mapped out who talked to whom regularly. Then they suggested that the management talk to Juan and have him talk to the Hispanic workers. It was a success, promptly everyone was on board with the new plan. Why?

Sawmill network: source Exploratory Social Network Analysis with Pajek

Why do it: gain understanding

- Gain understanding of networks
  - Discover communities of practice
  - Measure isolation of groups
  - Understand opinion dynamics / adoption

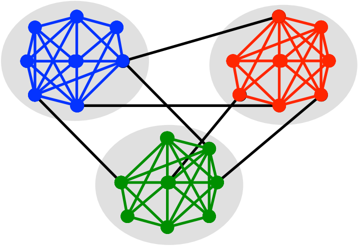


Why do it: visualize

- Communities help to "aggregate" network data

Unveiling network communities

- Nodes in real-world networks organize into communities
  Ex: families, clubs, political organizations, proteins by function, . . .



- Community (a.k.a. group, cluster, module) members are:
  $\Rightarrow$ Well connected among themselves
  $\Rightarrow$ Relatively well separated from the rest
- Exhibit high cohesiveness w.r.t. the underlying relational patterns
- Q: How can we automatically identify such cohesive subgroups?

Community detection and graph partitioning

- Community detection is a challenging clustering problem
  - C1) No consensus on the structural definition of community
  - C2) Node subset selection often intractable
  - C3) Lack of ground-truth for validation

- Useful for exploratory analysis of network data

  Ex: clues about social interactions, content-related web pages

Graph partitioning

Split V into given number of non-overlapping groups of given sizes

- **Criterion:** number of edges between groups is minimized (more soon)

  Ex: task-processor assignment for load balancing

- Number and sizes of groups unspecified in community detection

  $\Rightarrow$ Identify the natural fault lines along which a network separates

Graph partitioning is hard

- Ex: Graph bisection problem, i.e., partition V into two groups
  - Suppose the groups V_1 and V_2 are non-overlapping
  - Suppose groups have equal size, i.e., $|V_1| = |V_2| = N_v/2$
  - Minimize edges running between vertices in different groups
- Simple problem to describe, but hard to solve

$$\text{Number of ways to partition } V: \quad \binom{N_v}{N_v/2} \approx \frac{2^{N_v}}{\sqrt{N_v}}$$

$\Rightarrow$ Used Stirling's formula $N_v! \approx \sqrt{2\pi N_v}(N_v/e)^{N_v}$

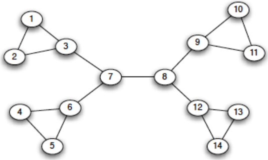
$\Rightarrow$ Exhaustive search intractable beyond toy small-sized networks

- No smart (i.e., polynomial time) algorithm, NP-hard problem

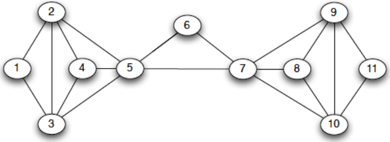
$\Rightarrow$ Seek good heuristics, e.g., relaxations of natural criteria

Strength of weak ties motivation

- Local bridges connect weakly interacting parts of the network



- Q: What about removing those to reveal communities?

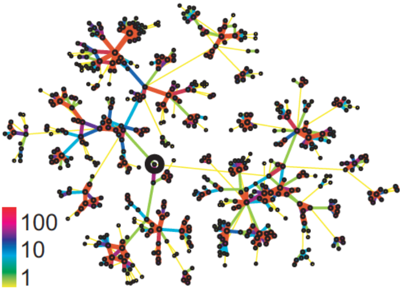


- **Challenges**
  - Multiple local bridges. Some better that others? Which one first?
  - There might be no local bridge, yet an apparent natural division

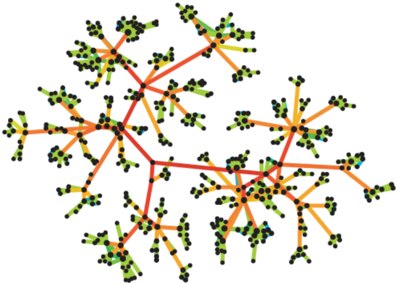
Edge betweenness centrality

- Idea: high edge betweenness centrality to identify weak ties
  - High $c_{Be}(e)$ edges carry large traffic volume over shortest paths
  - Position at the interface between tightly-knit groups

