

Answer Set Programming for the Semantic Web

Thomas Eiter

Institute of Information Systems
Vienna University of Technology (TU Wien)



eiter@kr.tuwien.ac.at

23rd Int'l Conference on Logic Programming, Sept 8-13, 2007, Porto, Portugal

Collaborators: Jos de Bruijn (Free U. Bolzano), Giovambattista Ianni (U. Calabria),
Thomas Lukasiewicz (U. Rome "La Sapienza"), Axel Polleres (DERI Galway)
Roman Schindlauer (TU Wien), Hans Tompits (TU Wien)

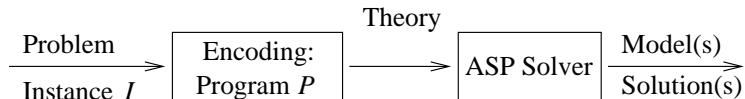
(IST REWERSE, FWF Project P17212-N04)

- Recall Answer Set Programming
- Semantic Web
- ASP for Semantic Web
- Focus: Combining Rules and Ontologies
- Research Issues

- Term was coined by Vladimir Lifschitz (1999)
- Roots in KR, logic programming, nonmonotonic reasoning
- Proposed by other people (Marek & Truszczyński, Niemelä,...) at about the same time
- Makes fruitful use of the (often criticized) fact that non-monotonic logic programs (LPs) have multiple stable models (aka *answer sets*)
- At an abstract level, relates to SAT solving and CSP.

General idea: stable models are solutions!

Reduce solving a problem instance I to computing stable models of an LP



- 1 **Encode** I as a (non-monotonic) logic program P , such that solutions of I are represented by models of P
- 2 **Compute** some model M of P , using an ASP solver
- 3 **Extract** a solution for I from M .

Variant: Compute multiple models (for multiple I / all solutions)

- A *normal logic program* P is a (finite) set of rules of the form

$$a \leftarrow b_1, \dots, b_m, \text{not } c_1, \dots, \text{not } c_n$$

where all a, b_i, c_j are literals of the form p or $\neg p$, where p is an atom.

- “ \neg ” is called strong negation (also written as “ \neg ”)
- a may be missing (constraint)
- HB_P is the set of all literals p and $\neg p$ with predicates and constants from P .
- In *disjunctive programs*, the rule head may be a disjunction $a_1 \mid \dots \mid a_k$ of literals

- Consider a consistent set of ground literals $M \subseteq HB_P$
- Recall the Gelfond-Lifschitz (GL) reduct P^M :
remove from the grounding of P , $Ground(P)$,
 - 1 every rule $a \leftarrow b_1, \dots, b_m, not\ c_1, \dots, not\ c_n$,
where some c_i is in M , and
 - 2 all literals $not\ c_j$ from the remaining rules.
- Then, M is an answer set of P iff M is the least model of P^M
- Such an M satisfies all rules, and intuitively P justifies each atom in M
- Easily generalized to disjunctive P : “the least” \rightsquigarrow “a minimal”

Example

$$P = \{ \text{person}(\text{joey}); \\ \text{male}(X) \mid \text{female}(X) \leftarrow \text{person}(X) \}$$

- $M_1 = \{ \text{person}(\text{joey}), \text{male}(\text{joey}), \text{bachelor}(\text{joey}) \}$ is stable
- $M_2 = \{ \text{person}(\text{joey}), \text{male}(\text{joey}), \text{married}(\text{joey}) \}$ is not stable

In general, no, one, or multiple stable models exist.

Further stable model:

- $M_3 = \{ \text{person}(\text{joey}), \text{female}(\text{joey}) \}$

- **ASP features “pure” declarative programming**

Under answer set semantics,

- the order of program rules does not matter;
- the order of subgoals in a rule does not matter;

- **Nondeterminism in ASP:** Means to make guesses

- **Only limited support of function symbols in current ASP solvers**
(more is emerging)

Simple Horn LPs with function symbols are undecidable, while ASP strives for decidability

Some decidable fragments of ASP with function symbols:

- ω -restricted programs [Syrjänen, LPNMR01]
- finitary programs [Bonatti, 04],
finitely recursive programs [Baselice et al., ICLP07]
- FDNC programs [Simkus & E_, LPAR07]

Problems in different domains (some with substantial amount of data), see

<http://www.kr.tuwien.ac.at/projects/WASP/report.html>

- information integration
- constraint satisfaction
- planning, routing
- biology
- diagnosis
- security analysis
- configuration
- computer-aided verification
- ...

