# Applied Cryptography
# Week 4: Hash Functions and Keyed Hashing

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M:ERSI, M:SI - 23

# What is a Hash Function?

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- Key derivation
- Digest for authentication
- Randomness extraction
- Password protection
- Proofs of work

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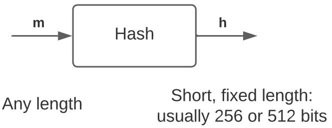
Not only in crypto:

- Indexing in version management
- Deduplication in cloud storage systems
- File integrity in intrusion detection



# Describing Hash Functions

THe hash output is short, aka hash, fingerprint or digest

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Signing $H(m)$ is **as secure** as signing $m$

Hash functions need to be deterministic and public

- Everyone should be able to recompute hash/identifier
- ... So what do we mean by security here?

# Secure Cryptographic Hash Functions

Efficient algorithms with nice properties

- Unpredictable outputs
- Hard to find pre-images
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## Efficient algorithms with nice properties

- Unpredictable outputs
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## Hash functions are validated heuristically

- Similar to process for AES
- International competition for select designs
- Competitors are scrutinized wrt security and performance
- Several rounds, so more eyes on small number of proposals
- Most recent one: SHA-3

# #1: Pre-image resistance

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## Pre-image experiment

- Let $\mathcal{S}$ be the set of pre-images (domain)
- Let $\mathcal{R}$ be the set of images (range)
- Attacker is given a value $y \in \mathcal{R}$
- Attacker guesses $x \in \mathsf{S}$ and wins if $h(x) = y$

# #2: Collision Resistance (CR)

- By definition, collisions **must exist**.
  - Recall that $|\mathcal{S}| >> |\mathcal{R}|$
- This can be argued from the *pidgeonhole principle*
  - If you have $m$ holes and $n$ pidgeons to put in these holes, if $n > m$, at least one hole will have more than one pidgeon!
- But can we find $m_0$ and $m_1$ s.t. $h(m_0) = h(m_1)$?

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Suppose we have the *best possible hash function*?
**Q1: What could that be?**

- Lets think of the probability of collision
- Outputs are random, so $1/2^n$ where $n$ is the output length
- Collision will be found if we check roughly $2^n$ pairs

**Q2: Is CR harder or easier then pre-image resistance?**

# Breaking Hash Functions

## Attack that finds a pre-image

- Search through all possible pre-images (brute-force)
- Consider a perfect hash function with output of $n$ bits
- Cost: $2^n$ operations!
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But what if we only want to find a collision?

# Finding Collisions

Collisions can be found with work $\sqrt{2^n}$, much better than $2^n$!

## Methodology

- Compute values like the brute-force attack
- Store them in a data structure *indexed by image value*
- Each new image value is searched in data structure
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- After $n$ values, we checked $n * (n-1)/2$ pairs **Q: why?**

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## How many operations?

- After $n$ values, we checked $n * (n-1)/2$ pairs **Q: why?**
- Checking $2^n$ pairs takes roughly $\sqrt{2^n}$ values
- Overall complexity is that of finding the pre-image of a hash with $n/2$ bits of output (only half of the range)

**The birthday paradox** (not very paradoxical, just counterintuitive)

# Implication of Birthday Attacks

**For CR, hash outputs must be 2x security parameter**

- 128-bit security $\rightarrow$ 256-bit hashes
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We can use security-parameter-sized hash outputs when:

- Security against arbitrary collisions is not required
- E.g. we might only need pre-image resistance
- Deriving a key from a secret input

# Building Hash Functions

Two main approaches that use iterative processes

- **Merkle-Damgård construction:** Used for MD4, MD5, SHA-1, SHA-256, SHA-512. Relies on a $m + n$-to-$n$ bits compression function to construct a hash function of output length $n$ for arbitrary input lengths

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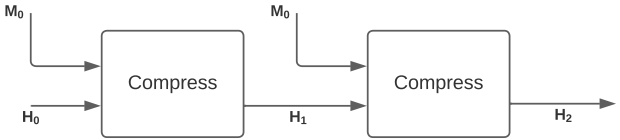
- **Sponge construction:** Used for SHA-3, uses a $l$-bit permutation to construct a hash function for arbitrary input and output lengths

# Merkle-Damgård Construction

All prominent hash functions from 80s-2000s.

