# Applied Cryptography Week 3: Block Ciphers

Bernardo Portela

M:ERSI, M:SI - 24

# Defining Block Ciphers

A block cipher is defined by two deterministic algorithms

## Encrypt: E(k, p)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a plaintext block  $p \in \{0,1\}^B$
- Outputs a ciphertext block  $c \in \{0, 1\}^B$

# Defining Block Ciphers

A block cipher is defined by two deterministic algorithms

## Encrypt: E(k, p)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a plaintext block  $p \in \{0,1\}^B$
- Outputs a ciphertext block  $c \in \{0, 1\}^B$

## Decrypt: D(k,c)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a ciphertext block  $c \in \{0, 1\}^B$
- Outputs a plaintext block  $p \in \{0, 1\}^B$

A block cipher is **invertible**: k defines a **permutation** 

Block cipher should be a **pseudorandom permutation** (PRP)

How can we define this concretely?

Block cipher should be a **pseudorandom permutation** (PRP)

How can we define this concretely? Using an experiment:

- Experiment samples uniformly at random:
  - $k \in \{0, 1\}^{\lambda}$
  - permutation  $\pi: \{0,1\}^B \Rightarrow \{0,1\}^B$
  - bit *b*

#### Block cipher should be a **pseudorandom permutation** (PRP)

#### How can we define this concretely? Using an experiment:

- Experiment samples uniformly at random:
  - $k \in \{0, 1\}^{\lambda}$
  - permutation  $\pi: \{0,1\}^B \Rightarrow \{0,1\}^B$
  - bit *b*
- Attacker can ask for encryptions:
  - Attacker selects  $p \in \{0, 1\}^B$
  - If b = 0, experiment returns E(k, p)
  - Otherwise, experiment returns  $\pi(p)$

#### Block cipher should be a **pseudorandom permutation** (PRP)

#### How can we define this concretely? Using an experiment:

- Experiment samples uniformly at random:
  - $k \in \{0, 1\}^{\lambda}$
  - permutation  $\pi: \{0,1\}^B \Rightarrow \{0,1\}^B$
  - bit *b*
- Attacker can ask for encryptions:
  - Attacker selects  $p \in \{0, 1\}^B$
  - If b = 0, experiment returns E(k, p)
  - Otherwise, experiment returns  $\pi(p)$
- Attacker outputs b' and wins if b = b'



#### Block cipher should be a **pseudorandom permutation** (PRP)

#### How can we define this concretely? Using an experiment:

- Experiment samples uniformly at random:
  - $k \in \{0, 1\}^{\lambda}$
  - permutation  $\pi: \{0,1\}^B \Rightarrow \{0,1\}^B$
  - bit *b*
- Attacker can ask for encryptions:
  - Attacker selects  $p \in \{0, 1\}^B$
  - If b = 0, experiment returns E(k, p)
  - Otherwise, experiment returns  $\pi(p)$
- Attacker outputs b' and wins if b = b'

## Q2: How do we calculate the adversarial advantage?

#### Block cipher should be a **pseudorandom permutation** (PRP)

#### **How can we define this concretely?** Using an experiment:

- Experiment samples uniformly at random:
  - $k \in \{0, 1\}^{\lambda}$
  - permutation  $\pi: \{0,1\}^B \Rightarrow \{0,1\}^B$
  - bit b
- Attacker can ask for encryptions:
  - Attacker selects  $p \in \{0, 1\}^B$
  - If b = 0, experiment returns E(k, p)
  - Otherwise, experiment returns  $\pi(p)$
- Attacker outputs b' and wins if b = b'

#### Q2: How do we calculate the adversarial advantage?

**Advantage:** 
$$|\Pr[b = b'] - \frac{1}{2}|$$

Our scheme is *indistinguishable* from a random permutation. What is a random permutation  $(\pi : \{0,1\}^B \Rightarrow \{0,1\}^B)$ , exactly?

## Implications of PRP Security

Our scheme is *indistinguishable* from a random permutation. What is a random permutation  $(\pi : \{0,1\}^B \Rightarrow \{0,1\}^B)$ , exactly?

- Huge table with 2<sup>B</sup> entries, indexed by plaintext p
- Each entry contains a corresponding ciphertext C
- Each C is sampled uniformly at random, without repeats
  - Q: Why must Cs never repeat?

Our scheme is *indistinguishable* from a random permutation. What is a random permutation  $(\pi : \{0,1\}^B \Rightarrow \{0,1\}^B)$ , exactly?