- Ex: cell-phone network with colored edge strength and betwenness



Edge strength

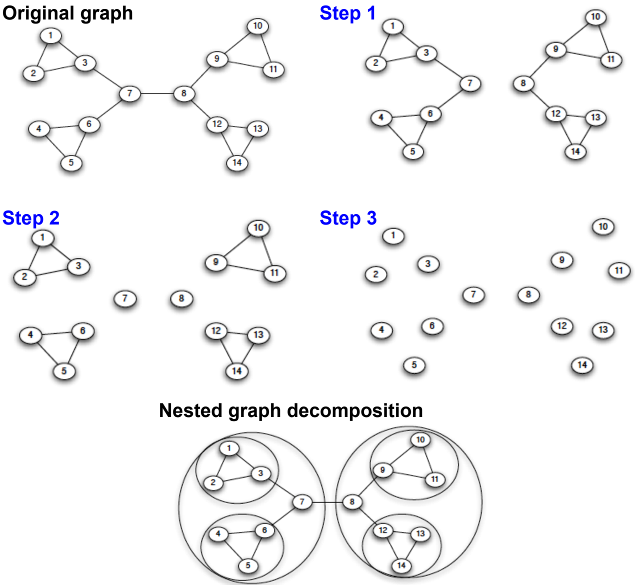


Edge betweenness

Girvan-Newman's method

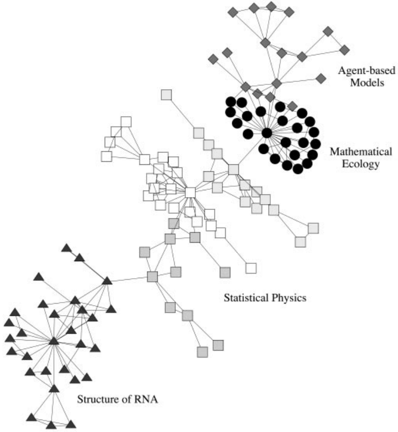
- Girvan-Newmann's method extremely simple conceptually
  - $\Rightarrow$ Find and remove "spanning links" between cohesive subgroups
- **Algorithm:** Repeat until there are no edges left
  - $\Rightarrow$ Calculate the betweenness centrality $c_{Be}(e)$ of all edges
  - $\Rightarrow$ Remove edge(s) with highest $c_{Be}(e)$
- Connected components are the communities identified
  - Divisive method: network falls apart into pieces as we go
  - Nested partition: larger communities potentially host denser groups
  - Recompute edge betweenness in $O(N_v N_e)$-time per step
- M. Girvan and M. Newman, "Community structure in social and biological networks," *PNAS*, vol. 99, pp. 7821-7826, 2002

Example: The algorithm in action



Scientific collaboration network

- Ex: Coauthorship network of scientists at the Santa Fe Institute



- Communities found can be traced to different disciplines

Hierarchical clustering

- Greedy approach to iteratively modify successive candidate partitions
  - Agglomerative: successive coarsening of partitions through merging
  - Divisive: successive refinement of partitions through splitting
- Per step, partitions are modified in a way that minimizes a cost
  - Measures of (dis)similarity x_{ij} between pairs of vertices v_i and v_j
  - Ex: Euclidean distance dissimilarity

$$x_{ij} = \sqrt{\sum_{k \neq i,j} (A_{ik} - A_{jk})^2}$$

- Method returns an entire hierarchy of nested partitions of the graph

$\Rightarrow$ Can range fully from $\{\{v_1\}, \ldots, \{v_{N_v}\}\}$ to V

Agglomerative clustering

- An agglomerative hierarchical clustering algorithm proceeds as follows

  S1: Choose a dissimilarity metric and compute it for all vertex pairs

  S2: Assign each vertex to a group of its own

  S3: Merge the pair of groups with smallest dissimilarity

  S4: Compute the dissimilarity between the new group and all others

  S5: Repeat from S3 until all vertices belong to a single group

- Need to define group dissimilarity from pairwise vertex counterparts
  - Single linkage: group dissimilarity $x^{SL}_{G_i,G_j}$ follows single most dissimilar pair

