Model Mini-Test - Network Science 2021/2022 (CC4063 & CC4095)

Duration: 1h30m+30m

 $\mathrm{DCC}/\mathrm{FCUP}$

Answer to only **five** of the following six groups

Essential Concepts (20%)

- 1 *Directed Networks*. Consider the right figure, representing a simple directed network:
 - a) What is the distance from node C to node E?
 - b) Identify all strongly connected components (SCCs) (and their correspondent nodes)
 - c) Redraw the network as a **DAG** (directed acyclic graph) of SCCs.

2 - Bipartite Networks.

- a) Give an example of a real world bipartite network, identifying what are the nodes and the edges.
- b) What is a bipartite network projection? Give an example projection of the bipartite network you chose.

Graph Models (20%)

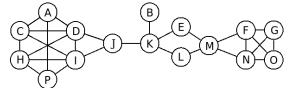
- **3** *Erdös-Rényi* $G_{(n,p)}$ model (*ER*). Imagine you generate an ER graph with n = 1000 and p = 0.01.
 - a) What is the average degree of its nodes?
 - b) Draw a plot showing what you expect to be its degree distribution.
 - c) How many connected components do you expect the graph to have? Why?

d) If you increase the amount of nodes to 1 million (keeping the same average degree), what happens to the **average path length?** Does it stay the same? decreases? increases? by how much?

- 4 Small-World Networks. Imagine you generate a small world network using the Watts-Strogatz (WS) model.
 - a) Which model generates a network with higher clustering coefficient: ER or WS? Why?
 - b) A purely regular network can have an high average path length. Explain how the WS model lowers this value.

Node Centrality (20%)

5 - Centrality Metrics. Consider the following network, and indicate:



- a) a node with high degree but low betweenness centrality
- b) a node with high closeness centrality but low betweenness centrality
- c) a node with high betweenness centrality, high closeness centrality and high degree

6 - The PageRank Algorithm.

a) **Problems.** Explain what a spider trap is, why it might be a problem for the PageRank algorithm and how we can avoid that problem.

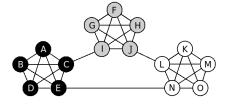
b) Customization. Explain what is a **Personalized PageRank**, what is different from the normal PageRank and in what kind of situation it should be used.

Community Structure (20%)

7 - Girvan-Newman and Louvain algorithms. Consider these two algorithms for community discovery.

- a) Classify both algorithms in terms of being divisive or agglomerative .
- **b**) Which one is faster in terms of **execution time**?

c) Consider the following network with three very natural and intuitive communities. Explain how one of the two algorithms would work on this network, by explaining a couple of its initial greedy iterations.



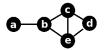
8 - Modularity.

a) Give a brief informal description of the concept of modularity, and what can we use it for.

Subgraph Patterns (20%)

9 - Subgraph census.

a) Calculate the **frequency of all possible induced subgraphs of size 3** on graph pf the following figure (draw the topology of each possible subgraph and indicate all occurrences as sets of nodes).



b) Explain the relationship between the concepts of network motifs and subgraph frequency.

10 - G-Tries.

a) Explain what G-Trie is. Explain what does it store, draw an example, indicate in what kind of task it might be useful and give a brief explanation of the intuition on why it is efficient.

Diffusion and Cascading Behavior (20%)

11 - Spreading in Random Trees. Consider a simple epidemic model based on random trees: a patient meets d people and infects each one with probability q.

- a) Suppose you start with a single infected patient and that d = 200 and p = 1%. What is the expected number of infected persons at depth level 1 and 2? Will the epidemic run forever or will it stop?
- **b)** For a general case, what is the **reproductive number**? How can we use it to determine if a disease will spread or die out? Does it matter the amount of people that initially have the disease?

12 - Spreading models of viruses.

- a) Describe the SIS model: indicate states, transition probabilities and dynamics on a complete graph.
- b) Give an example of a disease that fits the SIS model.