Applied Cryptography
Week 4: Stream Ciphers

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M:ERSI, M:SI - 24

Context

Historically associated with fragility

- Insecure mobile phone communications
- Weak wi-Fi protocols
- Broken smart card implementations

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Overview

- We will take a look at the reason for said fame
  - A5/1; GSM; RC4
- As well as the reason for their comeback
  - Grain-128A (HW); Salsa20 (SW)

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- k - Key; secret; 128/256 bits
- n - Nonce; public; 64/128 bits
- KS - Keystream; XOR'd with the plaintext
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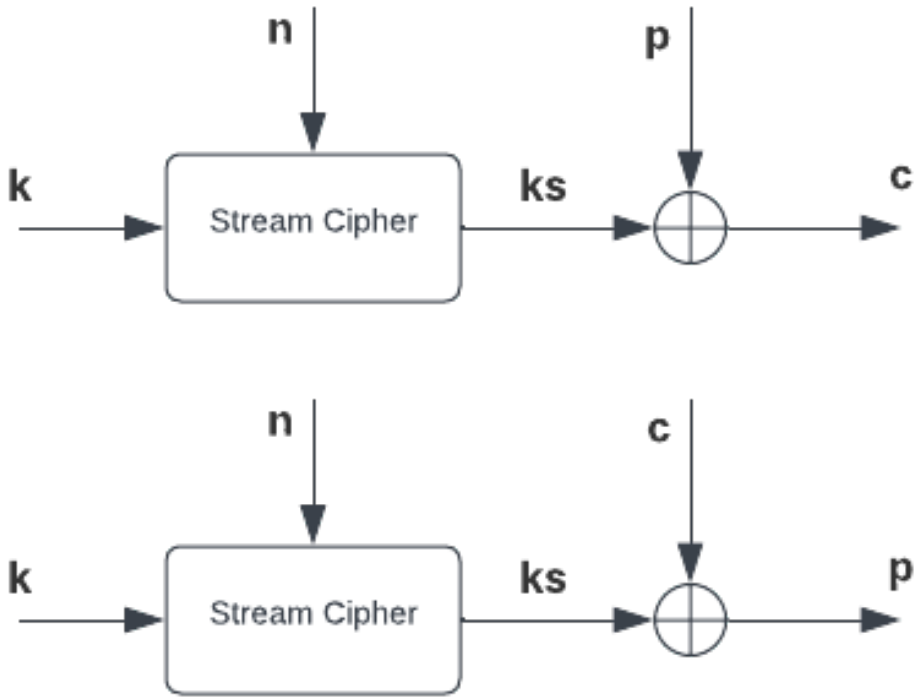
Q: How to decrypt? ... the same as encryption

Stream cipher behavior



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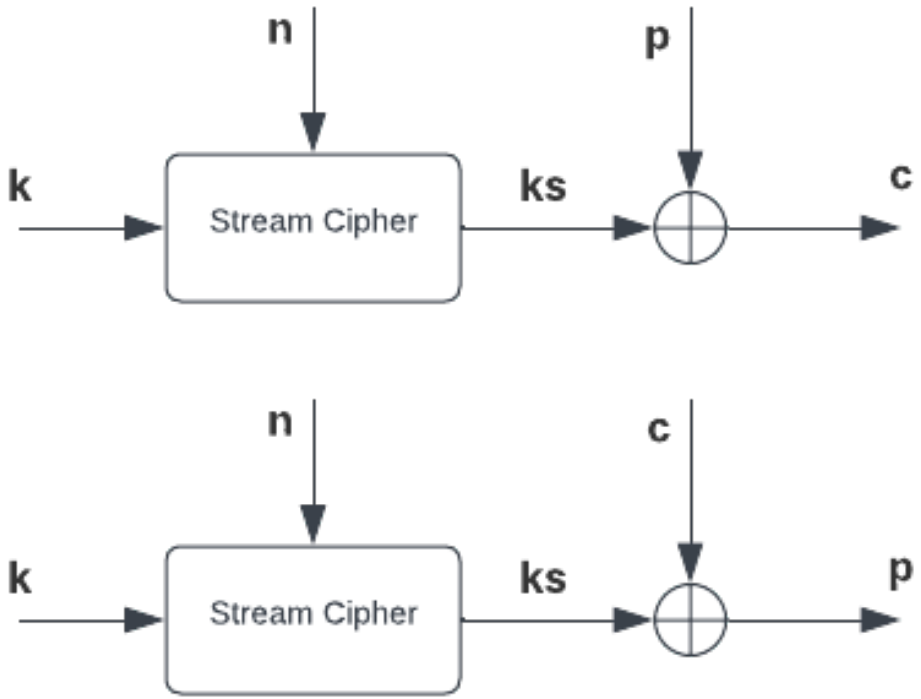