- Huge table with 2<sup>B</sup> entries, indexed by plaintext p
- Each entry contains a corresponding ciphertext C
- Each C is sampled uniformly at random, without repeats
  - Q: Why must Cs never repeat?
  - PRPs are invertible!
  - Different from purely random functions

# Implications of PRP Security

Our scheme is *indistinguishable* from a random permutation. What is a random permutation  $(\pi : \{0,1\}^B \Rightarrow \{0,1\}^B)$ , exactly?

- Huge table with 2<sup>B</sup> entries, indexed by plaintext p
- Each entry contains a corresponding ciphertext C
- Each C is sampled uniformly at random, without repeats
  - Q: Why must Cs never repeat?
  - PRPs are invertible!
  - Different from purely random functions

## **Implications**

- Ciphertext blocks look totally random
- Different inputs ⇒ independent outputs
- Must be impossible to recover key

E and D work on bitstrings of size B – the block size

Data Encryption Standard (DES, 70s-90s): B = 64 (8 bytes)

Advanced Encryption Standard (AES, 2000s-): B = 128 (16 bytes)

E and D work on bitstrings of size B – the block size

Data Encryption Standard (DES, 70s-90s): B = 64 (8 bytes)

Advanced Encryption Standard (AES, 2000s-): B = 128 (16 bytes)

- Block must be small for efficient SW/HW implementation
- Block cannot be too small
  - Constructions based on block ciphers
  - Kev space 2<sup>λ</sup>
  - Block size must be close to the security parameter  $B \approx \lambda$

Some encryption schemes based on block constructions are insecure if the block size is too small (64 can be problematic). More information here

#### Shorter descriptions and code/HW footprints:

- Simple and efficient round algorithm R
- Round algorithm is not as secure as a block cipher
- Block cipher iterates round algorithm *n* times

#### Shorter descriptions and code/HW footprints:

- Simple and efficient round algorithm R
- Round algorithm is not as secure as a block cipher
- Block cipher iterates round algorithm *n* times
- Each round takes a different key
  - Round key derived from block cipher key
  - Sequence of round keys called key schedule
- Decrypting follows the same method in reverse
- E.g. for a 3 round scheme:

$$c \leftarrow E(k,p) = R_3(k_3, R_2(k_2, R_1(k_1, p)))$$
$$p \leftarrow D(k,c) = R_1^{-1}(k_1, R_2^{-1}(k_2, R_3^{-1}(k_3, c)))$$

- Substitution: S-boxes are small lookup tables (4-8 bits) designed to introduce non-linearity in the round function. They create confusion
- Permutation: Bit-level transformations (e.g. switches) or algebraic functions that introduce dependencies across the whole block (diffusion)

- Substitution: S-boxes are small lookup tables (4-8 bits) designed to introduce non-linearity in the round function. They create confusion
- Permutation: Bit-level transformations (e.g. switches) or algebraic functions that introduce dependencies across the whole block (diffusion)

Q: Why is diffusion necessary?

- Substitution: S-boxes are small lookup tables (4-8 bits) designed to introduce non-linearity in the round function. They create confusion
- Permutation: Bit-level transformations (e.g. switches) or algebraic functions that introduce dependencies across the whole block (diffusion)

## Q: Why is diffusion necessary?

Consider the encryption of "Attack at dawn" and "Attack at dusk"

- Substitution: S-boxes are small lookup tables (4-8 bits) designed to introduce non-linearity in the round function. They create confusion
- Permutation: Bit-level transformations (e.g. switches) or algebraic functions that introduce dependencies across the whole block (diffusion)

#### Q: Why is diffusion necessary?

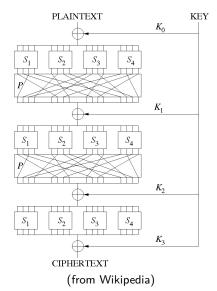
Consider the encryption of "Attack at dawn" and "Attack at dusk"

S-boxes heuristically designed to

- Create complex relations between input and output
- Minimize statistical bias in outputs

Example block cipher: AES

## Substitution-Permutation Networks - High-level View



## Round Functions #2: Feistel Networks

## Round function processes half of the block

- Input block seen as pair (I, r)
- Output block is  $(r \oplus R(k_i, l), l)$
- R is the round function