ASP Showcase: <http://www.kr.tuwien.ac.at/projects/WASP/showcase.html>

DLV	http://www.dbai.tuwien.ac.at/proj/dlv/
Smodels	http://www.tcs.hut.fi/Software/smodels/
GnT	http://www.tcs.hut.fi/Software/gnt/
Cmodels	http://www.cs.utexas.edu/users/tag/cmodels/
ASSAT	http://assat.cs.ust.hk/
NoMore	http://www.cs.uni-potsdam.de/~linke/nomore/
Platypus	http://www.cs.uni-potsdam.de/platypus/
clasp	http://www.cs.uni-potsdam.de/clasp/
XASP	distributed with XSB v2.6 http://xsb.sourceforge.net
aspps	http://www.cs.engr.uky.edu/ai/aspps/
ccalc	http://www.cs.utexas.edu/users/tag/cc/

- Some provide a number of extensions to the language described here.
- Answer Set Solver Implementation: see Niemelä's ICLP'04 tutorial.

- Many extensions have been proposed, partly motivated by applications
- Some are syntactic sugar, other strictly add expressiveness
- Incomplete list:
 - cardinality constraints (Smodels)
 - optimization: weight constraints, *minimize* (Smodels); weak constraints (DLV)
 - aggregates (Smodels, DLV)
 - templates (for macros)
 - preferences: e.g., PLP
 - KR frontends (diagnosis, planning,...) in DLV
- Comprehensive survey of extensions:

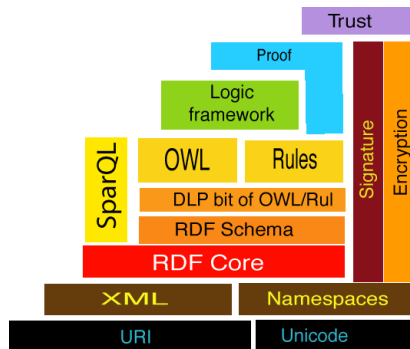
<http://www.tcs.hut.fi/Research/Logic/wasp/wp3/>

Why a “Semantic” Web?

- Undoubtedly, the World Wide Web is one of the most significant technical innovations of the last decades,
- The WWW will be (and already is) strongly impacting, and changing, our living, culture, and society.
- The WWW has been designed for human users, not for machines
- To process the data and information out on the Web, semantic annotation and description is needed.
- The Semantic Web is the vision of such an enriched, future generation Web
- Logic and logic-based formalisms (should/might) play an important role in this endeavor.

- Web services (description, discovery, invocation, composition, choreography, etc.)
- Knowledge management
- Trust, Privacy, Security
- Searching
- Data Annotation
- Ontology Management, Alignment
- Web Mining
- Reasoning on the Semantic Web
- ...

Building the Semantic Web (T. Berners-Lee, 04/2005)



- RDF is the data model of the Semantic Web
- RDF Schema semantically extends RDF by simple taxonomies and hierarchies
- OWL is a W3C standard, which builds on Description Logics
- Rule languages: *Rule Interchange Format (RIF)* WG of W3C

RDF/S (Resource Description Framework)

- **RDF data model:** labeled graph of resources (nodes) linked to other resources or literals by predicates.



- Usually represented in form of triples $\langle \textit{Subject}, \textit{Predicate}, \textit{Object} \rangle$ e.g.

```
http://www.polleres.net/index.html dc:creator
    http://www.polleres.net/foaf.rdf#me.
http://www.polleres.net/foaf.rdf#me foaf:name "Axel Polleres"
```

```
<rdf:Description rdf:about="http://www.polleres.net/index.html">
  <dc:creator>
    <rdf:Description rdf:about="http://www.polleres.net/foaf.rdf#me">
      <foaf:name>Axel Polleres</foaf:name>
    </rdf:Description>
  </dc:creator>
</rdf:Description>
```

- Resources identified by URIs
- **RDF Schema (RDFS):** simple taxonomies on RDF vocabularies using `rdf:type`, `rdf:subClassOf`, `rdfs:subPropertyOf`
- RDF semantics has some subtleties (blank nodes, XML literals, RDF keywords treated as normal resources, reification, etc.)

- Knowledge about concepts, individuals, their properties and relationships
- W3C Standard (04/2004): *Web Ontology Language (OWL)*
- Three increasingly expressive sublanguages

OWL Hierarchy

- OWL Lite: Concept hierarchies, simple constraint features. ($\equiv SHIF(\mathbf{D})$)
 - OWL DL : Basically, DAML+OIL. ($\equiv SHOIN(\mathbf{D})$)
 - OWL Full: Allow e.g. to treat classes as individuals.
- OWL Syntax is based on RDF
 - Most widely considered: OWL DL
 - Currently, OWL 1.1 (an extension of OWL DL) is a submitted to W3C

OWL / Description Logics offers more expressivity than RDF/S!