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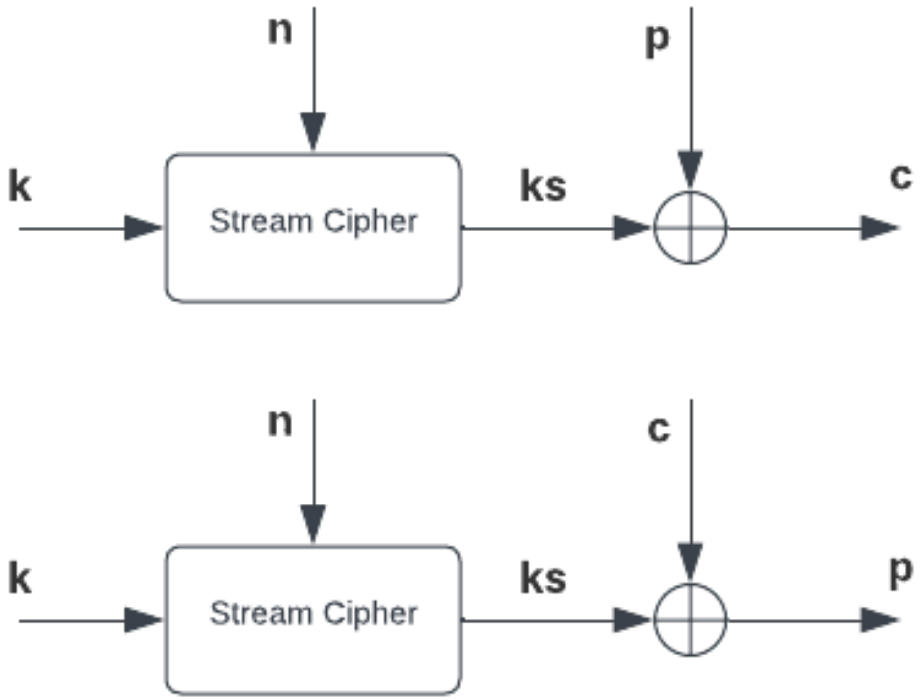

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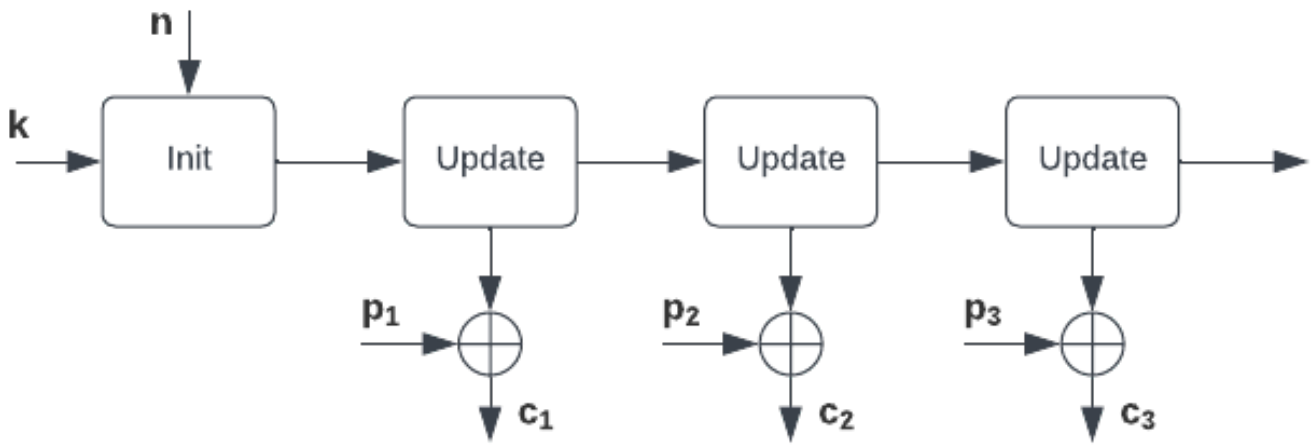
Q: Why? Reveals duplicates

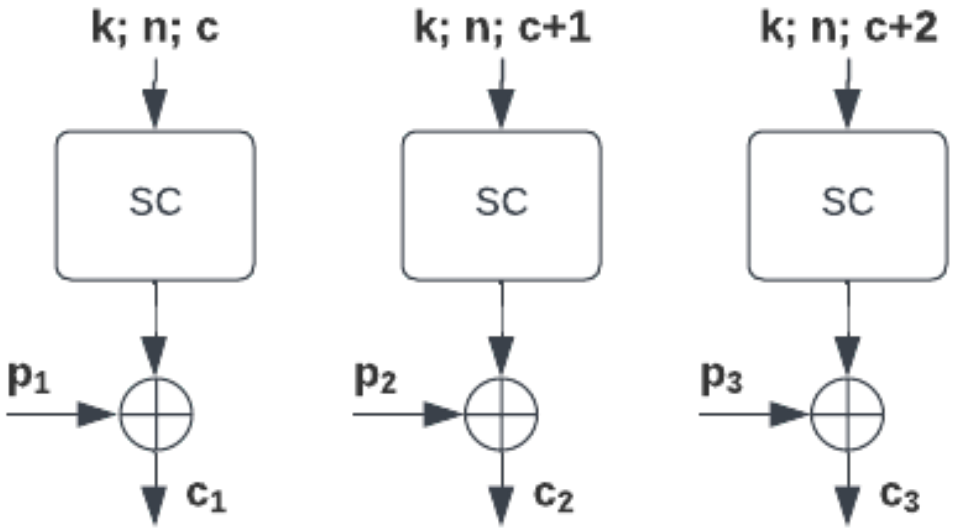
Types of Stream Ciphers - 1

Stateful Stream Ciphers (RC4)



Types of Stream Ciphers - 2

Counter-based Stream Ciphers (Salsa20)