## Round function processes half of the block

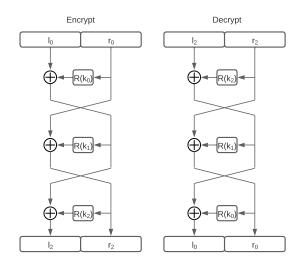
- Input block seen as pair (1, r)
- Output block is  $(r \oplus R(k_i, l), l)$
- R is the round function

Unprocessed half-block is masked to the next round

Decryption is identical to encryption

- Only key scheduling is inverted
- Very important for HW optimization in the 70s

Example block cipher: DES, GOST

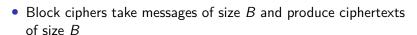


- These can be Pseudorandom Functions (PRFs)
- A PRF is similar to a PRP, but not necessarily invertible
- Input size can be different from output size
- Security experiment is similar to that of the PRP:

- These can be Pseudorandom Functions (PRFs)
- A PRF is similar to a PRP, but not necessarily invertible
- Input size can be different from output size
- Security experiment is similar to that of the PRP:
  - Experiment chooses a random f
  - Rather than a random permutation  $\pi$
  - Q: Is the domain space of random functions larger or smaller than that of all permutations?

- These can be Pseudorandom Functions (PRFs)
- A PRF is similar to a PRP, but not necessarily invertible
- Input size can be different from output size
- Security experiment is similar to that of the PRP:
  - Experiment chooses a random f
  - Rather than a random permutation  $\pi$
  - Q: Is the domain space of random functions larger or smaller than that of all permutations?
- If the round function is secure, 4 rounds ensure a PRP!
- Practical block ciphers use extra rounds
  - Round functions heuristically designed

• Block ciphers take messages of size B and produce ciphertexts of size B



- We want them to behave like pseudo-random permutations
  - The ciphertext might as well have been a random permutation
  - ... that has nothing to do with the key

- Block ciphers take messages of size B and produce ciphertexts of size B
- We want them to behave like pseudo-random permutations
  - The ciphertext might as well have been a random permutation
  - ... that has nothing to do with the key
- There are two main ways to build block ciphers
  - SPN Substitution-Permutation Networks
  - ... We substitute, then permute
  - Feistel Networks
  - ... We transform right side, then swap

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains  $E(k_1, D(k_2, E(k_3, p)))$

# Advanced Encryption Standard (AES)

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains  $E(k_1, D(k_2, E(k_3, p)))$
- Q1: Why not 2DES?

# Advanced Encryption Standard (AES)

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains  $E(k_1, D(k_2, E(k_3, p)))$
- Q1: Why not 2DES? Meet-in-the-middle: 2<sup>57</sup>!

# Advanced Encryption Standard (AES)

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains  $E(k_1, D(k_2, E(k_3, p)))$
- Q1: Why not 2DES? Meet-in-the-middle: 2<sup>57</sup>!
- Q2: Why EDE and not EEE?

# Advanced Encryption Standard (AES)

#### AES was standardized in 2000

- DES was still standard (56-bit keys)
- 3DES was a common solution for short keys (112-bit security)
- 3DES: use DES 3 times with 3 independent keys
- 3DES chains  $E(k_1, D(k_2, E(k_3, p)))$
- Q1: Why not 2DES? Meet-in-the-middle: 2<sup>57</sup>!
- Q2: Why EDE and not EEE?

AES is now the most used block cipher, by far

Available in mainstream CPUs as HW implementation

## Selected as a result of a competition

- 1997-2000 public competition run by NIST
- This process has since become the norm
- Criteria: performance and resistance to cryptanalysis

- Block size 128-bits and varying key size (128, 192, 256)-bits
- Keeps a 128-bit internal state: 4 x 4 array of 16-bits
- State is transformed using a substitution-permutation network



Substitutions/permutations have an algebraic description

#### The substitution-permutation network uses:

- AddRoundKey ⊕ with the state
- **SubBytes** Replace each byte using lookup table (S-Box)
- **ShiftRows** Matrix rows shifted 0..3 positions
- MixColumns Columns transformed

# Internals of AES - Explained

The substitution-permutation network uses:

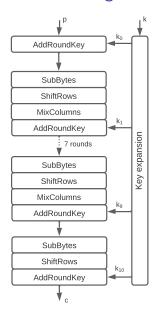
- AddRoundKey ⊕ with the state
- SubBytes Replace each byte using lookup table (S-Box)
- ShiftRows Matrix rows shifted 0..3 positions
- MixColumns Columns transformed