- $H_0$ is the initial value: constant and **public**
- $M$ is broken into blocks of size $m$, $M_1, M_2, \ldots$



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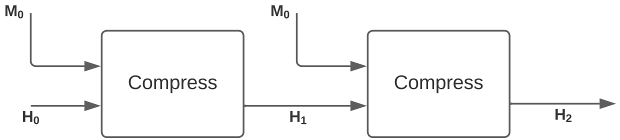


- SHA-256: block size 512, output size 256 bits
- SHA-512: block size 1024, output size 512 bits
- What if messages are not of the same size as the block?

# Merkle-Damgård Construction – Padding

## Padding is always added to the message

- Append the message with a 1 bit
- Fill with zeros up to 64/128 bits away from the block end
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Message is: 10101010**1**00000(...)000001000

**Q: Can't we just pad by adding 0s?**

# Merkle-Damgård Construction – Security

## Useful result

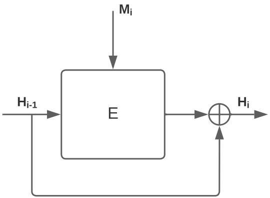
- Compression result is CR (for small inputs)
- Then the whole construction is CR (for arbitrary inputs)

To break the hash function you must break the compression function

So, does having a $2n$-to-$n$ CR compression function solve all our problems?

# Compression Functions: Davis-Meyer

All popular MD constructions use the Davis-Meyer construction:

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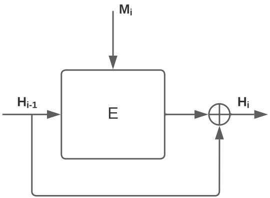


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$$H_i = E(M_i, H_{i-1}) \oplus H_{i-1}$$
$$H_i = E(M_i, D(M_i, 0)) \oplus D(M_i, 0)$$
$$H_i = H_{i-1}$$

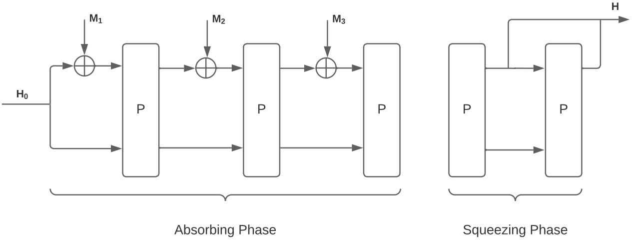
# Sponge Construction

A more recent alternative to the MD is the sponge construction

It relies on a fixed (non-keyed) permutation

## Very Versatile

- Varying input/output lengths
- PRGs and stream ciphers
- PRFs and keyed hashes



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- Fixed initial value $h_0$, gradually accumulate message into state
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## Squeeze

- Dual process iteratively constructs output
- Output constructed block by block
- Permutation computed over the entire state
- Block-sized part of the state is accumulated in the output

# MD5

- Broken! 128-bit output
- Most popular hash function until broken in 2005
- These days, it takes seconds to find collisions
- The SHA function family (next) uses a similar design

# Secure Hash Function (SHA)

Standardized by NIST in the US. International *de facto* standard

SHA-0 published in 93', replaced with SHA-1 in 95'

- Both with 160-bit outputs
- Vulnerability not public at the time
- Later discovered collision attack in $2^{60} << 2^{80}$ operations
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SHA-1 remained unbroken until quite recently – (**2017**) Most

applications currently use SHA-2 (256 or 512 bits)

- Same design principles; larger parameters

Future applications adopting SHA-3 evolve to the Sponge

- Flexible output size is very useful!