- The vocabulary of basic DLs comprises:
 - Concepts (e.g., *Wine*, *WhiteWine*)
 - Roles (e.g., *hasMaker*, *madeFromGrape*)
 - Individuals (e.g., *SelaksIceWine*, *TaylorPort*)
- Statements relate individuals and their properties using
 - logical connectives (\sqcap , \sqcup , \neg , \sqsubseteq , etc), and
 - quantifiers (\exists , \forall , $\leq k$, $\geq k$, etc)
- A DL knowledge base usually comprises
 - a T-Box (terminology, conceptualization), and
 - an A-Box (assertions, extensional knowledge)
- DLs are tailored for decidable reasoning (key task: satisfiability)

Example: The Wine Ontology

- Available at <http://www.w3.org/TR/owl-guide/wine.rdf>
- Some axioms from the T-Box

$Wine \sqsubseteq PotableLiquid \sqcap =1 hasMaker \sqcap \forall hasMaker. Winery;$
 $\exists hasColor. Wine \sqsubseteq \{ "White", "Rose", "Red" \};$
 $WhiteWine \equiv Wine \sqcap \forall hasColor. \{ "White" \}.$

- A wine is a potable liquid, having exactly one maker, who is a member of the class “*Winery*”.
- Wines have colors “*White*”, “*Rose*”, or “*Red*”.
- A *WhiteWine* is a wine with color “*White*”.
- The A-Box contains, e.g., $WhiteWine("StGenevieveTexasWhite")$, $hasMaker("TaylorPort", "Taylor")$

- The semantics is given by a mapping to First-Order Logic
Thus, DLs are (in essence) FO Logic in disguise

OWL property axioms as RDF Triples	DL syntax	FOL short representation
$\langle P \text{ rdfs:domain } C \rangle$	$\top \sqsubseteq \forall P^- . C$	$\forall x, y. P(x, y) \supset C(x)$
$\langle P \text{ rdfs:range } C \rangle$	$\top \sqsubseteq \forall P . C$	$\forall x, y. P(x, y) \supset C(y)$
$\langle P \text{ owl:inverseOf } P_0 \rangle$	$P \equiv P_0^-$	$\forall x, y. P(x, y) \equiv P_0(y, x)$
$\langle P \text{ rdf:type owl:SymmetricProperty} \rangle$	$P \equiv P^-$	$\forall x, y. P(x, y) \equiv P(y, x)$
$\langle P \text{ rdf:type owl:FunctionalProperty} \rangle$	$\top \sqsubseteq \leq 1P$	$\forall x, y_1, y_2. P(x, y_1) \wedge P(x, y_2) \supset y_1 = y_2$
$\langle P \text{ rdf:type owl:InverseFunctionalProperty} \rangle$	$\top \sqsubseteq \leq 1P^-$	$\forall x_1, x_2, y. P(x_1, y) \wedge P(x_2, y) \supset x_1 = x_2$
$\langle P \text{ rdf:type owl:TransitiveProperty} \rangle$	$P^+ \sqsubseteq P$	$\forall x, y, z. P(x, y) \wedge P(y, z) \supset P(x, z)$

OWL complex class descriptions	DL syntax	FOL short representation
owl:Thing	\top	$x = x$
owl:Nothing	\perp	$\neg x = x$
owl:intersectionOf ($C_1 \dots C_n$)	$C_1 \sqcap \dots \sqcap C_n$	$\bigwedge C_i(x)$
owl:unionOf ($C_1 \dots C_n$)	$C_1 \sqcup \dots \sqcup C_n$	$\bigvee C_i(x)$
owl:complementOf (C)	$\neg C$	$\neg C(x)$
owl:oneOf ($o_1 \dots o_n$)	$\{o_1 \dots o_n\}$	$\bigvee x = o_i$
owl:restriction (P owl:someValuesFrom (C))	$\exists P . C$	$\exists y. P(x, y) \wedge C(y)$
owl:restriction (P owl:allValuesFrom (C))	$\forall P . C$	$\forall y. P(x, y) \supset C(y)$
owl:restriction (P owl:value (o))	$\exists P . \{o\}$	$P(x, o)$
owl:restriction (P owl:minCardinality (n))	$\geq n P$	$\exists_{i=1}^n y_i . \bigwedge_{j=1}^n P(x, y_j) \wedge \bigwedge_{i \neq j} y_i \neq y_j$
owl:restriction (P owl:maxCardinality (n))	$\leq n P$	$\forall_{i=1}^{n+1} y_i . (\bigwedge_{j=1}^n P(x, y_i) \supset \bigvee_{i \neq j} y_i = y_j)$

- Different ways to exploit ASP and ASP techniques have been considered
 - **As a host language for Web/Semantics Web formalisms**
 - Mapping / encoding of ontologies and DLs into ASP
 - Encoding of web query languages
E.g., SPARQL [Polleres, WWW07], [Polleres & Schindlauer, ALPSWS07]
 - **For ad hoc problem solving.** E.g.,
 - Web service composition, e.g., [Rainer, KI-WS05]
 - Web service repair, e.g, [Friedrich et al., in progress]
 - **For combining rules and ontologies**