$$x^{SL}_{G_i,G_j} = \max_{u \in G_i, v \in G_j} x_{uv}$$

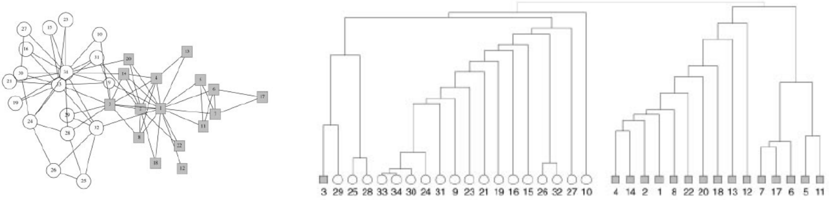
  - Complete linkage: every vertex pair highly dissimilar to have high $x^{CL}_{G_i,G_j}$

$$x^{CL}_{G_i,G_j} = \min_{u \in G_i, v \in G_j} x_{uv}$$

- Hierarchical partitions often represented with a dendrogram
- Shows groups found in the network at all algorithmic steps

  $\Rightarrow$ Split the network at different resolutions

- Ex: Girvan-Newman's algorithm for the Zachary's karate club



- Q: Which of the divisions is the most useful/optimal in some sense?
- A: Need to define metrics of graph clustering quality

Modularity

- Size of communities typically unknown $\Rightarrow$ Identify automatically
- Modularity measures how well a network is partitioned in communities
  - Intuition: density of edges in communities higher than expected
- Consider a graph G and a partition into groups $s \in S$. Modularity:

$$Q(G,S) \propto \sum_{s \in S} [(\# \text{ of edges within group } s) - \mathbb{E}[\# \text{ of such edges}]]$$

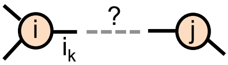
- Formally, after normalization such that $Q(G,S) \in [-1,1]$

$$Q(G,S) = \frac{1}{2N_e} \sum_{s \in S} \sum_{i,j \in s} \left[A_{ij} - \frac{d_i d_j}{2N_e} \right]$$

$\Rightarrow$ Null model: randomize edges, preserving degree distribution

Expected connectivity among nodes

- Null model: randomize edges preserving degree distribution in G
  - $\Rightarrow$ Random variable $A_{ij} := \mathbb{I}\left\{(i,j) \in E\right\}$
  - $\Rightarrow$ Expectation is $\mathbb{E}\left[A_{ij}\right] = \mathrm{P}\left((i,j) \in E\right)$
- Suppose node i has degree d_i, node j has degree d_j
  - $\Rightarrow$ Degree is "# of spokes" per node, $2N_e$ spokes in G



- Probability spoke i_k connected to j is $\frac{d_j}{2N_e - 1} \approx \frac{d_j}{2N_e}$, hence

$$\mathrm{P}\left((i,j) \in E\right) = \mathrm{P}\left(\bigcup_{i_k=1}^{d_i} \{\text{spoke } i_k \text{ connected to } j\}\right)$$

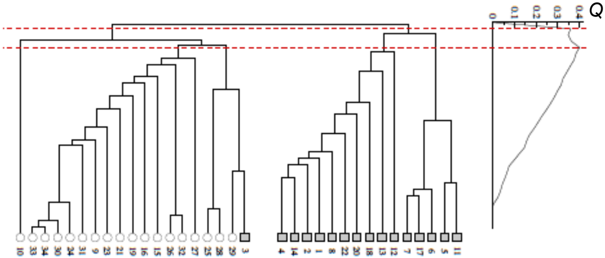
$$= \sum_{i_k=1}^{d_i} \mathrm{P}\left(\text{spoke } i_k \text{ connected to } j\right) = \frac{d_i d_j}{2N_e}$$

Assessing clustering quality

- Can evaluate the modularity of each partition in a dendrogram

  $\Rightarrow$ Maximum value gives the "best" community structure

- Ex: Girvan-Newman's algorithm for the Zachary's karate club



- Q: Why not optimize $Q(G, S)$ directly over possible partitions S?

Modularity

- **Modularity of partitioning S of graph G:**
  - **Q ∝ $\sum_{s \in S}$ [(# edges within group *s*) – (expected # edges within group *s*)]**
  - $$Q(G,S) = \underbrace{\frac{1}{2m}}_{\text{Normalizing const.: } -1<Q<1} \sum_{s \in S} \sum_{i \in s} \sum_{j \in s} \left(A_{ij} - \frac{k_i k_j}{2m} \right)$$

$A_{ij} = 1$ if $i \rightarrow j$, 0 else

- **Modularity values take range [−1,1]**
  - It is positive if the number of edges within groups exceeds the expected number
  - **Q** greater than **0.3-0.7** means **significant community structure**