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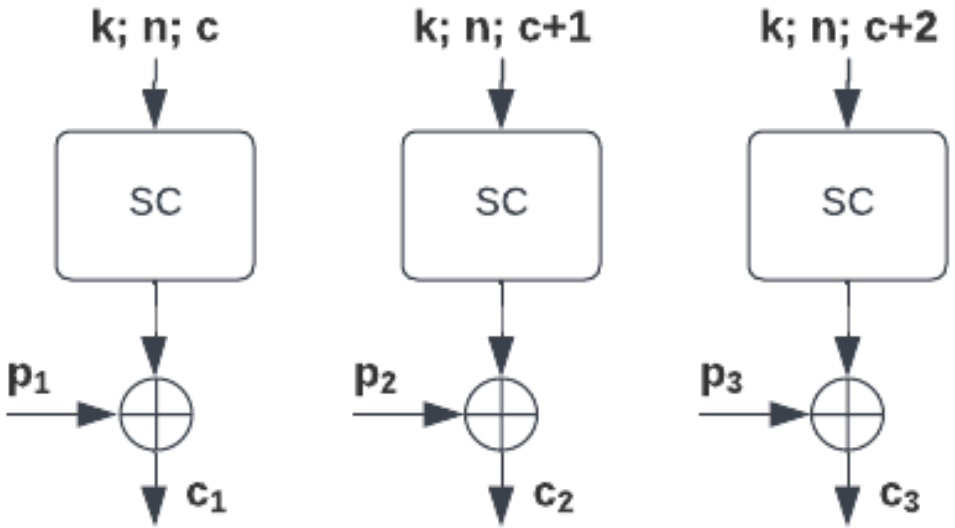

- **Q: What does this remind you of?**

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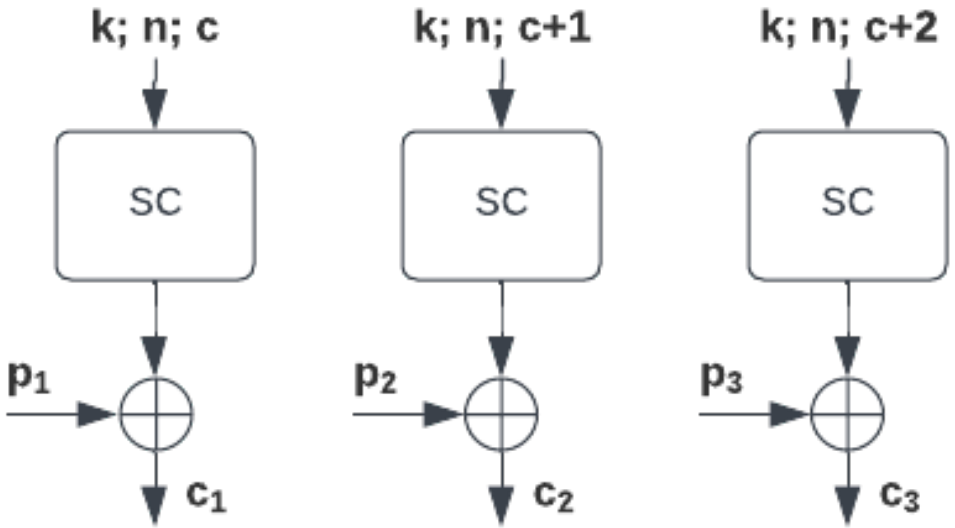

- **Q: What does this remind you of?**
- AES-CTR follows a similar structure

Stream Ciphers and Hardware

Hardware-based ciphers

- Cipher is an electronic circuit implementing the algorithm; can only be used for that
  - Application-specific integrated circuits (ASICs)
  - Programmable logic devices (PLDs)
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Software-based ciphers

- Algorithms tell the system what to do

- HW deals with bits; SW deals with words (32/64 bits)
- Traditionally, SCs were cheaper – used smaller circuits
- Nowadays, block ciphers are not that different
- Distinction is not that obvious nowadays

Feedback Shift Registers (FSRs)

- FSRs are popular because they are *simple* and *well-understood*
- Array of bits + feedback function f

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R_0 is the initial state R_1 is the next state, defined as:

- R_0 left-shifted by 1
- Leftmost bit of R_0 is the output
- Rightmost bit of R_1 filled by $f(R_0)$

FSR Example

4-bit FSR; f XORs all bits; initial state $R_0 = 1100$

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Output $= 1$
$R_1 = 1000$
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$f(R_1) = 1 \oplus 0 \oplus 0 \oplus 0 = 1$
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$R_2 = 0001$

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Output $= 0$
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Period of an FSR

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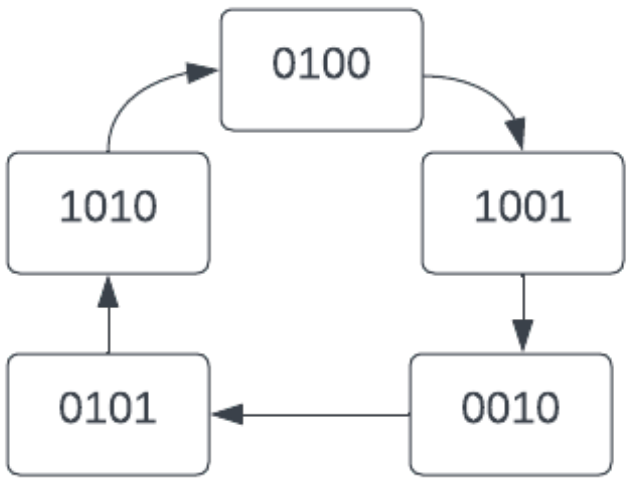