- Dealing with open worlds and domains
- Access to (semi-)structured data
cf. Frame-Syntax of HiLog, F-Logic
- Heterogeneity (integration)
- External sources, distributed computation
- Dynamics

Some Special ASP Extensions

- **Open logic programs** [Van Belleghem et al., 97], [Bonatti, ASP03]
Keep definition of some predicates open
- **Open Answer Set Programs** [Heymans & Vermeir, ASP03;DEXA03]
relax domain closure: add countably many anonymous elements
decidability via restrictions (e.g., conceptual LPs)
- **ASP-EX** [Calimeri & Ianni LPNMR05]
External sources of computation; impose safety
- **HEX-Programs** [E_ et al. IJCAI05]
HiLog ASP plus external sources
- **ONTODLV** [Dell'Armi et al., ASP07]
ASP + non-standard ontologies + OWL interface
- Several extensions to combine rules and ontologies

Expressing RDF/S in ASP

- Common: use a predicate `triple/3`:

```
triple("http://www.polleres.net/foaf.rdf#me",  
      "foaf:name", "Axel Polleres").
```

- RDFS semantics can be fully captured by DLV-EX, DLV-HEX [Ianni et al., ASP07]

E.g., subclass relation:

```
[...]  
triple(S,rdf:type,C2) :- triple(S,rdf:type,C1),  
                        triple(C1,rdfs:subClassOf,C2).
```

map triples to predicate notation (using higher-order features) and to frame constructs

```
P(S,O) :- triple(S,P,O).  
S : C   :- triple(S,rdf:type,C).  
S : C2 :- S : C1, rdfs:subClassOf(C1,C2).
```

- Technicalities for treating blank nodes and infinite axiomatic triples.

- ASP techniques for encoding OWL / DLs have been considered e.g. by
 - Alsac and Baral [2002]
 - Swift [LPNMR04]
 - Hustadt, Motik, and Sattler [KR04]
 - Heymans et al. [2003+]
- Works only under limitations
- Noticable: Hustadt et al.'s mapping of *SHIQ* to disjunctive datalog; basis for the KAON2 DL solver
- **Main problem:** important differences between ASP and OWL/DLs

- ASP and OWL/DLs have related yet different underlying settings
- At the heart, the difference is between LP and Classical logic
- **Main Differences:**
 - Closed vs. Open World Assumption
 - Negation as failure vs. classical negation
 - Strong negation vs. classical negation
 - Unique names, equality

LP / Classical Logic: CWA vs. Open World Assumption (OWA)

- LP aims at building a single model, by closing the world

Reiter's CWA:

If $T \not\models A$, then conclude $\neg A$, for ground atom A

- FO logic / description logics keep the world open
- In the Semantic Web, this is often reasonable
- However, taking the agnostic stance of OWA may be not helpful for drawing rational conclusions under incomplete information
- A mix of CWA and OWA may be appropriate [Damasio et al., PPSWR06], [Polleres et al., EWSC06]

P : *wine*(X) \leftarrow *whiteWine*(X).
 nonWhite(X) \leftarrow *not whiteWine*(X).
 wine(*myDrink*).

T : $\forall X. (WhiteWine(X) \supset Wine(X)) \wedge$
 $\forall X. (\neg WhiteWine(X) \supset NonWhite(X)) \wedge$
 Wine(*myDrink*).

- Query *nonWhite*(*myDrink*)?
 - Conclude *nonWhite*(*myDrink*) from P .
 - Do not conclude *nonWhite*(*myDrink*) from T .

P : $wine(X) \leftarrow whiteWine(X).$
 $\neg wine(myDrink).$

T : $\forall X. (WhiteWine(X) \supset Wine(X)) \wedge$
 $\neg Wine(myDrink).$

- Conclude $\neg WhiteWine(myDrink)$ from T ;
- Do not conclude $\neg whiteWine(myDrink)$ from P
- Note: no contraposition in LP!

$\neg whiteWine(X) \leftarrow \neg wine(X).$

is not equivalent to

$wine(X) \leftarrow whiteWine(X).$

- In LP, usually we have *Unique Names Assumption (UNA)*:
Syntactically different ground terms are different objects.
- Thus, usually only Herbrand interpretations are considered in LP
- Ontology languages like OWL don't make UNA, and allow to link objects (owl:sameAs)
- OWL considers also non-Herbrand interpretations
- Further, related problems with existential quantifiers:

$$\mathcal{T} : \forall X \exists Y. (Wine(X) \supset hasColor(X, Y))$$

(in DL Syntax, $Wine \sqsubseteq \exists hasColor$)

Simple skolemization does not work in general

Generic Settings for KB Combination

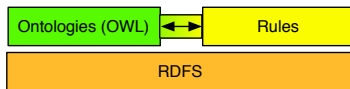
A hybrid knowledge base $\mathcal{KB} = \langle \mathcal{T}, P \rangle$ has

- a FO theory \mathcal{T} (the *classical component*)
 - an LP P (the *rules component*)
-
- Predicates are either “*classical*” or “*rules*” predicates
 - Occurrence of rules-predicates in a FO theory is usually restricted, function symbols are disallowed.
- Main reason:** Combinations of Horn logic and very simple DLs are undecidable [Levy & Rousset, AIJ98].
- Problems with recursion and *unsafety* of rules
 - L&R used *role-safety*: at least one of X, Y in a role atom $R(X, Y)$ in a rule r occurs in rules-predicate in r not occurring in any rule head.