Modularity

- Consider edges that fall within a community or between a community and the rest of the network
- Define modularity:

if vertices are in the same community

$$Q = \frac{1}{2m}\sum_{vw}\left[A_{vw} - \frac{k_v k_w}{2m}\right]\delta(c_v, c_w)$$

adjacency matrix

probability of an edge between
two vertices is proportional to their degrees

- For a random network, Q = 0
  - the number of edges within a community is no different from what you would expect

Finding community structure in very large networks
Authors: Aaron Clauset, M. E. J. Newman, Cristopher Moore 2004

RECAP: Modularity

$$Q(G,S) = \frac{1}{2m} \sum_{s \in S} \sum_{i \in s} \sum_{j \in s} \left(A_{ij} - \frac{k_i k_j}{2m} \right)$$

Equivalently modularity can be written as:

$$Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

- A_{ij} represents the edge weight between nodes i and j;
- k_i and k_j are the sum of the weights of the edges attached to nodes i and j, respectively;
- $2m$ is the sum of all of the edge weights in the graph;
- c_i and c_j are the communities of the nodes; and
- δ is an indicator function

Idea: We can identify communities by maximizing modularity

Louvain Modularity

Louvain Algorithm

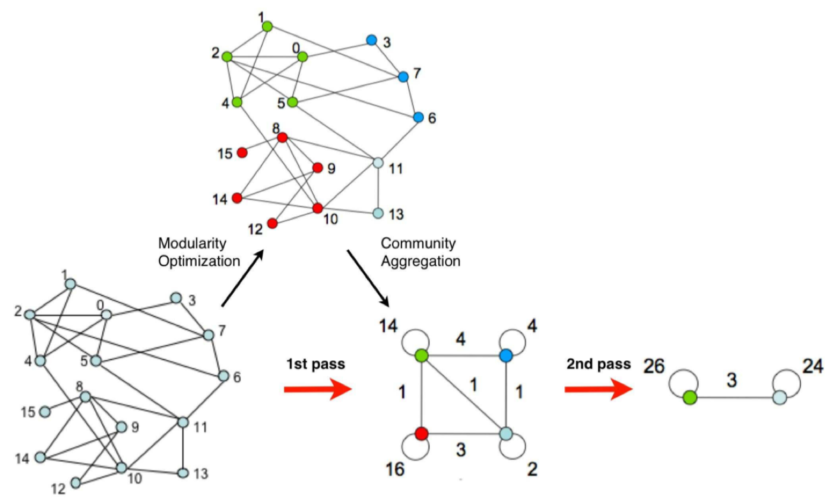
- **Greedy algorithm** for community detection
  - O(n log n) run time
- Supports weighted graphs
- Provides hierarchical partitions
- Widely utilized to **study large networks** because:
  - Fast
  - Rapid convergence properties
  - High modularity output (i.e., "better communities")

"Fast unfolding of communities in large networks" Blondel et al. (2008)

Louvain Algorithm: At High Level

- Louvain algorithm **greedily maximizes** modularity
- **Each pass is made of 2 phases:**
  - **Phase 1:** Modularity is optimized by allowing only local changes of communities
  - **Phase 2:** The identified communities are aggregated in order to build a new network of communities
  - **Goto Phase 1**

The passes are repeated **iteratively** until no increase of modularity is possible!



Louvain: 1st phase (partitioning)

- Put each node in a graph into a distinct community (one node per community)
- For each node *i*, the algorithm performs two calculations:
  - Compute the modularity gain (ΔQ) when putting node i into the community of some neighbor j
  - Move i to a community of node j that yields the largest gain ΔQ
- The loop runs until no movement yields a gain

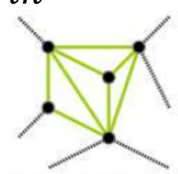
Louvain: Modularity Gain

What is ΔQ if we move node i to community C?