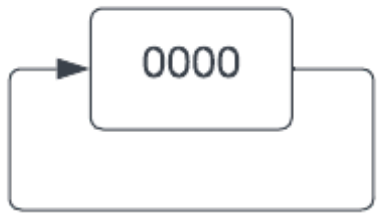

Period of an FSR

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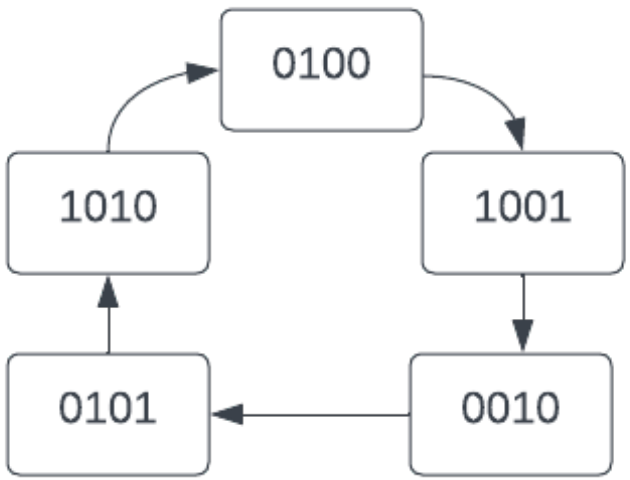

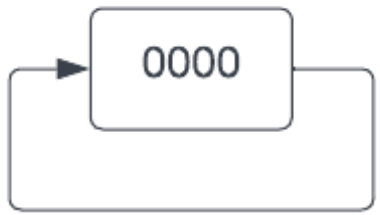

- All FSRs have a period
- Larger period $\Rightarrow$ more secure

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- X^i only included if the i-th bit is XORed by the function
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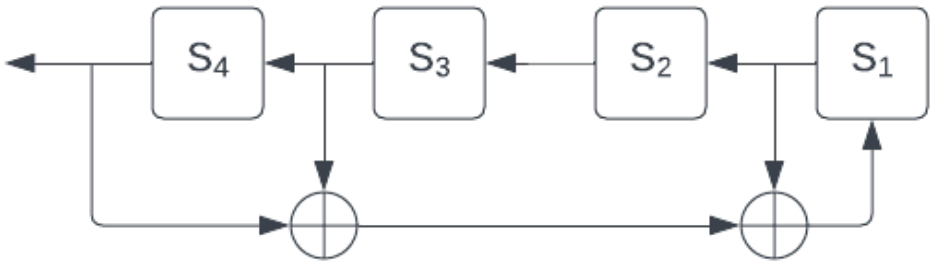
  $2^n - 1$! The state $000\ldots$ will always shift to $000\ldots$

Example #1 - LFSR

Initial state $R_0 = 0001$; Polynomial $1 + X + X^3 + X^4$



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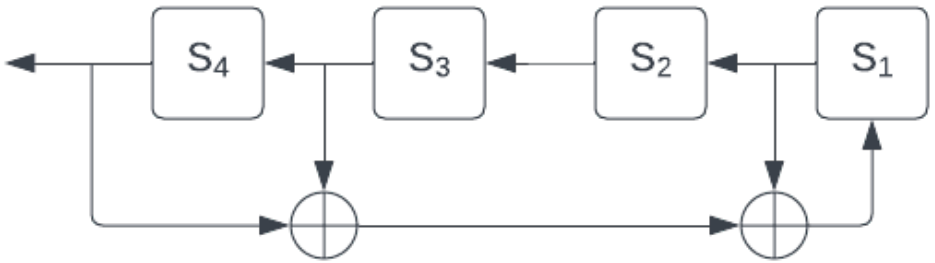

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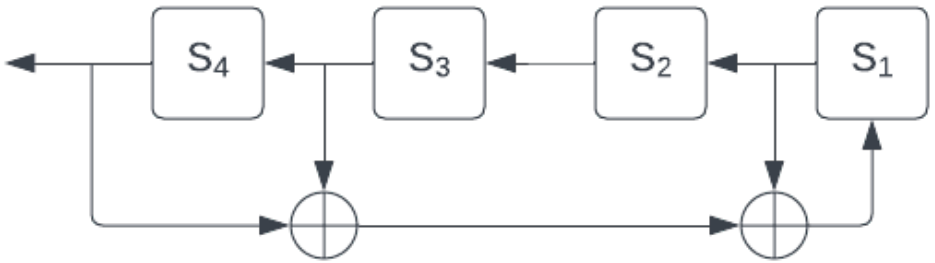

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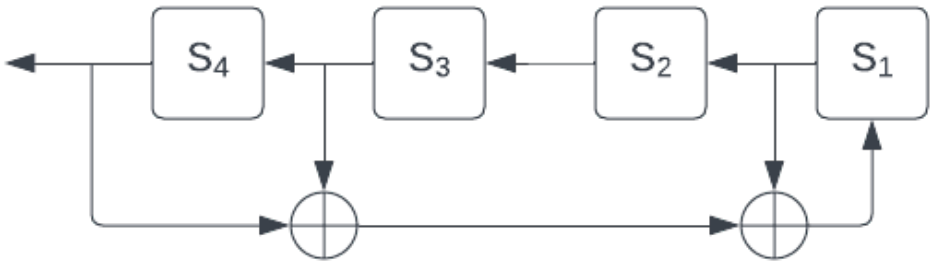