Generic Settings for KB Combination

- Safety (in several versions) is a key tool to decidability for several combination approaches.
- Different approaches to a semantics for $\mathcal{KB} = \langle \mathcal{T}, \mathcal{P} \rangle$
 - Strict semantic separation (loose coupling)
 - Tight integration
 - Full integration
- Surveys and Discussion
 - KNOWLEDGEWEB [Pan et al., 2004]
 - REVERSE [Antoniou et al., 2005]
 - Rosati, E_ et al. [ReasoningWeb2006]

- **Strict semantic separation between rules / ontology**



- View rule base and FO theory as separate components
- They are connected through a minimal interface for exchanging knowledge (formulas).

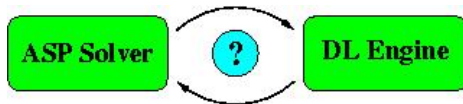
“safe interface”

Examples: TRIPLE [Sintek & Decker, 02]; non-monotonic dl-programs [E_ et al., 04]; defeasible logic+DLs [Antoniou et al., 04]

Loose Coupling: Non-monotonic dl-Programs

- An extension of answer set programs with *queries to DL knowledge bases* (through *dl-atoms*)
- dl-atoms allow to query a DL knowledge base differently

bidirectional flow of information, with clean technical separation of DL engine and ASP solver



- Use dl-programs as “glue” for combining inferences on a DL base.

Basic Idea:

- Query the DL base \mathcal{T} using the query interface of the DL engine
Query Q may be concept/role instance $C(X)/R(X, Y)$, subsumption $C \sqsubseteq D$ etc (recent extension: conjunctive queries)
- Allow to **modify** the extensional part (ABox) of \mathcal{T} , by adding positive (\oplus) or negative (\ominus) assertions
- Q evaluates to true iff the modified \mathcal{T} proves Q .

Examples: wine ontology

- $DL[Wine](\text{"ChiantiClassico"})$
- $DL[Wine](X)$
- $DL[RedWine \oplus my_red; Wine](X)$
add all assertions $RedWine(c)$ to \mathcal{T} , such that $my_red(c)$ holds.
- $DL[RedWine \ominus my_white; hasColor](X, \text{"Red"},)$
add all assertions $\neg RedWine(c)$ to \mathcal{T} , such that $my_white(c)$ holds.

Answer Sets:

- As usual, via grounding of the rules
- Consider a consistent set of ground literals $M \subseteq HB_{P^*}$ ($P^* = P$ with additional constants from \mathcal{T})
- A ground dl-atom $DL[\langle Add \rangle; Q](\mathbf{c})$ is true in M , iff $\mathcal{T} \cup \langle Add \rangle^M \models Q(\mathbf{c})$ for the modification $\langle Add \rangle^M$
- Use a reduct P^M akin to the Gelfond-Lifschitz reduct
- In building P^M , treat dl-atoms like ordinary atoms
- M is an answer set iff M is the least (resp. a minimal) model of P^M

Variants by different treatment of the dl-atoms.

Example: Wine Selection

is_redWine(" Chianti_21").

person(" axel"). *my_Wine*(" axel", " whiteWine").

person(" gibbi"). *my_Wine*(" gibbi", " redWine").

wineBottle(*X*) \leftarrow *DL*[*<Add>*; " Wine"](*X*).

ok_Bottle(*X*, *Z*) \leftarrow *my_Wine*(*X*, " RedWine"), *wineBottle*(*Z*), *DL*[*<Add>*; " RedWine"](*Z*).

ok_Bottle(*X*, *Z*) \leftarrow *my_Wine*(*X*, " WhiteWine"), *wineBottle*(*Z*), *DL*[*<Add>*; " WhiteWine"](*Z*).

takeBottle(*X*) | \neg *takeBottle*(*X*) \leftarrow *wineBottle*(*X*).

hasBottle(*X*) \leftarrow *takeBottle*(*X*), *ok_Bottle*(*X*, *Z*).

\leftarrow *person*(*X*), *not hasBottle*(*X*).