**SubBytes** performs the substitution part

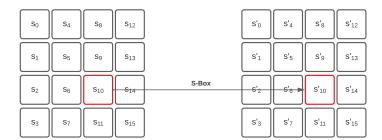
**ShiftRows** and **MixColumns** are the permutation

Last round has no MixColumns. Not necessary. Read more here

# Internals of AES - High Level View

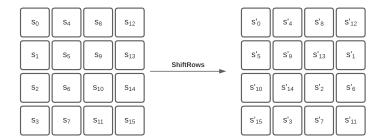


# Internals of AES - SubBytes

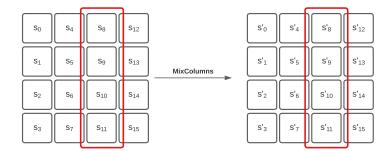


## Internals of AES - ShiftRows

Advanced Encryption Standard



## Internals of AES - MixColumns



000000000

Advanced Encryption Standard

## The not so good

- AES is hard to implement in software
- Naive implementations using tables leak via side-channels
- Removing side-channels in software is hard

## The not so good

- AES is hard to implement in software
- Naive implementations using tables leak via side-channels
- Removing side-channels in software is hard

## The good

- AES is super fast in mainstream processors
- AES-NI AES Native Instructions
- From SW one can resort to HW AES

00000000

Advanced Encryption Standard

There is no mathematical proof that AES is a PRP All practical applications based on AES assume this 00000000

Advanced Encryption Standard

There is no mathematical proof that AES is a PRP

All practical applications based on AES assume this

AES has been around for 25 years:

- No significant cryptanalysis progress
- AES scrutiny is an important area of research
- Direct attack on AES unlikely to be the weakest link

There is no mathematical proof that AES is a PRP

All practical applications based on AES assume this

AES has been around for 25 years:

- No significant cryptanalysis progress
- AES scrutiny is an important area of research
- Direct attack on AES unlikely to be the weakest link

Assuming AES is a PRP gives us provably secure and very efficient symmetric encryption schemes

Recall our secure PRP block cipher building block:

# Encrypt: E(k, p)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a plaintext block  $p \in \{0,1\}^B$
- Outputs a ciphertext block  $c \in \{0,1\}^B$

## Decrypt: D(k,c)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a ciphertext block  $c \in \{0,1\}^B$
- Outputs a plaintext block  $p \in \{0, 1\}^B$

Recall our secure PRP block cipher building block:

# Encrypt: E(k, p)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a plaintext block  $p \in \{0,1\}^B$
- Outputs a ciphertext block  $c \in \{0,1\}^B$

## Decrypt: D(k,c)

- Takes a key  $k \in \{0,1\}^{\lambda}$
- Takes a ciphertext block  $c \in \{0, 1\}^B$
- Outputs a plaintext block  $p \in \{0, 1\}^B$

Q: What problem arises in using this to encrypt messages?

# Modes of Operation

## Modern cryptography clearly defines these concepts

- Block-ciphers are a primitive
- On their own, they're not very useful
- There are **insecure** ways to encrypt with a block cipher
- Encryption schemes have their own security definitions
- Encryption schemes built from block ciphers
- We prove encryption secure assuming a block cipher PRP

## Syntax

- Key Generation: Often uniform sampling in  $\{0,1\}^{\lambda}$
- Encryption: Probabilistic algorithm  $c \leftarrow s E(k, m)$
- Decryption: Deterministic algorithm  $m/\bot \leftarrow D(k,c)$

## Syntax

- Key Generation: Often uniform sampling in  $\{0,1\}^{\lambda}$
- Encryption: Probabilistic algorithm  $c \leftarrow s E(k, m)$
- Decryption: Deterministic algorithm  $m/\bot \leftarrow D(k,c)$

## Security (IND-CPA): Semantic Security

- Experiment samples k and bit b uniformly at random
- Attacker can guery encryptions of chosen messages
- Attacker outputs  $(m_0, m_1)$  s.t.  $|m_0| = |m_1|$
- Attacker gets  $c \leftarrow s E(k, m_b)$
- Attacker outputs b' and wins if b = b'

# Defining Symmetric Encryption

## Syntax

- Key Generation: Often uniform sampling in  $\{0,1\}^{\lambda}$
- Encryption: Probabilistic algorithm  $c \leftarrow s E(k, m)$
- Decryption: Deterministic algorithm  $m/\bot \leftarrow D(k,c)$