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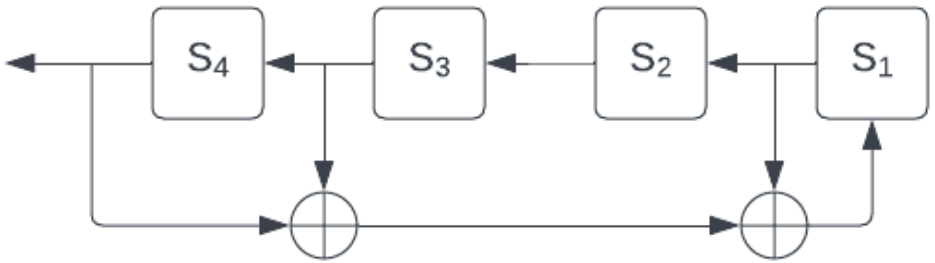

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$f(R_2) = 0$
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Example #1 - Continued

$R_0 = \mathbf{0001}$
$R_1 = 0011$
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$R_3 = 1110$
$R_4 = 1100$
$R_5 = 1000$
$R_6 = \mathbf{0001}$

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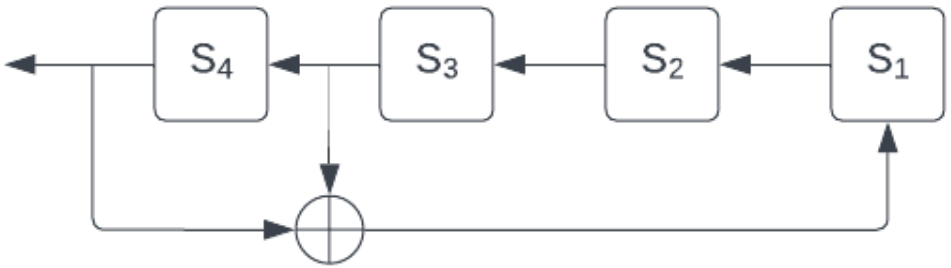
Q: What is the maximum period? $2^4 - 1 = 15$

Indeed, the polynomial is not primitive! It is reducible

$$(1 + X^3)(1 + X) = 1 + X + X^3 + X^4$$

Example #2 - LFSR

Initial state $R_0 = 0001$; Polynomial $1 + X^3 + X^4$



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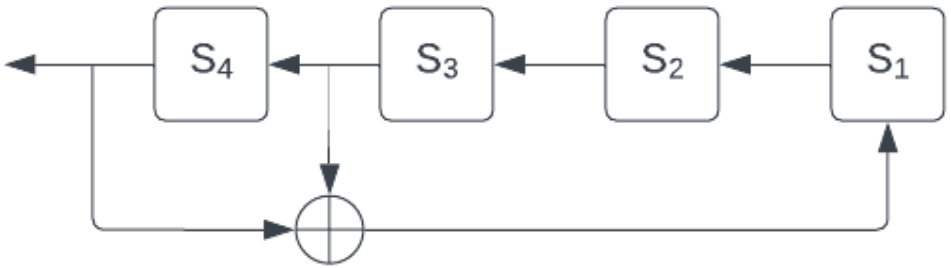

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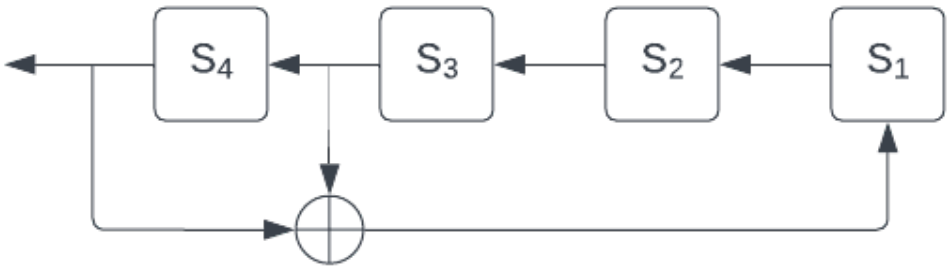

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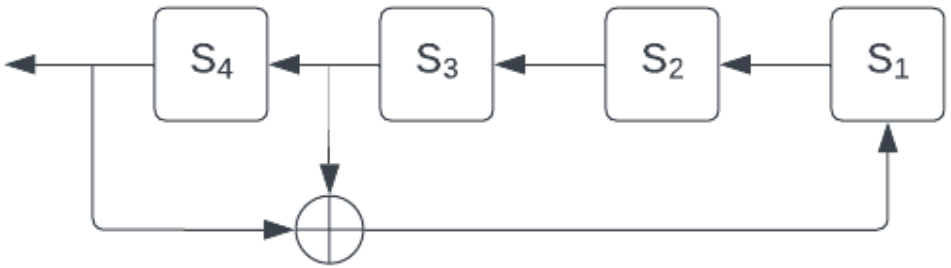

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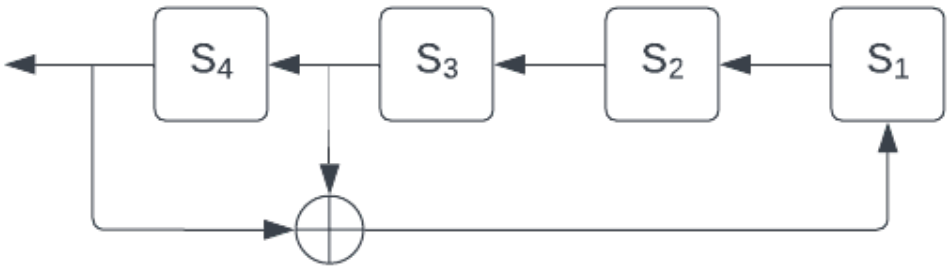