- Answer sets depend on the instances of *Wine*, *RedWine*, *WhiteWine*

E.g., { *wineBottle*(" SelaksIceWine"), *wineBottle*(" TaylorPort"), ...,
ok_Bottle(" axel", " SelaksIceWine"), *ok_Bottle*(" gibbi", " TaylorPort"), ...,
takeBottle(" SelaksIceWine"), *takeBottle*(" TaylorPort"), ... }

- Add knowledge *is_redWine*(" Chianti_21") to \mathcal{T} : *<Add>* = *RedWine* \uplus *is_redWine*
wineBottle(" Chianti_21") is in all answer sets

Example: Mutual Flow

- Existing network, encoded in OWL KB
- New nodes to be added

Condition: don't connect to nodes with high traffic (traffic depends on number of connections)

- Specify models for possibilities

connect(*X*, *Y*) \leftarrow *newNode*(*X*), *DL*[*Node*](*Y*), *not overloaded*(*Y*).
overloaded(*X*) \leftarrow *DL*[*wired* \uplus *connect*; *HighTrafficNode*](*X*).

- Mutual effects between the LP and the ontology!

- dl-programs facilitate some advanced reasoning tasks
 - **Closed World Reasoning**

Emulate *CWA* and *Extended CWA (ECWA)* on top of a DL knowledge base.
 - **Default Reasoning**

Poole's-style and Reiter's Default Logic over DL bases (for restricted fragments)

E.g., network example: connect new node X by default to node Y .
 - **Minimal Model Reasoning**

Single out “minimal” models of a DL base

- A DL base with disjunctive information:

$$\mathcal{T} = \{ \textit{Artist}(\textit{"Jody"}), \textit{Artist} \equiv \textit{Painter} \sqcup \textit{Singer} \}$$

- Single out “minimal” models (in the setting of Extended CWA):

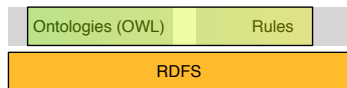
$$\begin{aligned} \bar{p}(X) &\leftarrow \textit{not } p(X). \\ \bar{s}(X) &\leftarrow \textit{not } s(X). \\ p(X) &\leftarrow \textit{DL}[\textit{Painter} \sqcup \bar{p}, \textit{Singer} \sqcup \bar{s}; \textit{Painter}](X). \\ s(X) &\leftarrow \textit{DL}[\textit{Painter} \sqcup \bar{p}, \textit{Singer} \sqcup \bar{s}; \textit{Singer}](X). \end{aligned}$$

Answer sets (corresponding to minimal models):

$$M_1 = \{p(\textit{"Jody"}), \bar{s}(\textit{"Jody"})\}, \quad M_2 = \{s(\textit{"Jody"}), \bar{p}(\textit{"Jody"})\}$$

- Extendible to keep concepts “fixed” $\rightsquigarrow \textit{ECWA}(\phi; P; Q; Z)$

KB Combination: Tight semantic integration



- Integrate FOL statements and the logic program to a large extent, but keep predicates separate.
- Build an integrated model as the union of two models, one for the FO theory and one for the rules, which share the same domain.

“safe interaction”

Examples: CARIN [Levy & Rousset, 98]; DLP [Grosz et al., 03]; SWRL [Horrocks et al., 04]; DL-safe rules [Motik et al., 05]; R-hybrid KBs, R⁺-hybrid KBs, $\mathcal{DL}+log$ [Rosati, DL99;05] [Rosati, PPSWR05;KR06]

Tight integration: $\mathcal{DL}+log$

[Rosati, KR06] Latest in chain of extensions of the DL \mathcal{ALC} with rules
[\mathcal{AL} -Log; R-, R⁺-hybrid KBs]

- Fixed countably infinite domain, *standard names* for the elements.
- Models of $\mathcal{KB} = \langle \mathcal{T}, P \rangle$ are of form $\mathcal{I} \cup M$, where \mathcal{I} is a model of the classical predicates, M of the rules-predicates
- No strong negation, weak negation limited to rules-predicates
- Uses *weak (DL-)safety*: each variable X occurs in some positive body atom, which is a rules atom if X occurs in the head.
- Weak safety allows to *access unnamed individuals* in classical atoms (not possible in dl-programs)

$$girl(X) \leftarrow kid(X), father(Y, X), not\ likes(X, cars);$$

- Decidable, if certain union of conjunctive queries containment (CQ/UCQ) in \mathcal{T} is decidable

- $\mathcal{DL}+log$ has a stable model (answer set) semantics
- Roughly, a 2-step reduction
- **Step 1:** Take some interpretation \mathcal{I} of the classical predicates.
 - Ground P and “reduce” it wrt. \mathcal{I} , by “evaluating” classical atoms in rules wrt. \mathcal{I} .
 - The resulting ground program $P_{\mathcal{I}}$ contains no classical predicates.
- **Step 2:** Build a stable model M of $P_{\mathcal{I}}$ as usual

