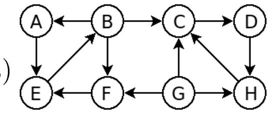
Essential Concepts

1 - *Directed Networks.* Consider the right figure, representing a simple directed network:



a) Identify all **strongly connected components** (SCCs) (and their correspondent nodes)

b) Redraw the network as a **DAG** (directed acyclic graph) of SCCs.

Graph Models

2 - Erdös-Rényi $G_{(n,p)}$ model (ER). Imagine you generate an ER graph with $n = 1000$ and $p = 0.01$.

a) What is the **average degree** of its nodes?

b) Draw a plot showing what you expect to be its **degree distribution**.

c) How many **connected components** do you expect the graph to have? Why?

d) Do you expect to find many big **cliques** on the network? Why?

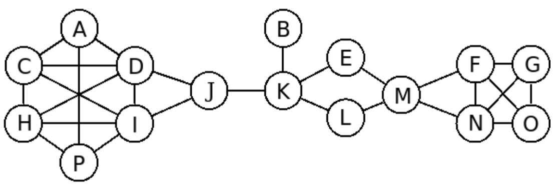
3 - Small-World Networks. Imagine you generate a small world network using the Watts-Strogatz (WS) model.

a) Which model generates a network with higher **clustering coefficient**: ER or WS? Why?

b) A purely regular network can have an high **average path length**. Explain how the WS model lowers this value.

Node Centrality

4 - Centrality Metrics. Consider the following network, and indicate:



a) a node with high degree but low betweenness centrality

b) a node with high closeness centrality but low betweenness centrality

c) a node with high betweenness centrality but low closeness centrality

d) two nodes with the same betweenness centrality

5 - The PageRank Algorithm.

a) Problems. Explain what a **spider trap** is, why it might be a problem for the PageRank algorithm and how we can avoid that problem.

Community Structure

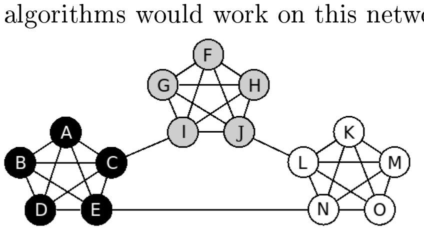
6 - Girvan-Newman and Louvain algorithms. Consider these two algorithms for community discovery.

a) Classify both algorithms in terms of being **divisive or agglomerative** .

b) Why can we say that both algorithms are **hierarchical**?

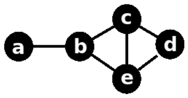
c) Which one is faster in terms of **execution time**?

d) Consider the following network with three very natural and intuitive communities. Explain how each of the two algorithms would work on this network, by explaining a couple of their initial greedy iterations.



Subgraph Patterns

For the following questions consider the graph G depicted in the following figure:



7 - *Subgraph census.* Calculate the **frequency of all possible induced subgraphs of size 3** on graph G (draw the topology of each possible subgraph and indicate all occurrences as sets of nodes).

8 - *Network Motifs.*

a) Explain the concept of **network motifs**. Indicate the necessary conditions for a subgraph to be considered a motif.

b) Explain how you could use network motifs to compare the **similarity of two networks**.

9 - *Graphlets and Orbits.*

a) Give a brief explanation of what a **orbit** is and indicate all possible orbits of graph G.

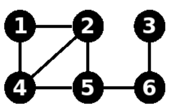
b) Calculate the **graphlet degree vector** of node c in graph G (consider graphlets up to size 3)

Diffusion and Cascading Behavior

10 - *Decision Based Model of Diffusion.* Imagine you have two different behaviors A and B, such that if two neighbor nodes adopt behavior A they each receive a payoff of b, if they both adopt B they each receive b and if they adopt different behaviors they both receive zero.

a) Imagine a node v with N_A neighbors adopting A and N_B neighbors adopting B. **Which behavior should** v **adopt?** Give an *explicit formula* using the described model (the global payoff of a node is the sum of all its payoffs).

b) Suppose $a = b$ and consider the following network where all nodes are initially adopting behavior A and will only change it there is an improvement in their payoff (if the payoff is the same, they will keep the same behavior).



Suppose that nodes 1 and 2 are hardwired to adopt behavior B and will always adopt it. Describe **what happens in this scenario**: indicate all iterations until the network stabilizes (with no more changes), indicate the size of the cascade and the activation threshold of all nodes that get activated.

c) Suppose you can only hardwire a single node of the previous network to adopt B and that $b > a$ (by a very small margin). **Which node should you choose** so that you maximize the nr. of nodes adopting B in the end? Why?

11 - *Spreading models of viruses.*

a) Describe the **SIR model**: indicate states, transition probabilities and dynamics on a complete graph.

b) Give an **example of a disease** that fits the SIR model.

Network Construction

12 - *Multimode Network Transformation.* Imagine you have a bipartite network of genes and diseases.

a) If you construct a one mode projection on the diseases you will loose some information. Exemplify by **drawing two different initial bipartite networks that will give origin to the same projection** (suppose that a single common neighbor will give origin to an edge in the projection).

b) Explain the concept of the **Jaccard Index** and how you could use it to build network projections.

13 - Time series and Network Science:

a) Suppose you are using sensors to measure the air pollution of different locations (creating a time series for each location). **How you could build a network from this data?** (what are the nodes? what do the edges represent?)

b) Draw the natural **visibility graph** for the following time series with 8 equally spaced points in time: $6, 3, 8, 5, 4, 6, 7, 10$