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Example #2 - Continued

0001 0011 0101 1110
0010 0110 1011 1100
0100 1101 0111 1000
1001 1010 1111 **0001**

Example #2 - Continued

0001	0011	0101	1110
0010	0110	1011	1100
0100	1101	0111	1000
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The polynomial $1 + X^3 + X^4$ is primitive

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- For size n, attacker needs n output bits to recover the initial state and feedback polynomial
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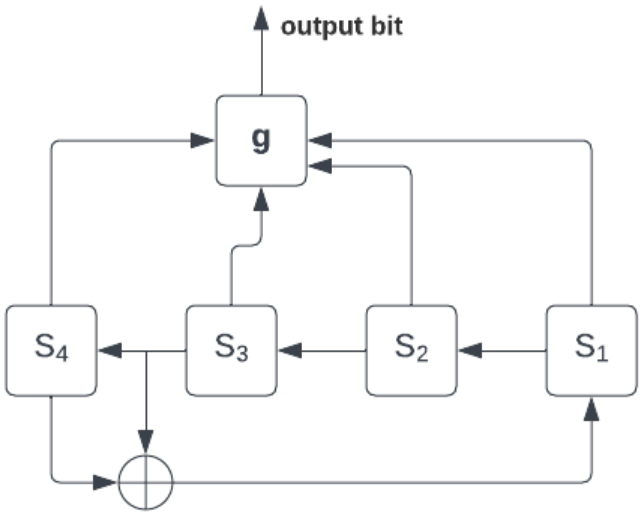
The issue is that LFSRs are **linear** – short equations, solved by easy math

Filtered LFSRs

One way to hide linearity is to filter outputs via a **nonlinear** function. These are called filtered LFSRs



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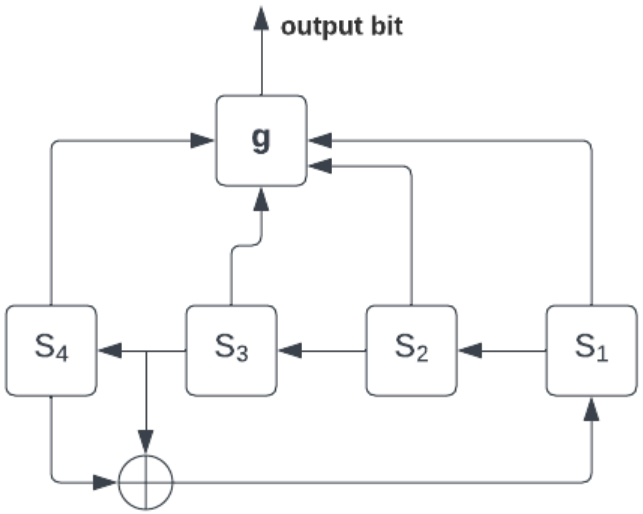

Straightforward attacks thwarted. But...

- Still vulnerable to
  - Algebraic attacks
  - Cube attacks (compute derivatives to reduce degree)
  - Fast correlation attacks (some behave like linear functions)

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We now replace X_1 with the new bit:
$(X_1+X_2+X_1X_2+X_3X_4)+X_1+(X_1+X_2+X_1X_2+X_3X_4)X_1+X_2X_3$
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Equation has algebraic degree 3 and 6 terms. This quickly scales to incredibly complex equations.

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Modern stream ciphers combine LFSRs with NFSRs to get the best of both worlds, which incidentally is what Grain-128a does.

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- Linear Feedback Shift Registers...
  - Have a linear feedback function
  - If they follow a primitive polynomial have maximum period
  - $2^n - 1$ for size n
  - Cryptographically weak

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  - Cryptographically weak
- Nonlinear Feedback Shift Registers...
  - Have a nonlinear feedback function
  - Impossible to know if they have the maximum period
  - Cryptographically challenging to solve

Key Takeaways

- Stream ciphers generate keystream, to $\oplus$ over the message
- Historically, stream ciphers were fragile and a better fit for HW-based solutions. Nowadays, neither is necessarily true
- FSR consist in an internal state, and repeated shifts over it
- Linear Feedback Shift Registers...
  - Have a linear feedback function
  - If they follow a primitive polynomial have maximum period
  - $2^n - 1$ for size n
  - Cryptographically weak
- Nonlinear Feedback Shift Registers...
  - Have a nonlinear feedback function
  - Impossible to know if they have the maximum period
  - Cryptographically challenging to solve
- Modern approaches combine LFSRs with NFSRs

Grain-128a

- Proposed as part of a public competition
- Combines LFSR and NFSR (as promised)
- Works bit-by-bit; Good for hardware optimizations