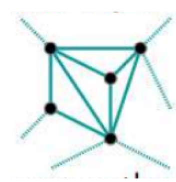
$$\Delta Q(i \rightarrow C) = \left[\frac{\sum_{in} + k_{i,in}}{2m} - \left(\frac{\sum_{tot} + k_i}{2m}\right)^2\right] - \left[\frac{\sum_{in}}{2m} - \left(\frac{\sum_{tot}}{2m}\right)^2 - \left(\frac{k_i}{2m}\right)^2\right]$$

- where:
  - Σ_{in}… sum of link weights between nodes in C
  - Σ_{tot}… sum of all link weights of nodes in C
  - $k_{i,in}$… sum of link weights between node i and C
  - k_i… sum of all link weights (i.e., degree) of node i

Σ_{in}:



Σ_{tot}:

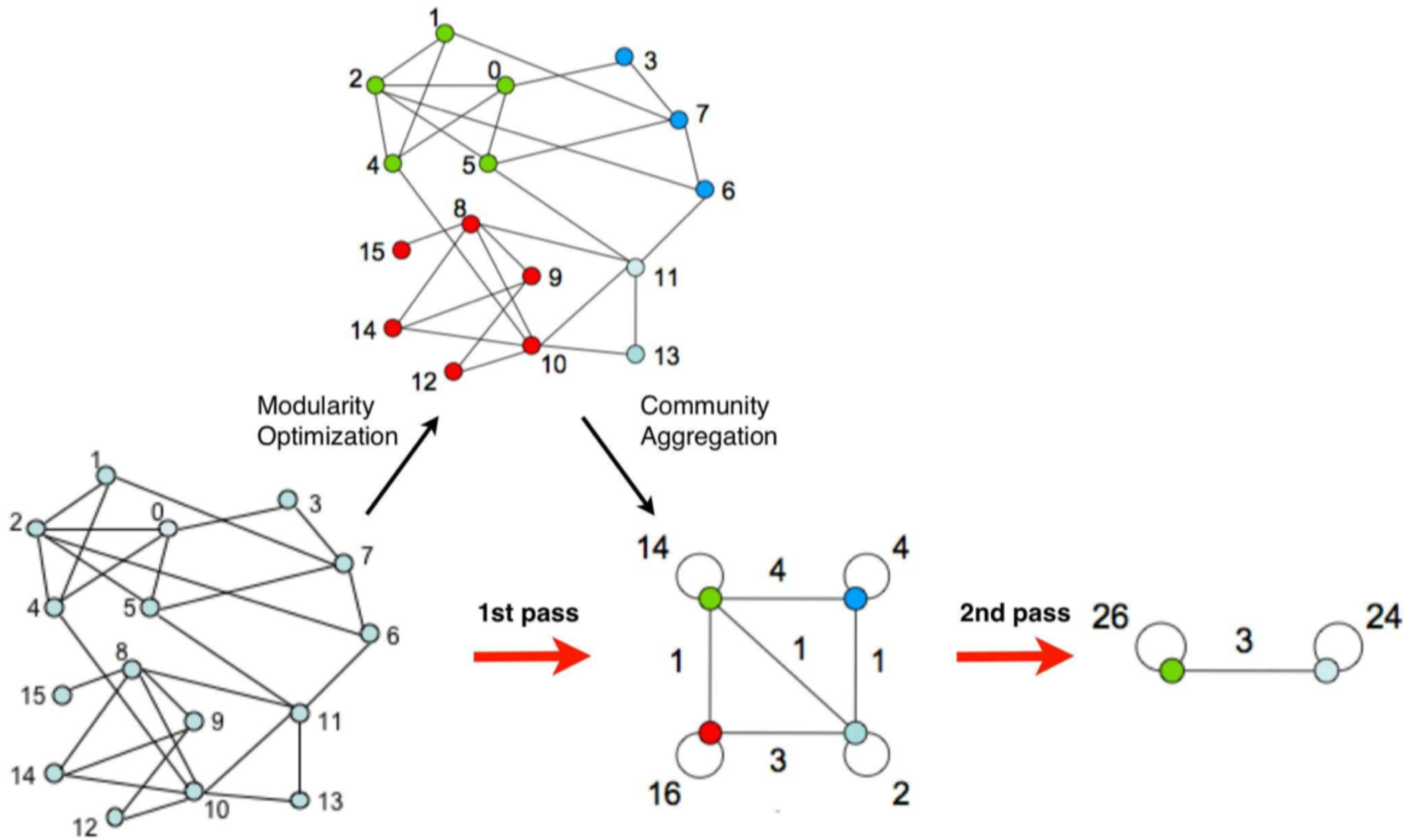


- Also need to derive $\Delta Q(D \rightarrow i)$ of taking node i out of community D.
- And then: $\Delta Q = \Delta Q(i \rightarrow C) + \Delta Q(D \rightarrow i)$