# SHA-1 Internals

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```
SHA1-blockcipher(a, b, c, d, e, M) {
  W = expand(M);
  for i = 0 to 79 { // K are constants
    new = (a <<< 5) + f(i, b, c, d) + e + K[i] + W[i]
    (a, b, c, d, e) = (new, a, b >>> 2, c, d)
  }
  return (a, b, c, d, e)
}
```

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No non-generic attacks exist on these hash functions

- Still SHA-3 was (prudently) developed with different design
- Also has the benefit of varying sized outputs
- Good to generate keys!

# SHA-3

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## Keccack is very different and very flexible

- Sponge based with 1600-bits permutation (in SHA-3)
- Blocks can be 1152, 1088, 832 or 576 bits
- Corresponding to 224, 256, 384 or 512 bit outputs
- As a bonus we get the SHAKE functions
    - SHAKE128 and SHAKE256
    - eXtendable Output Functions (XOFs)
    - You can specify output length

# MACs as Keyed Hashes

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## Message Authentication Codes – MACs

- Symmetric Authentication $t \leftarrow \mathrm{MAC}(k, m)$
- $t$ guarantees that $m$ was produced by someone that knows $k$
- Implies message $m$ was not changed since its creation
- Digital signatures in the symmetric paradigm!

# Message Authentication Codes

## Typical use of MACs – SSH, IPSec, TLS

- Two parties was message authentication and integrity
- Some form of set-up/agreement to establish common key $k$
- Sender computes $t \leftarrow \mathrm{MAC}(k, m)$ and sends $(m, t)$
- Receiver gets $(m, t)$, recomputes $t' \leftarrow \mathrm{MAC}(k, m)$
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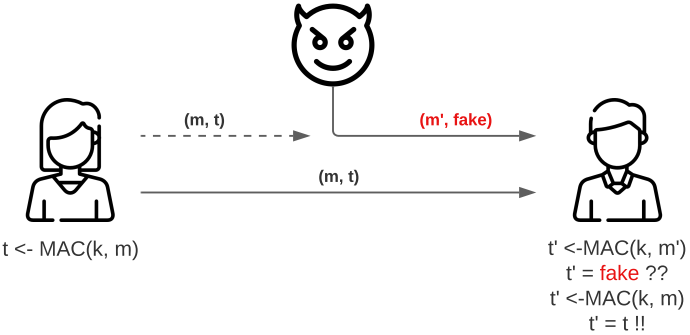
Acceptance means $m$ was produced while knowing $k$

In this process, message is public!

MACs do not give confidentiality. They provide integrity

Its orthogonal to encryption. In real-world applications, we will need to combine these

# Authentication and Message Integrity




- No possibility of computing $t$ without $k$ implies
- Adversary cannot change the message
- Adversary cannot conjure new messages

# MAC Security

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## Security Experiment

- Experiment generates a key $k$
- Adversary (adaptively) sends $m$ to get $t \leftarrow \text{MAC}(k, m)$
- Eventually, attacker outputs $(m^*, t^*)$

Attacker wins if $t^* = \text{MAC}(k, m^*)$, and if $t^*$ was not produced by the experiment. Contrary to IND-CPA, a victory here implies a broken MAC scheme.

# MAC Security Nuances

- MAC on its own does not protect against replay attacks
- Suppose a network scenario
  - Attacker sees authenticated message $(m, t)$
  - Delivers $(m, t)$ multiple times
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# MAC Security Nuances

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- Suppose a network scenario
  - Attacker sees authenticated message $(m, t)$
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  - MAC will verify every time!
- Simple technique: impose message never repeats in network
- Sequence numbers
  - Prepend counter and keep counter as state in both sides
  - Prepend timestamp (local clock reading)
  - How should the receiver operate in both cases?

# Some Context

MACs constructed from hash functions and block ciphers

Simplest construction: prefix key