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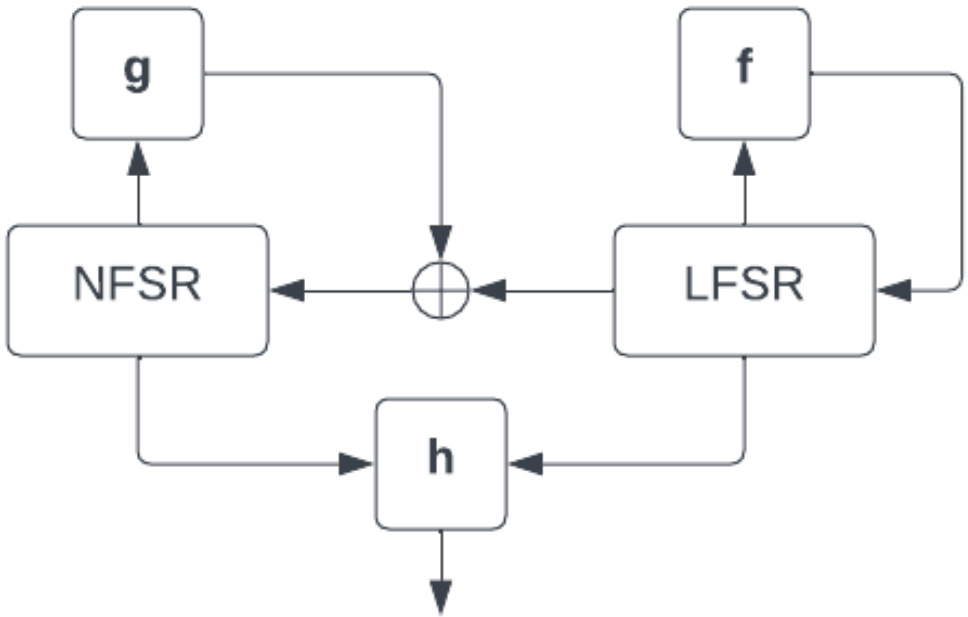
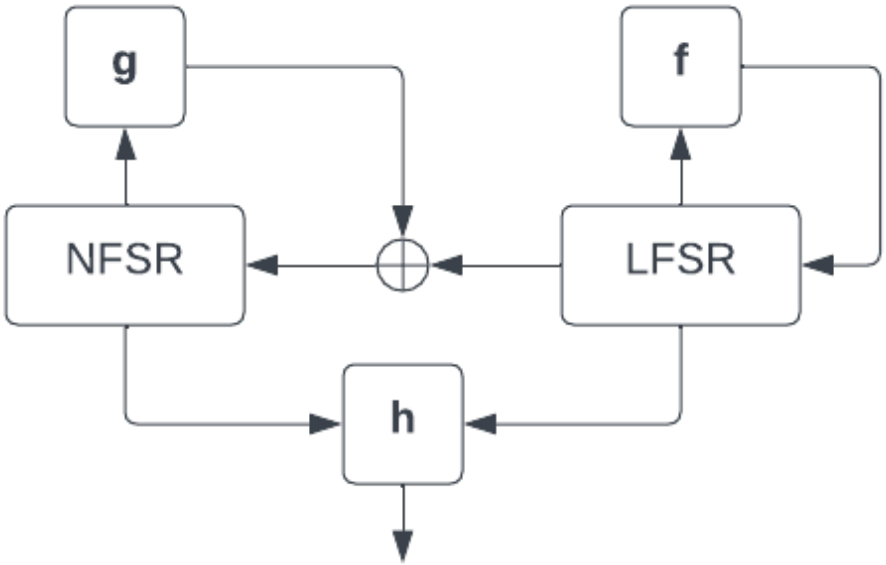


- No known devastating attacks on Grain-128a
  - Statistical attacks on chosen IV
  - Some related-key attacks
- Used in some smart cards. Compact and fast stream cipher

Grain-128a - Continued



Configuration

- LFSR has maximal period of $2^{128} - 1$
- NFSR and nonlinear function h add cryptographic strength
- 128 bit key; 96 bit nonce
  - Key goes to NFSR internal state
  - Nonce goes to the LFSR, alongside 36 zero bits

A5/1

- Used to encrypt voice communications over 2G
- Standard - 1987; Published - 1990; Attacks - 2000s
- Eventually broken in a practical sense
  - Some attacks are theoretical. They do not imply one can decrypt in real-time, or even in (pragmatically) short time

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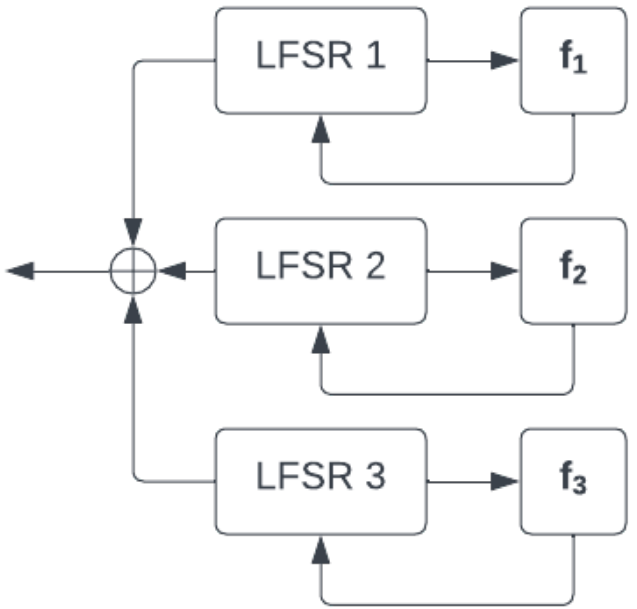
Configuration

- 64-bit key; 22-bit nonce
- Three LFSRs with 19, 22 and 23 bits
- Smart update mechanism with clocking rule
- Used to circumvent lack of NFSR

A5/1 - Continued

- LFSRs have *clocking* bits
- Clocks registers whose bits are the majority value
- 2/3 clocked each update



