Applied Cryptography
Week 3: Block Ciphers

Bernardo Portela

M:ERSI, M:SI - 23

Defining Block Ciphers

A block cipher is defined by two deterministic algorithms

Encrypt: $E(k, p)$

- Takes a key $k \in \{0, 1\}^\lambda$
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A block cipher is **invertible**: k defines a **permutation**

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Advantage: $|\Pr[b = b'] - \frac{1}{2}|$

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Implications

- Ciphertext blocks look totally random
- Different inputs $\Rightarrow$ independent outputs
- Must be impossible to recover key

Selecting the Block Size

E and D work on bitstrings of size B – the *block size*

Data Encryption Standard (DES, 70s-90s): $B = 64$ (8 bytes)

Advanced Encryption Standard (AES, 2000s-): $B = 128$ (16 bytes)

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- Block must be small for efficient SW/HW implementation
- Block cannot be too small
    - Constructions based on block ciphers
    - Key space 2^λ
    - Block size must be close to the security parameter $B \approx \lambda$

Some encryption schemes based on block constructions are insecure if the block size is too small (64 can be problematic). More information **here**

Iterated Ciphers: Rounds

Shorter descriptions and code/HW footprints:

- Simple and efficient round algorithm R
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- Block cipher iterates round algorithm n times

- Each round takes a different key
  - Round key *derived* from block cipher key
  - Sequence of round keys called *key schedule*
- Decrypting follows the same method in reverse
- E.g. for a 3 round scheme:

$$c \leftarrow E(k, p) = R_3(k_3, R_2(k_2, R_1(k_1, p)))$$
$$p \leftarrow D(k, c) = R_1^{-1}(k_1, R_2^{-1}(k_2, R_3^{-1}(k_3, c)))$$

Round Functions #1: Substitution-Permutation Networks

- **Substitution:** S-boxes are small lookup tables (4-8 bits) designed to introduce non-linearity in the round function. They create *confusion*
- **Permutation:** Bit-level transformations (e.g. switches) or algebraic functions that introduce dependencies across the whole block (*diffusion*)

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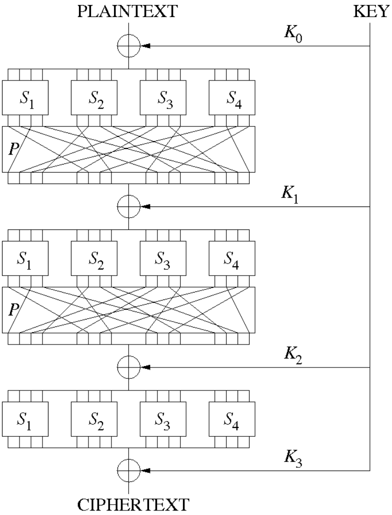
S-boxes heuristically designed to

- Create complex relations between input and output
- Minimize statistical bias in outputs

Example block cipher: AES

Substitution-Permutation Networks - High-level View



(from Wikipedia)

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Round function processes half of the block

- Input block seen as pair (l, r)
- Output block is $(r \oplus R(k_i, l), l)$
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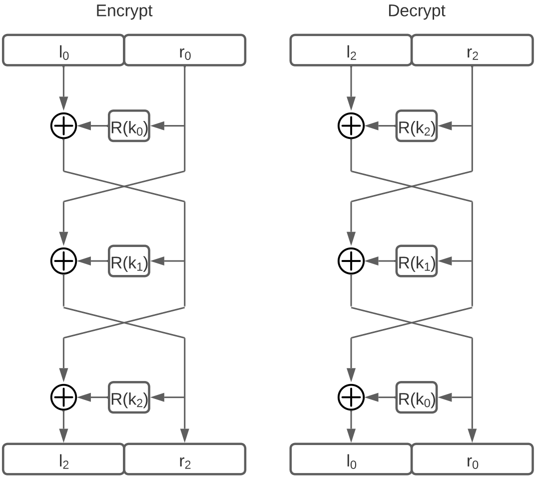
Unprocessed half-block is masked to the next round

Decryption is identical to encryption

- Only key scheduling is inverted
- Very important for HW optimization in the 70s

Example block cipher: DES, GOST

Feistel Networks - High-level View



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- If the round function is secure, 4 rounds ensure a PRP!
- Practical block ciphers use extra rounds
  - Round functions heuristically designed

Advanced Encryption Standard (AES)

AES was standardized in 2000

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains $E(k_1, D(k_2, E(k_3, p)))$

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AES is now the most used block cipher, by far

- Available in mainstream CPUs as HW implementation

Selected as a result of a competition

- 1997-2000 public competition run by NIST
- This process has since become the norm
- Open to proposals, scrutinized by the community
- Criteria: performance and resistance to cryptanalysis