$\mathcal{DL}+\log$: Example

$$\begin{aligned} \mathcal{T} = & \{ \text{Female} \sqsubseteq \neg \text{Male}; \text{Female} \sqcup \text{Male} \sqsubseteq \text{Person} \\ & \text{Person} \sqsubseteq \exists \text{father}^- . \text{Male}; \text{Person}(\text{joey}) \quad \} \\ \mathcal{P} = & \{ \text{boy}(X) \mid \text{girl}(X) \leftarrow \text{kid}(X); \\ & \text{Male}(X) \leftarrow \text{boy}(X); \\ & \text{Female}(X) \leftarrow \text{girl}(X); \\ & \text{girl}(X) \leftarrow \text{kid}(X), \text{father}(Y, X), \text{not likes}(X, \text{cars}); \\ & \text{kid}(\text{joey}) \quad \} \end{aligned}$$

- Classical predicates *Male*, *Female*, *father* occur in rules.
- Take \mathcal{I} where *joey* belongs to *Female*, *Person*:

$M = \{\text{kid}(\text{joey}), \text{girl}(\text{joey})\}$ is a stable model

$M = \{\text{kid}(\text{joey}), \text{boy}(\text{joey})\}$ is not a stable model

- Take \mathcal{I} where *joey* belongs to *Male*, *Person*:
No stable model (in any such M , *likes(joey, cars)* must be false)

- No separation between vocabularies

Examples:

- Hybrid MKNF knowledge bases [Motik & Rosati, IJCAI07];
 - G-hybrid knowledge bases [Heymans et al., ALPSWS06]
 - Quantified Equilibrium Logic [Pearce et al., RR07]
 - Autoepistemic Logic [de Bruijn et al., IJCAI07]
 - Open Answer Set Programs [Heymans et al., 03+]
-
- Related:
 - Terminological Default Logic [Baader and Hollunder, 1995]
 - Description Logics of Minimal Knowledge [Donini et al., 2002]

Full Integration: Quantified Equilibrium Logic (QEL)

- Builds on FOL version of Equilibrium Logic (EL) [Pearce, 96;06]
- Reconstructs Answer Sets in terms of the standard, nonclassical Logic of here & there (resp. 3-valued Gödel Logic).
- Models $\mathcal{M} = \langle (D, \sigma), I_h, I_t \rangle$ of the func.-free FO Logic **QHT**^s correspond to intuitionistic Kripke models for two worlds, $h \leq t$, with domain D .
- Axiomatization by first-order intuitionistic logic plus further axioms

equality axioms	$x = x, x = y \rightarrow (F(x) \rightarrow F(y))$ (y substitutable for x in formula $F(X)$)
Hosoi's axiom	$\alpha \vee (\neg\beta \vee (\alpha \rightarrow \beta))$
SQHT	$\exists x(F(x) \rightarrow \forall xF(x))$
DE	$x = y \vee x \neq y$

- A model \mathcal{M} of \mathcal{T} is *in equilibrium*, if $I_h = I_t$ and I_h is minimal, i.e., can not be decreased.

- QEL provides a useful logical foundation for different combination approaches:
 - The equilibrium models of a logic program P correspond to Heymans et al.'s [ALPSWS2006] generalised open answer sets of P .
 - Models of $\mathcal{KB} = \langle \mathcal{T}, P \rangle$ in several other combination approaches (G-hybrid, R-hybrid, R^+ -hybrid KBs) can be elegantly captured
 - For that, take $\mathcal{T} \cup P \cup st(\mathcal{T})$, where

$$st(\mathcal{T}) = \{\forall x p(x) \vee \neg p(x) \mid p \in \Sigma_{\mathcal{T}}\}.$$

Note: $st(\mathcal{T}) \models \neg\neg\phi \rightarrow \phi$, for each $\phi \in \mathcal{T}$.

- QEL has further usages (e.g., *strong equivalence* of hybrid KBs)

Full Integration: Hybrid MKNF

- Builds on a FO-version of Lifschitz's modal logic MKNF [IJCAI91]
- Rules of form

$$\mathbf{K}h_1 \vee \dots \vee \mathbf{K}h_l \leftarrow \mathbf{K}b_1, \dots, \mathbf{K}b_m, \text{not } b_{m+1}, \dots, \text{not } b_n$$

where the h_i, b_j are function-free FO atoms

- $\mathbf{K}\phi \approx \phi$ is known to hold under the values of the *not* -atoms, Kripke-based semantics (“maximal” S5-models wrt. \mathbf{K})
- Faithfully extends LP and DL; generalizes GARIN, \mathcal{AL} -Log
- Allows “closed world glasses” on classical predicates, stating exceptions
- Extensions with both modal and non-modal atoms in rules allow to generalize $\mathcal{DL}+log$ (equi-satisfiable)
- Decidable under DL-safety: each variable occurs in some positive rule atom in the body.