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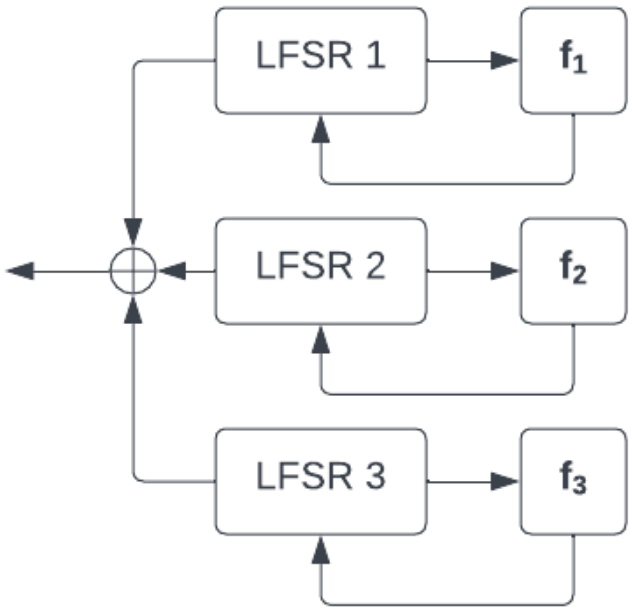

Broken

- Subtle attacks
  - Internal linearity of A5/1
  - Simple irregular clocking system
- Brutal attacks: short key of A5/1

RC4

- Takes 32/64-bit words instead of bits
- Proposed in 1987; reverse engineered in 1994
- Most widely used stream cipher
- Stil used in numerous systems
  - ... despite being completely broken

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Configuration

- One of the simplest ciphers ever created
- 128-bit key; No nonce $\leftarrow$ **???**
- No crypto-like operations
- No XORs
- No multiplications
- We just swap bytes around

RC4 - Continued

Key Scheduling

```
j = 0
# set S to the array S[0] = 0, S[1] = 1, . . . , S[255] = 255
S = range(256)
# iterate over i from 0 to 255
for i in range(256):
    # compute the sum of v
    j = (j + S[i] + K[i % n]) % 256
    # swap S[i] and S[j]
    S[i], S[j] = S[j], S[i]
```

From *Serious Cryptography: A Practical Introduction to Modern Encryption*

- Initial state set to $0, 1, \ldots, 255$
- Afterwards, we run the keystream generation
- ... Which also consists in swaps

RC4 - Continued

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Looks absolutely trivial, but it took 20 years of cryptanalysis to find exploitable flaws.

Attacks on RC4

Two main issues of RC4 have been exploited. No nonce in its original specification; and statistical biases on keystream generation

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RC4 in WEP

- First generation Wi-Fi security protocol
- 24-bit nonce included in the wireless frame header
- **Problem #1!** $2^{24/2} = 2^{12}$ (some megabytes of traffic) is usually enough to find two packets with the same nonce
- **Problem #2!** First keystream byte strongly dependent on first secret key byte – exploitable bias

RC4 in TLS

Attacks on RC4

Two main issues of RC4 have been exploited. No nonce in its original specification; and statistical biases on keystream generation

RC4 in WEP

RC4 in TLS

- Most important protocol on the internet
- 128-bit session key as nonce. Not as easily broken
- **Problem!** Statistical bias on the keystream. E.g. second keystream byte is zero with probability $1/128 \neq 1/256$ (ideal)
- Statistically crucial given many ciphertexts of the same byte

Salsa20

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- eSTREAM competition in 2005
- Counter-based stream cipher
- Used in many popular crypto libraries
  - Google Tink
  - LibSodium
  - OpenSSH

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Configuration

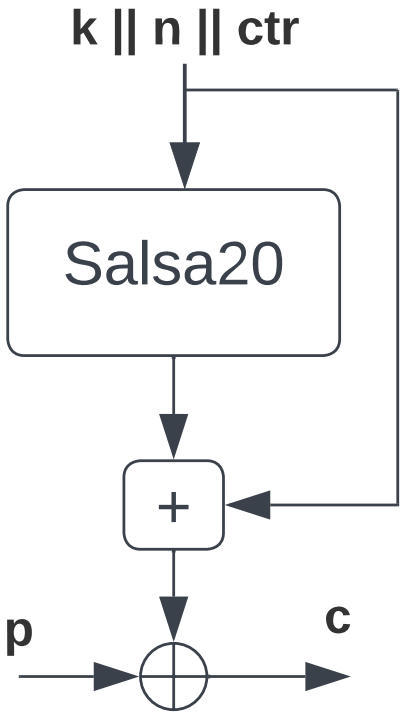
- Encrypts 512-bit plaintext blocks
- 256-bit key
- 64-bit nonce
- 64-bit counter
- 128-bit fixed constant words

Salsa20

- Core transforms 512 blocks
- Adds the result to original value
- Otherwise insecure – invertible

Quarter-Round Function

- Core transforms 32-bit words
- Structurally similar to AES
  - Column transformation
  - Row transformation



Key Takeaways

- Hardware-Oriented; bit-by-bit
- Good for HW optimization
  - Grain 128a
  - Combines LFSR and NFSR
  - Secure; Used in IoT
  - A5/1
  - Combines multiple LFSR
  - Very insecure; cracked after two decades

Key Takeaways

- Hardware-Oriented; bit-by-bit
- Good for HW optimization
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  - Combines LFSR and NFSR
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  - Combines multiple LFSR
  - Very insecure; cracked after two decades
- Software-Oriented; word-by-word
- Better for servers/browsers
  - RC4
  - Bare specification; does not include nonce
  - WEP and TLS. Broken (for different reasons)
  - Salsa20
  - Modern cipher permuting 32-bit words
  - Very fast; accepted in TLS 1.3

Applied Cryptography
Week 4: Stream Ciphers

Bernardo Portela

M:ERSI, M:SI - 24