Internals of AES

- Block size 128-bits and varying key size (128, 192, 256)-bits
- Keeps a 128-bit internal state: 4 x 4 array of 16-bits
- State is transformed using a substitution-permutation network

s_0	s_4	s_8	s_{12}
s_1	s_5	s_9	s_{13}
s_2	s_6	s_{10}	s_{14}
s_3	s_7	s_{11}	s_{15}

Substitutions/permutations have an algebraic description

Internals of AES - Explained

The substitution-permutation network uses:

- **AddRoundKey** - $\oplus$ with the state
- **SubBytes** - Replace each byte using lookup table (S-Box)
- **ShiftRows** - Matrix rows shifted 0..3 positions
- **MixColumns** - Columns transformed

Internals of AES - Explained

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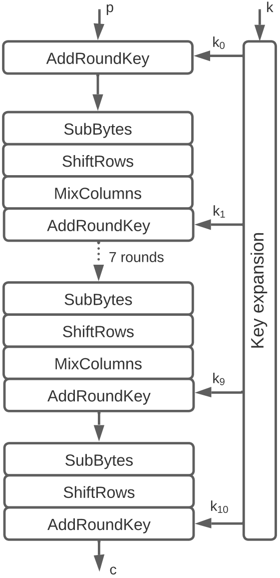
SubBytes performs the substitution part

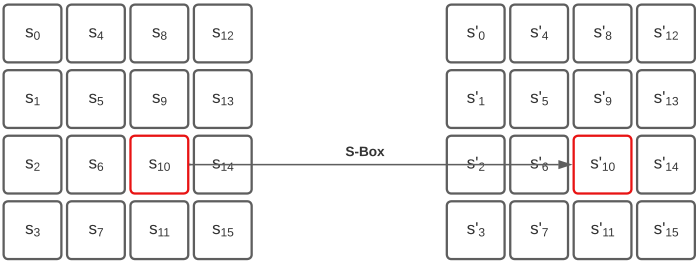
ShiftRows and **MixColumns** are the permutation

Last round has no **MixColumns**. Not necessary. Read more **here**

Internals of AES - High Level View



Internals of AES - SubBytes



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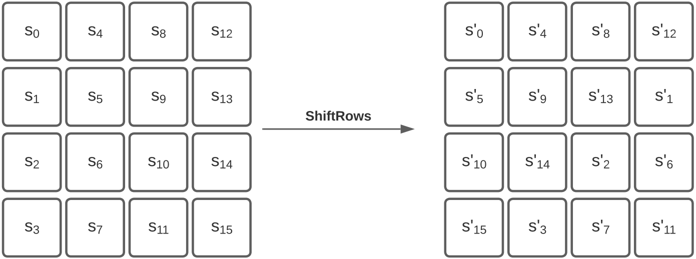

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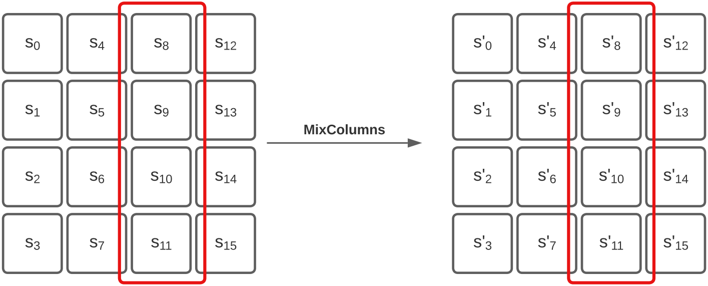

Implementing AES

The not so good

- AES is hard to implement in software
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The good

- AES is super fast in mainstream processors
- AES-NI - AES Native Instructions
- From SW one can resort to HW AES

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Assuming AES is a PRP gives us provably secure and very efficient symmetric encryption schemes

Using Block Ciphers Directly

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Q: What problem arises in using this to encrypt messages?

Modes of Operation

Modern cryptography clearly defines these concepts

- Block-ciphers are a **primitive**
- On their own, they're not very useful
- There are **insecure** ways to encrypt with a block cipher
- Encryption schemes have their own security definitions
- Encryption schemes built from block ciphers
- We prove encryption secure assuming a block cipher PRP

Defining Symmetric Encryption

Syntax

- Key Generation: Often uniform sampling in $\{0,1\}^\lambda$
- Encryption: Probabilistic algorithm $c \leftarrow_{\$} E(k,m)$
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Security (IND-CPA)

- Experiment samples k and bit b uniformly at random
- Attacker can query encryptions of chosen messages
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Insecure Encryption from Secure Block Ciphers

Electronic-Code-Book Mode (ECB)

- Break message into plaintext blocks $p_0, \ldots, p_n$
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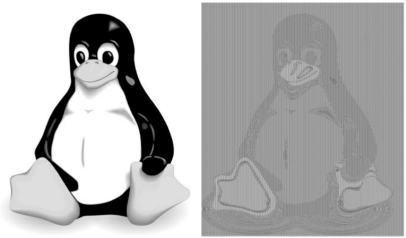

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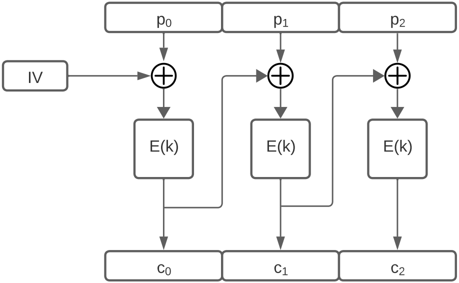
Q2: Can we prove it is insecure not querying exactly m_0/m_1?