## Security (IND-CPA): Semantic Security

- Experiment samples k and bit b uniformly at random
- Attacker can guery encryptions of chosen messages
- Attacker outputs  $(m_0, m_1)$  s.t.  $|m_0| = |m_1|$
- Attacker gets  $c \leftarrow s E(k, m_b)$
- Attacker outputs b' and wins if b = b'

**Advantage:**  $|\Pr[b=b']-\frac{1}{2}|$ 

# Electronic-Code-Book Mode (ECB)

- Break message into plaintext blocks  $p_0, \ldots, p_n$
- Last block may need padding
  - That's a can of worms in and of itself
  - More on that later
- Independently encrypt each block  $c_i \leftarrow E(k, p_i)$

# Electronic-Code-Book Mode (ECB)

- Break message into plaintext blocks  $p_0, \ldots, p_n$
- Last block may need padding
  - That's a can of worms in and of itself
  - More on that later
- Independently encrypt each block  $c_i \leftarrow E(k, p_i)$
- Q: Why is this insecure?

# Electronic-Code-Book Mode (ECB)

- Break message into plaintext blocks  $p_0, \ldots, p_n$
- Last block may need padding
  - That's a can of worms in and of itself
  - More on that later
- Independently encrypt each block  $c_i \leftarrow E(k, p_i)$
- Q: Why is this insecure?

ECB is broken because you can see the penguin!

# Insecure Encryption from Secure Block Ciphers

## Electronic-Code-Book Mode (ECB)

- Break message into plaintext blocks  $p_0, \ldots, p_n$
- Last block may need padding
  - That's a can of worms in and of itself
  - More on that later
- Independently encrypt each block  $c_i \leftarrow E(k, p_i)$
- Q: Why is this insecure?

ECB is broken because you can see the penguin!





#### What is the issue?

- Equal input blocks ⇒ Equal output blocks
- Preserves patterns that vary slower than block size

#### What is the issue?

- Equal input blocks ⇒ Equal output blocks
- Preserves patterns that vary slower than block size

Q1: Can we prove it is insecure (win the game)?

# Breaking ECB

#### What is the issue?

- Equal input blocks ⇒ Equal output blocks
- Preserves patterns that vary slower than block size

## Q1: Can we prove it is insecure (win the game)?

- Output  $m_0 \neq m_1$ ,  $|m_0| = |m_1|$ , get c
- Request an encryption of m<sub>0</sub> to get c\*
- If b' = 0 iff  $c = c^*$

This attack works against **all** deterministic encryption schemes

# Breaking ECB

#### What is the issue?

- Equal input blocks ⇒ Equal output blocks
- Preserves patterns that vary slower than block size

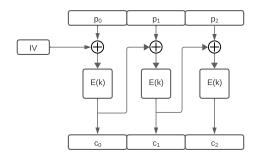
## Q1: Can we prove it is insecure (win the game)?

- Output  $m_0 \neq m_1$ ,  $|m_0| = |m_1|$ , get c
- Request an encryption of m<sub>0</sub> to get c\*
- If b' = 0 iff  $c = c^*$

This attack works against **all** deterministic encryption schemes

## Q2: Can we prove it is insecure not querying exactly $m_0/m_1$ ?

Engineers designed a secure encryption scheme before security proofs were well understood

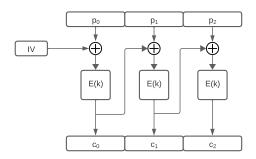


- Main difference to ECB is the Initialization Vector (IV)
- Blocks depend on each other

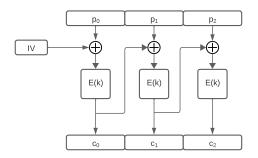
## Intuition of CBC security

- Random IV makes first block-cipher input random
- Block cipher security implies c<sub>1</sub> looks random and independent
- CBC uses  $c_1$  as the IV for the second block
- Same argument for  $c_2$
- Two encryptions of the same plaintext look independent

Symmetric Encryption



• Q1: How can we do decryption?



- Q1: How can we do decryption?
- Q2: Can we speed encrypt/decrypt with parallelism?