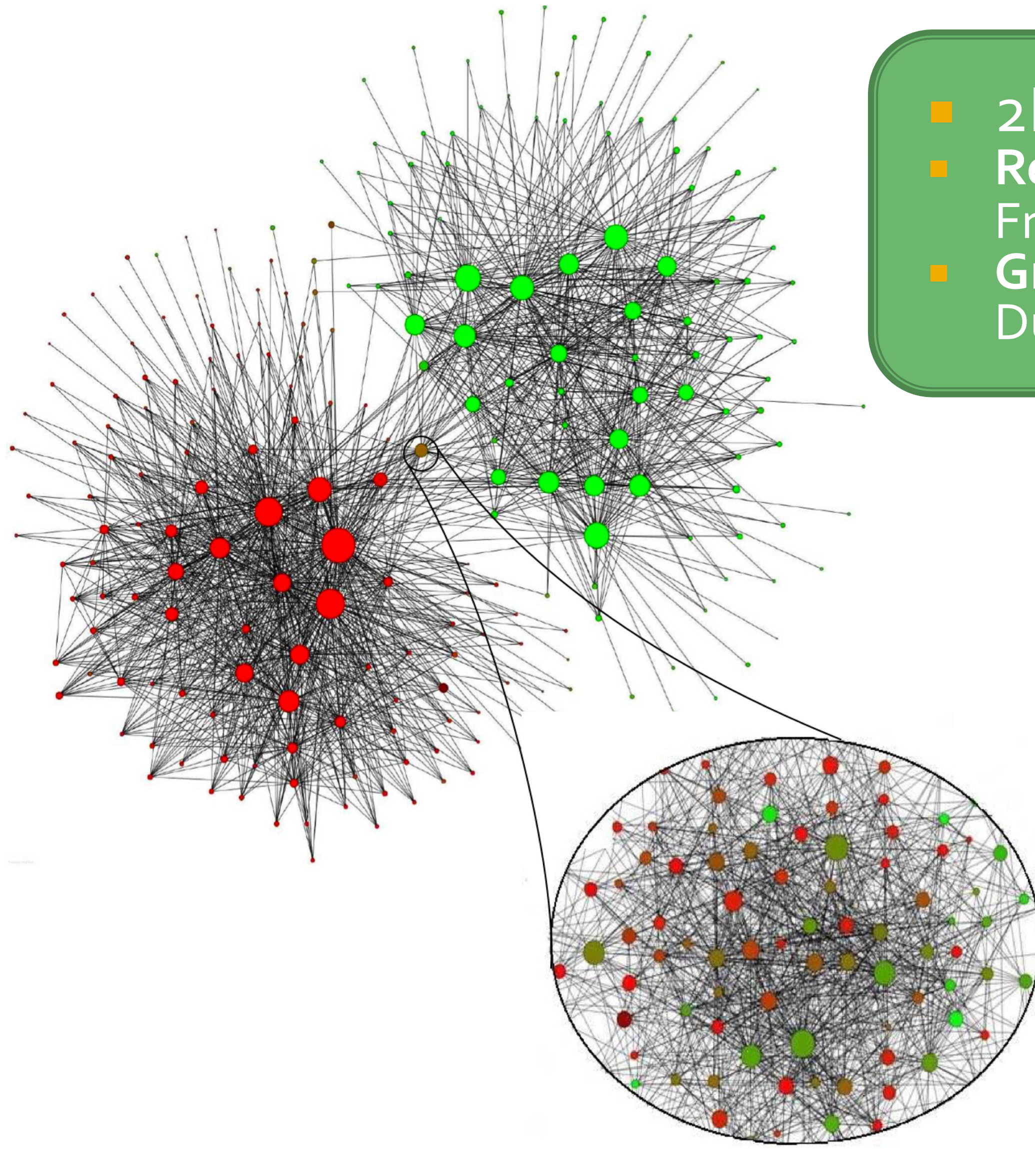
Louvain: 2nd phase (restructuring)

- The partitions obtained in the first phase are contracted into **super-nodes**, and the network is created accordingly
  - Super-nodes are connected if there is at least one edge between nodes of the corresponding partitions
  - The weight of the edge between the two super-nodes is the sum of the weights from all edges between their corresponding partitions
- **The loop runs until the community configuration does not change anymore**

Louvain Algorithm



Belgian Mobile phone network



- 2M nodes
- **Red nodes**: French speakers
- **Green nodes**: Dutch speakers