Hybrid MKNF: Example

$$\begin{aligned} T = & \{ \text{Female} \sqsubseteq \neg \text{Male}; \text{Female} \sqcup \text{Male} \sqsubseteq \text{Person} \\ & \text{Person} \sqsubseteq \exists \text{father} \neg .\text{Male}; \text{Person}(\text{joey}); \text{Diabetic} \sqsubseteq \text{Person} \} \\ P = & \{ \mathbf{K} \text{boy}(X) \vee \mathbf{K} \text{girl}(X) \leftarrow \mathbf{K} \text{kid}(X); \\ & \mathbf{K} \text{Male}(X) \leftarrow \mathbf{K} \text{boy}(X); \\ & \mathbf{K} \text{Female}(X) \leftarrow \mathbf{K} \text{girl}(X); \\ & \mathbf{K} \text{girl}(X) \leftarrow \mathbf{K} \text{kid}(X), \mathbf{K} \text{father}(Y, X), \text{not likes}(X, \text{cars}); \\ & \mathbf{K} \text{kid}(\text{joey}); \\ & \text{Koffer}(X, \text{cake}) \leftarrow \mathbf{K} \text{Person}(X), \text{not Diabetic}(X) \} \end{aligned}$$

- Offer cake to persons, with exception of diabetics
- Assume that, by default, a person is not suffering from diabetes
- Not expressible in $\mathcal{DL}+log$ in this way

Assessment

	dl-programs	<i>DL+log</i>	QEL	hybrid MKNF
Distinguish classical and rule predicates	+	+	-	-
<i>Domain of Discourse for P</i>				
Herbrand Universe of <i>P</i>	-	+/-	-	+
Combined Signature	+	+/-	-	+
Arbitrary domains	-	-	+	-
<i>Uniqueness of Names</i>				
unique names in HU of <i>P</i>	+	+/- ¹	-	+/- ¹
Special equality predicate	- ²	- ²	-	\approx
No uniqueness	-	+/-	+	+/-
<i>Interaction from FO Theories to Rules</i>				
Single models	-	+	+	+
Multiple models / Entailment	+	-	-	-
<i>Interaction from Rules to FO Theories</i>				
Single models	-	+	+	-
Multiple models / Entailment	+	-	-	+
Decidability	+ ³	+ ⁴	-	+ ³

- 1 The setting (standard names, Herbrand interpretations) implies the UNA.
- 2 Extendible with axioms for a congruence relation.
- 3 Essentially, satisfiability for the underlying DL must be decidable.

Systems / Implementations of Combinations

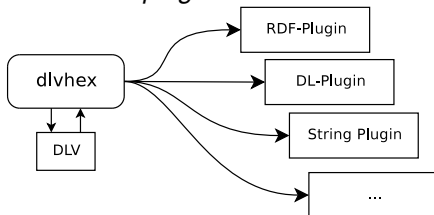
- NLP-DL implements dl-programs (prototype)

<http://con.fusion.at/nlpdl/>

- DLVHEX implements fragments of HEX-programs (prototype)

<http://con.fusion.at/dlvhex/>

Feature: external software *plugins*



Used for various applications (ontology merging, bio-ontologies, e-government, web querying, policy management, ...)

- ...

- **Probabilistic ASP for SW** [Lukasiewicz, ESQUARU05;07]
- **Fuzzy ASP for SW** [Lukasiewicz, RuleML06],
[Lukasiewicz & Straccia, RR07]
- **Rules plus RDF/S:**
Stable model theory for extended RDF/S [Analyti et al., ISWC05]
- **Well-founded semantics**
[E_ et al., RuleML04], [Drabent & Maluszynski; RR07] [Knorr et al., DL07]

- ASP has potential for Semantic Web, for different uses
- Promising applications and usages (e.g., SPARQL, TRIPLESTORES)
- **Pro's and Con's**
 - + Declarativity
 - + Expressiveness
 - + Rich suite of constructs and extensions
 - Mismatch to standard ontologies
 - Scalability (for some tasks)
 - Less commercialized

- **Semantics for rules plus ontologies?**

Only small scale KBs; larger examples / use cases are missing

- **Computational and semantic properties**

- expressiveness

- **Efficient implementations, algorithms**

- scalable complexity

- **Knowledge combination/integration beyond ontologies**

E.g., with rule bases under different semantics, cf. [Baral et al., RuleML06]

- **Language extensions of ASP**

E.g. need for function symbols, value invention (increasing domains)

- **Further applications**

More in depth tutorials on “Answer Set Programming for the Semantic Web,” covering different aspects and providing hand-on examples:

- Tutorial at ESWC 2006, available at
<http://rease.semanticweb.org/>,
<http://asptut.gibbi.com/>,
<http://con.fusion.at/asptut>
- Tutorial at ESTC 2007
- Tutorial at “Giornata GULP: Applications of LP and related CL paradigms” March 9th, 2007 (slides are available on the Web)

Background: https://www.mat.unical.it/ianni/wiki/Answer_Set_Programming_for_the_Semantic_Web



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