#### There are several padding methods

- Some schemes require message size as multiple of block size
- Padding schemes re-encode message so that is true
- To avoid ambiguity: padding is always added

#### There are several padding methods

- Some schemes require message size as multiple of block size
- Padding schemes re-encode message so that is true
- To avoid ambiguity: padding is always added

The most common padding scheme is specified in PKCS#7:

- Let k > |M| be the next multiple of B (in bytes)
- Add k |M| bytes with value k |M|
- The last byte always reveals how much padding was added
  - 0x01 means 1 byte of padding with that value
  - 0x03 means 3 bytes of padding with that value

## There are several padding methods

- Some schemes require message size as multiple of block size
- Padding schemes re-encode message so that is true
- To avoid ambiguity: padding is always added

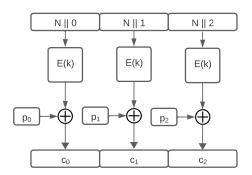
The most common padding scheme is specified in PKCS#7:

- Let k > |M| be the next multiple of B (in bytes)
- Add k |M| bytes with value k |M|
- The last byte always reveals how much padding was added
  - 0x01 means 1 byte of padding with that value
  - 0x03 means 3 bytes of padding with that value

## Q: What is the minimum and maximum of added padding?

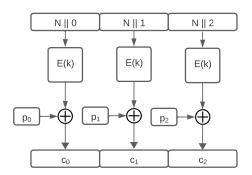
## Counter Block Mode

Often Counter Block Mode (CTR) is used in Nonce-based form



- N must be unique, but not necessarily random
- Encryption becomes stateful

## Often Counter Block Mode (CTR) is used in Nonce-based form



- N must be unique, but not necessarily random
- Encryption becomes stateful
- Q: How can this be faster than CBC?

# Advantages of CTR

## Counter mode is very efficient

- Key stream can be pre-processed
- Block cipher not applied to the message!
- Any part of the data can be accessed efficiently
- This includes read/write access
- Decryption/encryption can be parallelized

As such, many modern protocols rely on CTR mode

# Errors in Designing Modes of Operation

#### Recall the guarantees of IND-CPA

- Attacker has access to encryptions
- Can't extract any information about messages
- What if it has access to side information on decryption?
- No guarantee that modified ciphertext is rejected: what leaks?

#### Recall the guarantees of IND-CPA

- Attacker has access to encryptions
- Can't extract any information about messages
- What if it has access to side information on decryption?
- No guarantee that modified ciphertext is rejected: what leaks?

## A (very real) practical example:

- Padding oracle attacks against AES-CBC (TLS 1.\*)
- Attacker gets to observe padding check error
- This is enough to recover plaintext (e.g. cookies)

# Errors in Designing Modes of Operation

#### Recall the guarantees of IND-CPA

- Attacker has access to encryptions
- Can't extract any information about messages
- What if it has access to side information on decryption?
- No guarantee that modified ciphertext is rejected: what leaks?

### A (very real) practical example:

- Padding oracle attacks against AES-CBC (TLS 1.\*)
- Attacker gets to observe padding check error
- This is enough to recover plaintext (e.g. cookies)

At the root of the problem: allowing non-authenticated ciphertexts

- AES selected via public competition
  - ... as all modern ciphers are

- AES selected via public competition
  - ... as all modern ciphers are
- SubBytes; ShiftRows; MixColumns; AddRoundKey

- AES selected via public competition
  - ... as all modern ciphers are
- SubBytes; ShiftRows; MixColumns; AddRoundKey
- Currently the de facto standard for block ciphers



- AES selected via public competition
  - ... as all modern ciphers are
- SubBytes; ShiftRows; MixColumns; AddRoundKey
- Currently the *de facto* standard for block ciphers
- Block ciphers by themselves are **insecure**

- AES selected via public competition
  - ... as all modern ciphers are
- SubBytes; ShiftRows; MixColumns; AddRoundKey
- Currently the *de facto* standard for block ciphers
- Block ciphers by themselves are **insecure**
- Symmetric encryption requires two ciphertexts to be indistinguishable

- AES selected via public competition
  - ... as all modern ciphers are
- SubBvtes: ShiftRows; MixColumns; AddRoundKey
- Currently the *de facto* standard for block ciphers
- Block ciphers by themselves are **insecure**
- Symmetric encryption requires two ciphertexts to be indistinguishable
- So we rely on modes of encryption: ECB, CBC, CTR

# Applied Cryptography Week 3: Block Ciphers

Bernardo Portela

M:ERSI, M:SI - 24