$$\mathsf{MAC}(k, m) = \mathsf{H}(K||M) \text{ or } \mathsf{PRF}(k, m) = \mathsf{H}(K||M)$$

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MD yields insecure MAC and PRF!

- Given $(m, t)$, attacker outputs $\mathrm{H}(K||M||pad||M')$
- This can be computed just from $t'$ and $m'$
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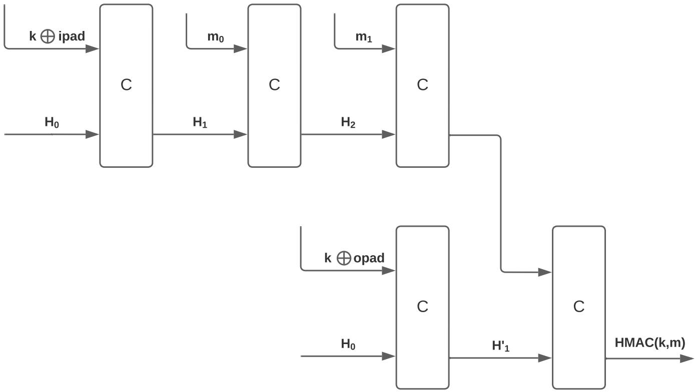
## A consideration in SHA-3 construction

- Abandon MD construction
- Include explicit keyed hash

# HMAC Construction

## When instantiated with MD construction

- Compression function is PRF → Secure MAC
- HMAC is simply $\mathrm{H}((K \oplus opad)||\mathrm{H}((k \oplus ipad)||m))$
- *ipad* and *opad* are constraints: align to block size



# On Collision-based Forgeries

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## Collisions in MAC also yield forgeries

- True for **any** MAC
- Collision occur when $\sqrt{2^n}$ MACs are issued

# Building MACs from Block Ciphers

We have seen block ciphers $\rightarrow$ hash functions $\rightarrow$ MACs

But there are also direct constructions: block ciphers $\rightarrow$ MACs

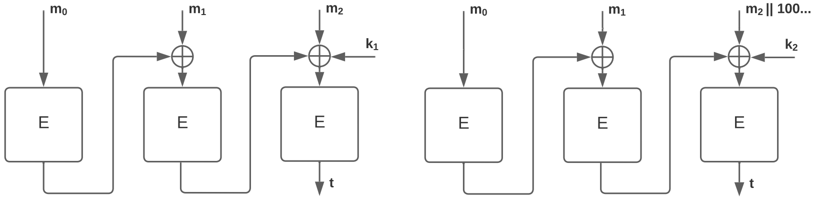
## CMAC

- Used in IPSec
- CMAC improves on CBC-MAC (which was broken!)
- Use CBC mode of operation
- Fix IV to all zero blocks
- Take the last ciphertext block as a tag

# CMAC Internals

## CMAC fixes CBC-MAC by processing last block differently

- All blocks except last are processed like CBC-MAC
- Keys $k_1$ and $k_2$ derived from $k$
  - $l \leftarrow E(k, 0)$
  - $k_1 = (l << 1) \oplus (0x00..0087 * LSB(l)))$
  - $k_2 = (k_1 << 1) \oplus (0x00..0087 * LSB(k_1)))$



# Custom MAC Constructions

More efficient MAC constructions are designed from scratch

Poly1305 is one such construction by D. J. Bernstein

Based on

- Universal Hash Functions
- Wegman-Carter construction

# Universal Hash Functions

## UHF are a Weak form of Hashing

- Don't need to be collision resistance
- Parametrised by a key $\mathrm{UH}(k, m)$
- Guarantee that, for two fixed messages $m_0 \neq m_1$:

$$\Pr[\mathrm{UH}(k, m_0) = \mathrm{UH}(k, m_1)] \leq \epsilon$$

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No other security experiment $\rightarrow$ easy to construct

We can use a universal hash function as a MAC

Provided that we only authenticate **one message!**

# Wegman-Carter Construction

## How to circumvent this limitation?

- Use a PRF to strengthen the UH
- Converts a UH into a fully secure MAC
- AES can fill the PRF role!

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## Intuition: Encrypt Universal Hash Value

$$\mathsf{UH}(k_1, m) \oplus \mathsf{PRF}(k_2, n)$$

- The full MAC key is $(k_1, k_2)$
- $n$ is a public value that must never repeat
    - A.k.a. a nonce
- This can be kept as a counter, or generated at random

# Poly1305-AES: Wegman-Carter in Practice

- Initial proposal used AES as the Wegman-Carter PRF
- The universal hash function uses prime $p^{130} - 5$

$$\text{Poly1305}((k_1, k_2), m) = (m_1 k + \ldots + m_n k^n \pmod{p}) + \text{AES}(k_2, n)$$

- Blocks are 128 bits and last block is padded with 100
- All blocks set bit 129, so MSB is 1
- The final addition is performed modulo $2^{128}$
- TLS recommends Poly1305 with ChaCha20, rather than AES

# Applied Cryptography
# Week 4: Hash Functions and Keyed Hashing

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