Cipher Block Chaining

Engineers designed a secure encryption scheme before security proofs were well understood



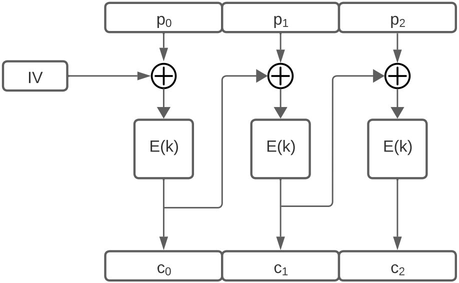
- Main difference to ECB is the Initialization Vector (IV)
- Blocks depend on each other

Cipher Block Chaining: Performance and Security

Intuition of CBC security

- Random IV makes first block-cipher input random
- Block cipher security implies c_1 looks random and independent
- CBC uses c_1 as the IV for the second block
- Same argument for c_2
- Two encryptions of the same plaintext look independent

Working with CBC



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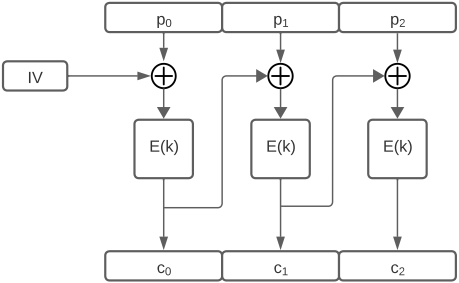

- **Q1: How can we do decryption?**
- **Q2: Can we speed encrypt/decrypt with parallelism?**

CBC: Padding

There are several padding methods

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The most common padding scheme is specified in PKCS#7:

- Let $k > |M|$ be the next multiple of B (in bytes)
- Add $k - |M|$ bytes with value $k - |M|$
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  - $0x01$ means 1 byte of padding with that value
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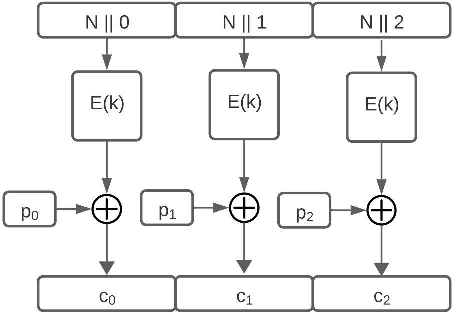
Q: What is the minimum and maximum of added padding?

Counter Block Mode

Often Counter Block Mode (CTR) is used in Nonce-based form



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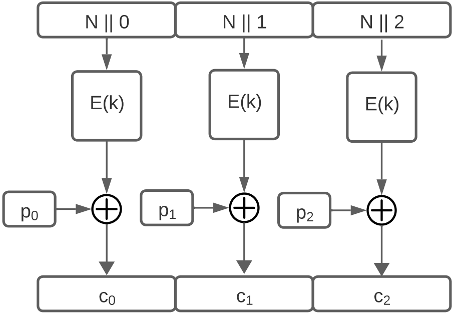

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- Encryption becomes stateful
- **Q: How can this be faster than CBC?**

Advantages of CTR

Counter mode is very efficient

- Key stream can be pre-processed
  - Block cipher not applied to the message!
- Any part of the data can be accessed efficiently
- This includes read/write access
- Decryption/encryption can be parallelized

As such, many modern protocols rely on CTR mode

Errors in Designing Modes of Operation

Recall the guarantees of IND-CPA

- Attacker has access to encryptions
- Can't extract any information about messages
- What if it has access to side information on decryption?
- No guarantee that modified ciphertext is rejected: what leaks?

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A (very real) practical example:

- Padding oracle attacks against AES-CBC (TLS 1.*)
- Attacker gets to observe padding check error
- This is enough to recover plaintext (e.g. cookies)

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- Attacker has access to encryptions
- Can't extract any information about messages
- What if it has access to side information on decryption?
- No guarantee that modified ciphertext is rejected: what leaks?

A (very real) practical example:

- Padding oracle attacks against AES-CBC (TLS 1.*)
- Attacker gets to observe padding check error
- This is enough to recover plaintext (e.g. cookies)

At the root of the problem: allowing non-authenticated ciphertexts

Applied Cryptography
Week 3: Block Ciphers

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